

Mathematical Modeling Applied to Study the Effects of Wastage Produced from the Coal-Based Power Plant on Marine Ecosystem

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Abstract

Mathematical Modeling in terms of a system of ordinary differential equations has been used to study the effects of wastage produced from coal based power plant on marine ecosystem. It is considered that a coal based power plant is one of the main sources of the emission of carbon dioxide, which is the principal reasonable agent for water pollution and thus poses a great threat to the marine ecosystem near those power stations. Since a huge amount of harmful gasses and chemical wastages emitted from any coal based power plant which is the most dangerous threat to the marine resources and ecosystem. A 1000 MW coal-based power plant uses 9000 tons of coal per day, which produces a large amount of wastage and these wastages are mixed up with water and water is getting more polluted day by day. Actually coal-based power plant produces a huge amount of Lead (Pb), Mercury (Hg), Cadmium (Cd), Particulate Matter, Organic Compounds, etc. and it is mixed up with water in many ways and thus hampered the marine ecosystem. For studying the effects of wastage produced from the coal-based power plant on the marine ecosystem we've proposed a non-linear mathematical model using a system of three nonlinear differential equations. The model has been analyzed in order to describe the dynamics of emission and concentration of wastage from coal based power plant. The analysis carried out with the equilibrium and stability of the model. Boundedness of the solution of the model is discussed. The model has been analyzed by finding the existence of equilibrium points and also the conditions of stability and instability of the system have been derived. Finally, the reliability of the analytical model was confirmed with the numerical simulations.

Keywords

Mathematical modeling; wastage; coal based power plant; marine ecosystem; prey; predator.

1. Introduction

Water is one of the most important substances on earth. With around 70% of the world is covered by water, it certainly winds up plainly one of our most noteworthy assets [1]. Water is used practically in every vital human tasks and procedures. It is a vital component in both household and additionally mechanical and industrial purposes. However, a nearer investigation of our water assets today, gives us an impolite result [1]. National geography states that 'In developing countries, 70 percent of industrial wastes are dumped untreated into waters, polluting the useable water supply. Average, 22 million tons of fertilizers and chemicals are used each year [2]. A typical 500-megawatt coal-based power plant creates more than 125,000 tons of ash and 193,000 tons of sludge each year which contain arsenic, mercury, chromium, and cadmium etc [1]. For more than 50 years, global production and consumption of wastage released from coal-based power plant have continued to rise. An estimated 299 million tons of coal ash were produced in 2013. In 2015, it has been estimated at 260 million tons. This huge amount of waste mixed with water frequently and marine ecosystem is damaged. Among this entire problem, one of the most affective problems of water pollution is the wastage of coal-based power plant.

2. Average Emission of Gases from Coal Burning

Two types of coal Lignite and Anthracite are mainly used in coal-based power plant. When they burn CO_2 , SO_2 , NO_x , CO, organic components and particulate material are produced. Here, Figure 1 shows average emission of gases from coal burning.

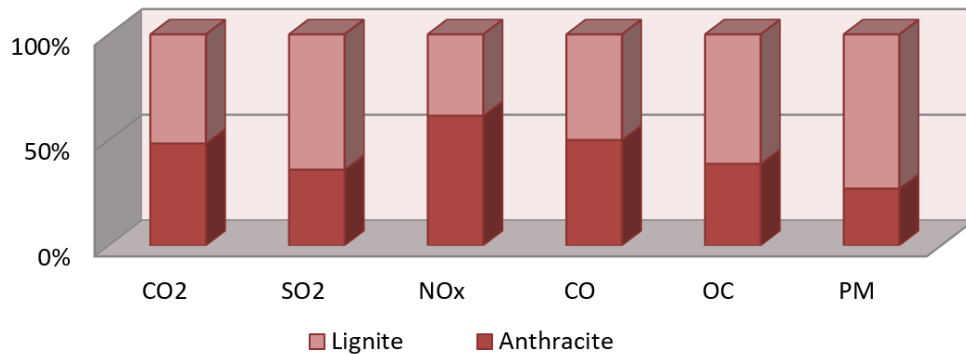


Figure 1: Average emission of gases of coal based power industry (Source: [4])

3. Biological Model

Pollution from coal-based power plant comes from the emission of Hg, CO_2 , As, SO_x , NO_x , Cl_2 etc. into the water. These elements react with the water to generate acidic components like H_2SO_3 , H_2SO_4 , HNO_2 , and HNO_3 . Sulfur (S) is emitting from the coal based plant and then it is mixed with the oxygen (O_2) of nature then produce SO_2 and it is mixed with water and produce H_2SO_3 which is harmful for the prey and predator of marine or biological ecosystem. Marine ecosystem is affected due to coal based power plant is shown in Figure 2.

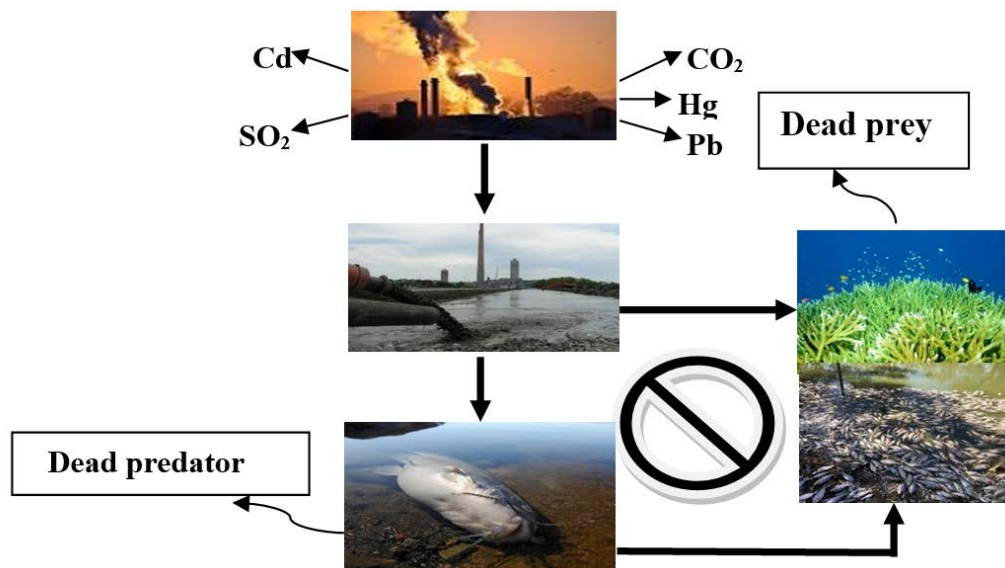


Figure 2: Marine ecosystem is affected due to coal-based power plant

4. Mathematical Model Formulation

The coal-based power plant emitted lot of harmful elements such as CO_2 , SO_2 , H_2CO_3 , HNO_3 , Pb, Hg, As, Cd. We can consider it as a source of waste of coal-based power plant. The density of waste products released from coal-based power plant is denoted as z . Density of prey species is denoted as x . the waste mixed up with water as a rate of ρ_1 . Density of predator species is denoted as y . the waste mixed up with water as a rate of ρ_2 . The affected rate of prey population by predator population is α . The density of prey species and predator species and density of wastage

are interconnected relation to each other. The rate of waste fixed with nature and lost from the marine ecosystem is denoted as δ_0 . The schematic diagram is given below in Figure 3.

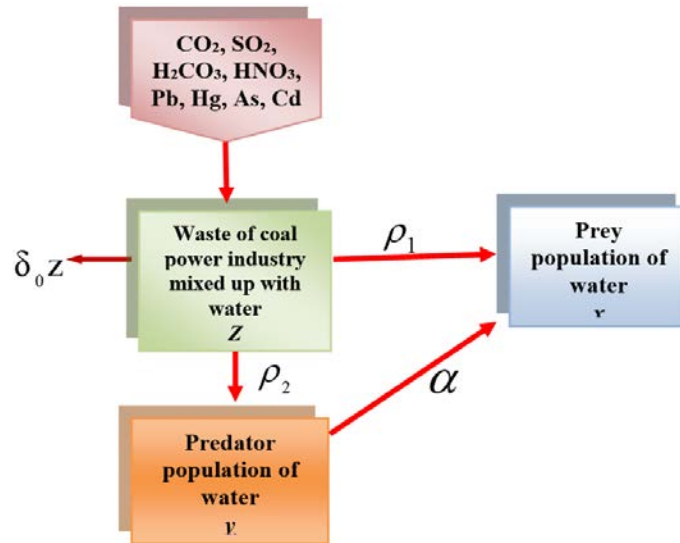


Figure 3: Schematic diagram of waste products released from coal-based power plant

We have formulated the dynamical model system

$$\frac{dx}{dt} = r_1 x \left(1 - \frac{x}{k_1} \right) - \alpha xy - \rho_1 xz \quad (4.1)$$

$$\frac{dy}{dt} = r_2 y \left(1 - \frac{y}{k_2} \right) + \alpha xy - \rho_2 yz \quad (4.2)$$

$$\frac{dz}{dt} = Q - (\rho_1 x + \rho_2 y) z - \delta_0 z \quad (4.3)$$

$$x(0) = x_0, y(0) = y_0, z(0) = z_0$$

Table 4.1: Description of state variables and parameters

Parameters	Descriptions of parameters
x	Density of prey species
y	Density of predator species
α	Maximum number of prey that can be eaten by each predator per unit
r_1	Prey intrinsic growth rate
r_2	Predator intrinsic growth rate
δ_0	The rate of waste fixed with nature and lost from the marine ecosystem
ρ_1	The rate of waste mixed with the prey population
ρ_2	The rate of waste mixed with the predator population
z	Density of wastage
Q	Increase of the wastage concentration into the water
k_1	Carrying capacity of prey species
k_2	Carrying capacity of predator species

4.1 Preliminary Result

Here we will apply following lemma to establish the positivity, the boundedness and the permanence of the proposed model.

Lemma 4.1 If $a, b > 0$ and $\frac{dx}{dt} \leq (\geq) x(t)(a - bx(t))$ with $x(t) > 0$, then $\limsup_{t \rightarrow \infty} x(t) \leq \left(\liminf_{t \rightarrow \infty} x(t) \geq \frac{a}{b} \right)$

By using this above lemma 4.1 then from the equation (4.1) we get,

Here, $r_1 = a$ and $\frac{r_1}{k_1} = b$

$$\limsup_{t \rightarrow +\infty} x(t) \leq \left(\lim_{t \rightarrow \infty} x(t) \geq k_1 \right) \quad (4.4)$$

Again from (4.2), we have,

$$\limsup_{t \rightarrow +\infty} x(t) \leq \left(\liminf_{t \rightarrow \infty} y(t) \geq \frac{k_2(r_2 + \alpha k_1 y)}{r_2} \right) \quad (4.5)$$

$$\limsup_{t \rightarrow +\infty} y(t) \leq \left(\liminf_{t \rightarrow \infty} y(t) \geq L \right) \quad (4.5)$$

$$\limsup_{t \rightarrow +\infty} z(t) \leq \left(\liminf_{t \rightarrow \infty} z(t) \geq Q \right) \quad (4.6)$$

Lemma 4.2 If $a, b > 0$ and $\frac{dx}{dt} \leq (\geq) x(t)(a - bx(t))$ with $x(t) > 0$. Then for all $(t \geq 0)$, $x(t) \leq \frac{a}{b - Ce^{-at}}$

with $C = b - \frac{a}{x(0)}$.

In particular, $x(t) \leq \max \left\{ x(0), \frac{a}{b} \right\}$ for all $t \geq 0$

By using this above lemma 4.2 then from the equation (4.1) we get,

$$\frac{dx}{dt} \leq (\geq) x(t) \left(r_1 - \frac{r_1(t)}{k_1} \right); x(t) > 0 \text{ for all } t > 0$$

$$x(t) \leq \frac{r_1}{\frac{r_1}{k_1} - Ce^{-at}}; C = \frac{r_1}{k_1} - \frac{r_1}{x(0)}$$

$$\text{So, } x(t) \leq \max \left\{ x(0), \frac{r_1}{\frac{r_1}{k_1}} \right\}$$

$$x(t) \leq \max \{x(0), k_1\} \quad (4.7)$$

Similarly,

$$y(t) \leq \max \{y(0), L\} \quad (4.8)$$

$$z(t) \leq \max \{z(0), Q\} \quad (4.9)$$

4.3 Boundedness

Here we will apply following lemma and theorem to establish boundedness of the proposed model (4.1)- (4.3).

Theorem 4.1 All the solution $(x(t), y(t), z(t))$ of the system in \mathbb{R}_3^+ are always positive i.e. $x(t) > 0, y(t) > 0$ and $z(t) > 0$.

Proof:

Theorem 4.1 is true so that we can write,

$$x(t) = x(0) \exp \int_0^t \left[r_1 \left(1 - \frac{x(s)}{k_1} \right) - \frac{\alpha y}{\frac{1}{x(s)} + m} - \rho x(s) z \right] ds \quad (4.10)$$

$$y(t) = y(0) \exp \int_0^t \left[r_2 \left(1 - \frac{y(s)}{k_2} \right) - \frac{\alpha y}{\frac{1}{y(s)} + m} - \rho y(s) z \right] ds \quad (4.11)$$

$$z(t) = z(0) \exp \int_0^t \left[\frac{Q}{z(s)} - (\rho_1 (x(s) + \rho_2 y(s)) (x(s) + y(s))) \right] ds \quad (4.12)$$

Before analyzing the model system let us prove that the solutions to the system (4.1)- (4.3) corresponding the following theorem (4.1).

Theorem 4.2

All the solutions of the model system with initial conditions are always bounded, for all $t \geq 0$.

From equation (4.1) we have,

$$x(t) \leq \max \{x(0), k_1\} \equiv M_1 \text{ for all } t \geq 0.$$

$$y(t) \leq \max \{y(0), L\} \equiv M_2 \text{ for all } t \geq 0.$$

From equation (4.3) we have,

$$z(t) \leq \max \{z(0), Q\} \equiv M_3$$

for all $t \geq 0$.

The proof is complete for the boundedness of the model system under the consideration is dissipative.

4.6 Equilibrium Analysis

In this section, we establish the conditions for the existence of the four equilibrium points of the model system (4.1)- (4.3) namely,

4.6.1 Trivial Equilibrium Point

When the affected prey and predator of a marine-ecosystem and wastage of coal based power are not exists i.e. $x=y=z=0$ thus the equilibrium point is denoted as, $E_0(x_0, y_0, z_0)$ then from required system (4.1)- (4.3) we get,

$$E_0(x_0, y_0, z_0) = E_0(0, 0, 0)$$

4.6.2 Boundary Equilibrium Point

When the affected prey and predator of a marine-ecosystem are not exist and wastage of coal based power plant are exist i.e., $x = y \neq 0$ and $z = 0$, thus the equilibrium point is denoted as $E_1(x_1, y_1, z_1)$ then from the system (4.1)-

$$(4.3) \text{ we get, } E_1(x_1, y_1, z_1) = E_1 \left(\frac{k_1(r_1 - \alpha y_1)}{r_1}, \frac{k_2(r_2 + \alpha x_1)}{r_2}, 0 \right)$$

4.6.3 Axial Equilibrium Point

Where there are no affected prey and predator i.e., $x = y = 0$ and wastage of coal based power plant are exist $z \neq 0$ thus the equilibrium point is denoted as, $E_2(x_2, y_2, z_2)$

So, the equilibrium point is obtained from the equation

$$E_2(x_2, y_2, z_2) = \left(0, 0, \frac{Q}{\partial_0} \right)$$

4.6.4 Interior Equilibrium Point

When the affected prey and predator of a marine-ecosystem and wastage of coal based power plant are exists that means all the state variables of the model system are co-existing i.e., $x \neq 0$, $y \neq 0$ and $z \neq 0$ thus the equilibrium point is denoted as,

$$E_3(x_3, y_3, z_3) = \left(\frac{k_1(r_1 - \alpha y_3 - \rho_1 z_3)}{r_1}, \frac{k_2(r_2 + \alpha x_3 - \rho_2 z_3)}{r_2}, \frac{Q}{(\rho_1 x_3 + \rho_2 y_3 + \partial_0)} \right)$$

4.7 Dynamic Behavior of the Mathematical Model

4.7.1 Local Stability Analysis

In this section we have analyzed the stability properties of the equilibrium points $E_0(x_0, y_0, z_0)$, $E_1(x_1, y_1, z_1)$, $E_2(x_2, y_2, z_2)$ and $E_3(x_3, y_3, z_3)$. The local stability is established through Jacobian matrix of the system and finding the eigenvalues to evaluate at each equilibrium point. For linearized system the Jacobian matrix is given by,

Let,

$$\dot{x}(t) = f(t, x)$$

$$x(0) = x_0$$

4.7.2 Behavior of the System at E_0

We have to find the behavior of the system at $E_0(0, 0, 0)$. The Jacobian matrix J_0 at $E_0(0, 0, 0)$ is,

$$J(E_0) = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & -\partial_0 \end{bmatrix}$$

Hence the equilibrium point $E_0(0, 0, 0)$ is asymptotically unstable.

4.7.3 Behavior of the System around E_1

We have to find the behavior of the system around $E_1(x_1, y_1, z_1)$. The Jacobian matrix J_1 at the point

$E_1(x_1, y_1, z_1)$ is,

$$J(E_1) = \begin{bmatrix} r_1 \left(1 - \frac{2x_1}{k_1} \right) - \alpha y_1 & -\alpha x_1 & -\rho_1 x_1 \\ -\alpha y_1 & r_2 \left(1 - \frac{2y_1}{k_2} \right) + \alpha x_1 & -\rho_2 y_1 \\ 0 & 0 & -\rho_1 x_1 - \rho_2 y_1 \end{bmatrix}$$

Eigen values corresponding to the point $E_1(x_1, y_1, z_1)$ are the roots of the equation,

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0$$

From Routh-Hurwitz criterion, E_1 is locally asymptotically stable if and only if $A_1 > 0$, $A_3 > 0$ and $A_1A_2 > A_3$.

$$A_1 = \alpha y_1 - \alpha x_1 + \rho_1 x_1 + \rho_2 y_1 + r_1 \left(1 + \frac{2x_1}{k_1}\right) + r_2 \left(1 + \frac{2y_1}{k_2}\right)$$

$$A_2 = (\rho_1 x_1 + \rho_2 y_1) \left(\alpha y_1 + r_1 \left(\frac{2x_1}{k_1} - 1 \right) \right) + \left(\alpha y_1 + r_1 \left(\frac{2x_1}{k_1} - 1 \right) \right) \left(\alpha x_1 - r_2 \left(\frac{2y_1}{k_2} - 1 \right) \right) \left(\alpha y_1 + \rho_1 z_1 + r_1 \left(\frac{2x_1}{k_1} - 1 \right) \right) \\ - \left(\alpha x_1 - r_2 \left(\frac{2y_1}{k_2} - 1 \right) \right) (\rho_1 x_1 + \rho_2 y_1) + \alpha^2 x_1 y_1$$

$$A_3 = \alpha^2 x_1 y_1 (\rho_1 x_1 + \rho_2 y_1) - \left(\alpha y_1 + r_1 \left(\frac{2x_1}{k_1} - 1 \right) \right) \left(\alpha x_1 - r_2 \left(\frac{2y_1}{k_2} - 1 \right) \right) (\rho_1 x_1 + \rho_2 y_1)$$

It is easy to examine that $A_1A_2 > A_3$ if $c_{11} < 0$, $c_{22} < 0$.

if $r_1 < \frac{\alpha y_1 k_1}{k_1 - 2x_1}$ and $r_2 < \frac{-\alpha x_1 k_2}{k_2 - 2y_1}$ then $c_{11} < 0$, $c_{22} < 0$ will be negative.

The Routh-Hurwitz criterion is satisfied. Hence we can consider that the equilibrium point $E_1(x_1, y_1, z_1)$ is locally asymptotically stable.

4.7.4 Behavior of the System at E_2

We have to find the behavior of the system at $E_2(x_2, y_2, z_2)$. The Jacobian matrix J_3 at co-existence point $E_2(x_2, y_2, z_2)$ is

$$J(E_2) = \begin{bmatrix} -\rho_1 z_2 & 0 & 0 \\ 0 & -\rho_2 z_2 & 0 \\ -\rho_1 z_2 & -\rho_2 z_2 & -\delta_0 \end{bmatrix}$$

Here, Eigen value $\lambda = -\rho_1 z_2, -\rho_2 z_2, -\delta_0$. All Eigen values are negative here. So, we can consider that the equilibrium point $E_2(x_2, y_2, z_2)$ is asymptotically stable.

4.7.5 Behavior of the System at E_3

We have to find the behavior of the system at $E_3(x_3, y_3, z_3)$. The Jacobian matrix J_3 at co-existence point $E_3(x_3, y_3, z_3)$ is

$$J(E_3) = \begin{bmatrix} r_1 \left(1 - \frac{2x_3}{k_1}\right) - \alpha y_3 - \rho_1 z_3 & -\alpha x_3 & \rho_1 x_3 \\ \alpha y_3 & r_2 \left(1 - \frac{2y_3}{k_2}\right) + \alpha x_3 - \rho_2 z_3 & -\rho_1 z_3 \\ -\rho_1 z_3 & -\rho_2 z_3 & -\rho_1 x_3 - \rho_2 y_3 - \delta_0 \end{bmatrix}$$

Eigen values corresponding to the point $E_3(x_3, y_3, z_3)$ are the roots of the equation,

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0$$

From Routh-Hurwitz criterion, E_3 is locally asymptotically stable if and only if $A_1 > 0, A_3 > 0$ and $A_1 A_2 > A_3$.

It is easy to examine that $A_1 A_2 > A_3$ if $c_{11} < 0, c_{22} < 0$.

if $r_1 < \frac{(\alpha y_3 + \rho_1 z_3)}{k_1 - 2x_3}$ and $r_2 < \frac{(\rho_2 z_3 - \alpha x_3)}{k_2 - 2y_3}$ then $c_{11} < 0, c_{22} < 0$ will be negative.

The Routh-Hurwitz criterion is satisfied. Hence we can that the equilibrium point $E_3(x_3, y_3, z_3)$ is locally asymptotically stable.

4.8 Global Stability Analysis

To show the global stability, we consider $[(x, y, z) \in \square_3^+, x \leq 0, y \leq 0, z \leq 0]$ and the function

$J(E_3) : \square_3^+ \rightarrow \square^3$, then construct a Lyapunov-Lasalle's function as follows,

$$J(E_3) = L_1(x - x_3 - x_3 \ln(x)) + L_2(y - y_3 - y_3 \ln(y)) + L_3(z - z_3 - z_3 \ln(z))$$

Where L_1, L_2 and L_3 be positive constants to be determined. We can easily verify that the function $J(E_3) = 0$ at $E_3(x_3, y_3, z_3)$ and positive for all other values of x, y, z . Then time derivation of $L(E_3)$ along the solution of the system given by,

$$\begin{aligned} \frac{dL}{dt} &= L_1 \left(1 - \frac{x_3}{x} \right) \frac{dx}{dt} + L_2 \left(1 - \frac{y_3}{y} \right) \frac{dy}{dt} + L_3 \left(1 - \frac{z_3}{z} \right) \frac{dz}{dt} \\ \frac{dL}{dt} &= L_1 \left(-\frac{r}{k_1} (x - x_3)^2 - 2\rho_1 (x - x_3)(z - z_3) \right) + L_2 \left(-\frac{r}{k_2} (y - y_3)^2 - 2\rho_2 (y - y_3)(z - z_3) \right) \\ &\quad + L_3 \left(2\rho_2 (y - y_3)(z - z_3) - \frac{Q}{zz_3} (z - z_3)^2 \right) \end{aligned} \quad (4.37)$$

We know $\frac{y}{y_3} > \frac{x}{x_3} > 1$ and $\frac{z}{z_3} > 1$

$$(x - x_3)(z - z_3) > 0 \text{ and } (y - y_3)(z - z_3) > 0$$

Again $\frac{dL}{dt} = 0$ when $(x, y, z) = (x_3, y_3, z_3)$ which satisfy the Lyapunov-Lasalle's principle.

Hence E_3 globally asymptotically stable.

5. Numerical Simulation

For demonstrating the analytical results, we have taken some hypothetical data. The parameters of the dynamical model are not based on real-world observation. Our main goal is to illustrate the result by numerical simulations considered from a qualitative, rather than a quantitative point of view. Along with the verification of our analytical observations, these numerical results are very much important from the practical point of view. The numerical result shows different behavior of our dynamical model. We have used a set of suitable parameter values.

$r_1 = 0.125, r_2 = 0.125, k_1 = 15, k_2 = 15, Q = 4.078, \alpha = 0.0130, \rho_1 = 0.0156, \rho_2 = 0.0123, \delta_0 = 0.127$ Here we perform the numerical simulations for time $t=0$ to $t=10$. The result of the combined classes is presented in Figure 5.1.

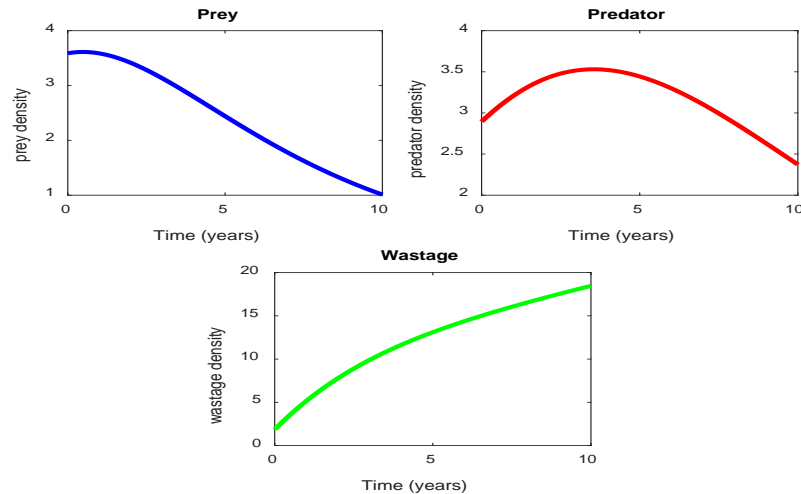


Figure 5.1: Graph of prey, predator and wastage density for our model (4.1)-(4.3) where time

$$r_1 = 0.125, r_2 = 0.125, k_1 = 15, k_2 = 15, Q = 4.078, \alpha = 0.0130, \rho_1 = 0.0156, \rho_2 = 0.0123, \delta_0 = 0.127$$

From Figure 5.1, we observe that the density of waste product of coal-based power plant is increasing for 10 years where the density of prey and predator are decreasing for 10 years. If again we perform numerical simulations of the equation of model (4.1)-(4.3) by using

$$r_1 = 0.125, r_2 = 0.125, k_1 = 15, k_2 = 15, Q = 4.078, \alpha = 0.0030, \rho_1 = 0.00156, \rho_2 = 0.00123, \delta_0 = 2.127$$

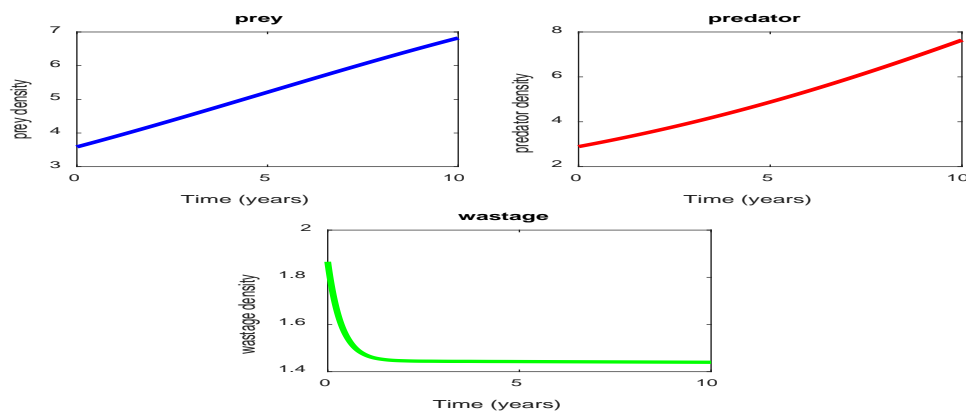


Figure 5.2: Graph of prey, predator and wastage density of a marine ecosystem for our model (4.1)-(4.3) where time is 10 year. $r_1 = 0.125, r_2 = 0.125, k_1 = 15, k_2 = 15, Q = 4.078, \alpha = 0.0030, \rho_1 = 0.00156, \rho_2 = 0.00123, \delta_0 = 2.127$

From Figure 5.2, we see that the density of wastage decreases for a given 10 years. Then the density of prey and density of predators gradually increases up to 10 years. It will increase from the initial point and last up to 10 years it will increase. Again we perform numerical simulations of the equation of our dynamical model (4.1)-(4.3) by using value of parameters where the time span 0 to 20.

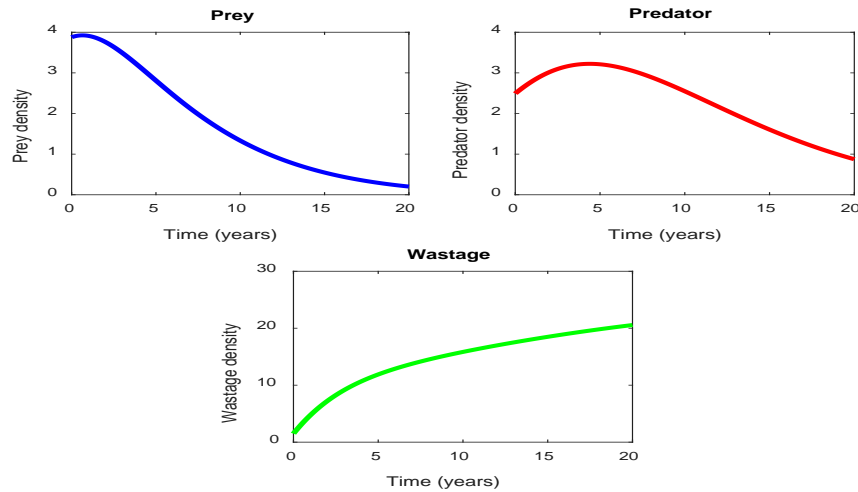


Figure 5.3: Graph of prey-predator model (4.1)-(4.3) of the population where time span 0 to 20. $r_1 = 0.125$, $r_2 = 0.125, k_1 = 15, k_2 = 15, Q = 4.078, \alpha = 0.0130, \rho_1 = 0.0156, \rho_2 = 0.0123, \delta_0 = 0.127$

From Figure 5.3, we see that the density of wastage of a coal-based power plant is increasing for 20 years. Again, we perform numerical simulations of the equation of model (4.1)-(4.3) by using value of parameters $r_1 = 0.125, r_2 = 0.125, k_1 = 15, k_2 = 15, Q = 4.078, \alpha = 0.0130, \rho_1 = 0.0156, \rho_2 = 0.0123, \delta_0 = 0.127$.

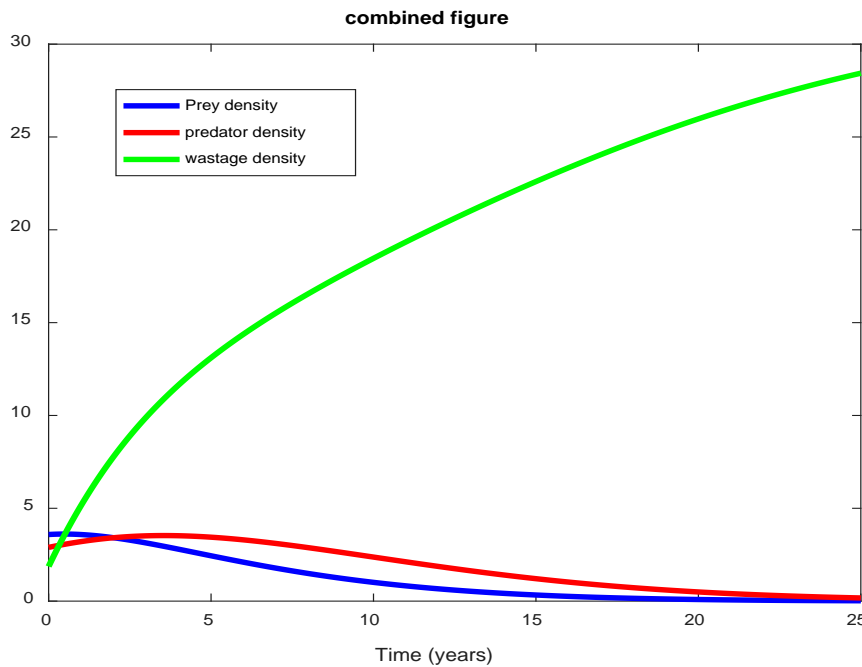


Figure 5.4: Graph of prey, predator and wastage density of a marine ecosystem for our model (4.1)-(4.3) where time span 0 to 25 and $r_1 = 0.125, r_2 = 0.125, k_1 = 15, k_2 = 15, Q = 4.078, \alpha = 0.0030, \rho_1 = 0.00156, \rho_2 = 0.00123, \delta_0 = 2.127$

Now we solve the model (4.1)-(4.3) numerically simultaneous parameter is given here. $r_1 = 0.125, r_2 = 0.125, k_1 = 15, k_2 = 15, Q = 4.078, \alpha = 0.0130, \rho_1 = 0.0156, \rho_2 = 0.0123, \delta_0 = 0.127$

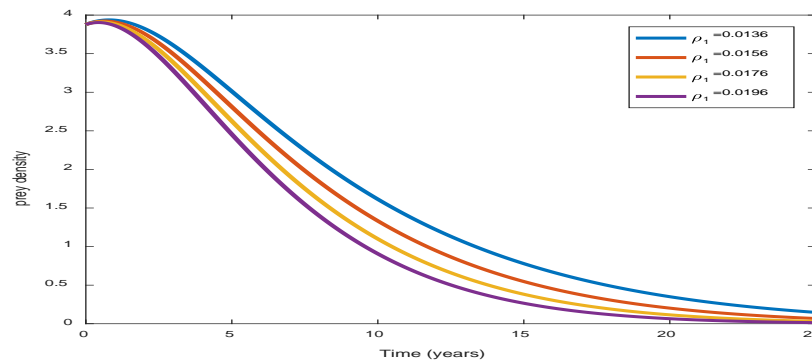


Figure 5.5: Variation of density of predator of a marine ecosystem for the parameter ρ_1 where time is 0 to 25 years

Figure 5.5 represents the density of the prey population over time. Here, we can see different prey density rates for 25 years against the parameter named rate of waste mixed with the prey population which is denoted by ρ_1 . The value of ρ_1 are 0.0136, 0.0156, 0.0176, 0.0196. Again we solve the model (4.1)-(4.3) numerically simultaneous parameter that is given here, Here, we execute the program for different value of ρ_2 .

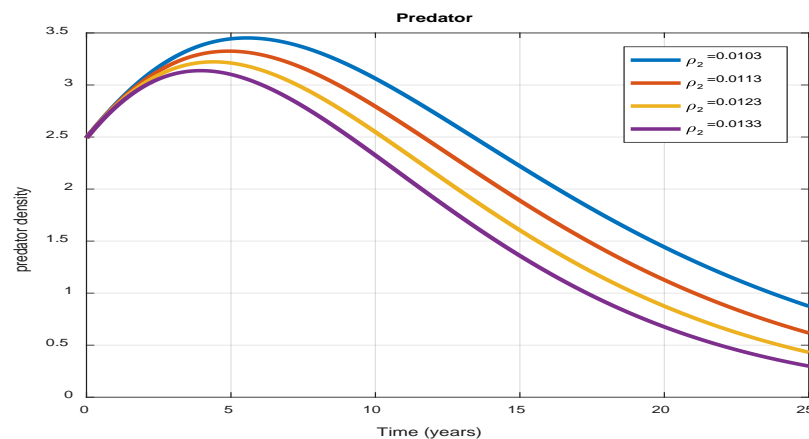


Figure 5.6: Variation of density of predator of a marine ecosystem for the parameter ρ_2 where time is 0 to 25 years

6. Conclusions

In this paper, we have analyzed the behavior of wastage produced from coal based power plant. The mathematical model used in this paper is a system of nonlinear differential equations which includes the parameter named density of prey, predator and wastage of coal based power plant. A dynamical model of wastage, produced from coal based power plant, has been introduced at first. Then the equilibrium points of the model have been determined. The stability analysis at the interior points has been tested which ensures the validity of the model. The analytical analysis is included with positivity test, equilibrium point, stability at steady state point and stability at interior point. In the schematic diagram, we used three compartments. Those compartments are called variables. Here variables are density of prey population, density of predator population and density of wastage produced from coal based power plant. These three variables are interconnected. We have presented the numerical results with its discussions which show that if wastage of coal based power plant increases, prey and predator population will be decreased.

References

- Arefin M. A. and Mallik A., Sources and Causes of Water Pollution in Bangladesh: A Technical Overview, *Bibechana*, 15: 97-112, (2018).
- Biswas M. H. A., Hasan M. N. and Uddin M. S., A mathematical model for fish management in the Sundarbans ecosystem, *Open J. Math. Anal.*, 3(2): 42-49, (2019).

- Biswas M. H. A., On the Evaluation of AIDS Treatment: An Optimal Control Approach, *Current HIV Research*, 12(1): 1-12, (2014).
- Biswas M. H. A., Optimal Control of Nipa Virus Infections A Bangladesh Scenario, *Pure and Applied Mathematics Advances and Applications*, 12(1): 77-104, (2014).
- Biswas M. H. A., Hossain M. R. and Mondal M. K., Mathematical Modeling Applied to Sustainable Management of Marine Resources, *Procedia Engineering*, 194: 337-344, (2017).
- Biswas M. H. A., Rahman T. and Haque N., Modeling the Potential Impacts of Global Climate Change in Bangladesh: An Optimal Control Approach, *J FundamApplSci*, 8(1): 1-19, (2016).
- Biswas M. H. A., Pal A. and Hasan M. N., Modeling the Potential Impact of Climate Change on the Natural Resources of the Coastal Region of Sundarban Ecosystem in Bangladesh, Proceedings on 17th CTWF September 17-19, 2018 in China CAS-TWAS-WMO Forum, (2018).
- Chowdhury A. H., Environmental Impact of Coal Based Power Plant of Rampal on the Sundarbans (World Largest Mangrove Forest) and Surrounding Areas, *Moj Eco Envi Sci*, 2(3):85-98, (2017).
- Ekvall T. and Finnveden G., The Application of Life Cycle Assessment to Integrated Solid Waste Management-Part II: Perspectives on Energy and Material Recovery, *JChemE*, 78(2): 288-294, (2000).

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