

# Mathematical Modeling Applied to Assess the Environmental Effect of Smog Concentration in Dhaka City

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Mathematics Discipline

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## Abstract

Smog is the combination of fog or haze intensified by smoke or other atmospheric pollutants and it is considered as one of the main threats to human and animal health in long and short term. Various diseases are caused by this increased amount of smog. Since Dhaka city is one of the most densely populated city in the world, human and animal health diseases are caused by increased amount of smog increasing at a concerning rate. In this study, we have formulated a non-linear three compartmental prey-predator model for describing dynamics of smog emission and concentration in Dhaka city by considering all the reasonable causes and constraints. Three compartments of this prey-predator model are Plants density, Human density & smog concentration. For validating and evaluating the system we did necessary qualitative and quantitative analysis like equilibria and stability of the model. The boundedness of the model is discussed. The model has been analyzed by finding the existence of equilibrium points & also the conditions of local stability, instability & global stability of the system has been derived. Finally the reliability of the analytical model has been confirmed with the numerical simulations.

## Keywords

Smog, Intensify, Compartment, Dynamic, Emission, Concentration, Constraint, Stability, Reliability Simulation

## 1. Introduction

Smog is a serious problem in most big urban areas. The emissions from vehicles and industries as well as the combustion of wood and coal together with the buildup of certain weather conditions are the main causal agents of smog. The terminology refers to a mixture of liquid and solid fog and smoke particles. Smog triggered due to air pollutants and fog. Deep Learning techniques are applied to predict the smog severity. The gaseous emissions are the main elements that form smog when acted upon by the sun's ultraviolet light together with particulate matter and volatile organic compounds. Dense urban areas suffer more from smog because of huge numbers of traffic, industries and combustion of different types of fuel. Smog has serious negative effects on people, plants, and animals. Smog pollution has now become a national crisis, too serious to be ignored by the public and the government.

## 2. Background of Smog Concentration

Smog is a kind of air pollutant. The word smog was first coined in the early 20th century as a portmanteau of the words smoke and fog to refer to "smoky fog". Air pollution is one of the major manmade environmental problems that have gained importance all over the world. In 2009, the 27 members of the European Union reported a cumulative drop in sulfur oxides by roughly 80% since 1990. Despite major gains in lowering pollution throughout the European Union, some major metropolitan cities like London are currently suffering from some of the worst air quality they've ever seen. The same report by the World Health Organization states that an average of 2 million people are killed worldwide every year due to air pollution. The air pollutants ( $CO_x$ ,  $NO_x$ ,  $PM_{2.5}$ ,  $PM_{10}$ ,  $SO_x$ ,  $CH_4$ ,  $O_3$ ) assorted with the meteorological factors forming the smog. Ambient national air quality standards in Bangladesh (2018) and comparison with neighboring countries including WHO and US and Worldwide pollutants gas represents is given in Table 2.1.

Table 2.1: Representation of Worldwide Pollutant Gases [18]

Pollutant	Unit	Bangladesh	India	Pakistan	USA	WHO guideline
$SO_2$ -24hr	ppm	.14	.031	.046	.14	.0077
$NO_2$ -Annual	ppm	.053	.0212	.0212	.053	.023
$O_3$ -8hr	ppm	.08	.092	.066	.08	.051
$CO$ -8hr	ppm	9	1.8	4.5	9	9
$PM_{10}$ -24hr	$ugm^{-3}$	150	100	150	150	50
$PM_8$ -24hr	$ugm^{-3}$	65	60	35	35	25

### 3. Smog Pollution in Dhaka City

Dhaka, the capital city of Bangladesh, is a densely populated mega-city in the world. About 16 million inhabitants are living within an area of 390 square kilometers. It is expanding very rapidly due to high influx of people from all over the country. Air pollution is a pressing issue in our country as Bangladesh ranks 169th (out of 178 countries) at the Environmental Performance Index for Air Quality (APT, 2018). Several causes have been identified for air pollution in Dhaka.

- Population Density
- Unplanned Industrialization and Urbanization Traffic Pressure
- Brick Fields
- Industrial particle
- Vehicle smog emission etc.

Now Dhaka City is overpopulated mega city in the world. This city is polluted day by day which is air quality pollution data from 2009-2019 is shown in Figure 3.1.

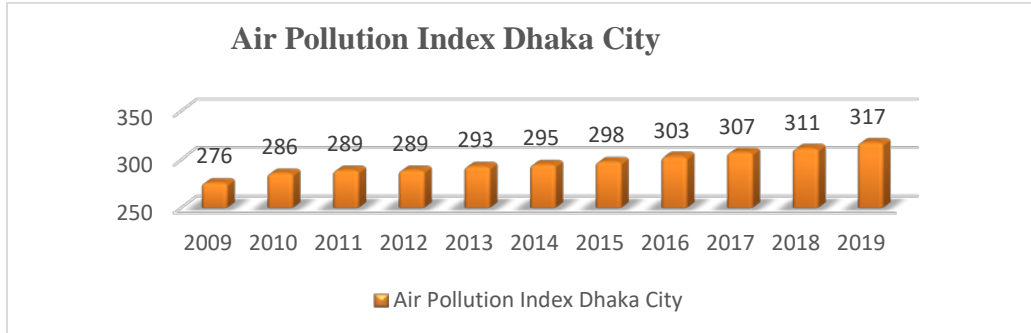


Figure 3.1: Pollution of Air Quality in Dhaka City from 2009-2019

### 4. Mathematical Model Formulation

In this project thesis, we will analyze the effect of emitted atmospheric smog concentration and the effect of particulate matters in Dhaka city. The enhanced emission of smog released from sources like breathing system, uses of vehicles, industrial pollution, gas stove etc. are absorbed by the plants and human beings. This survey based on mathematical modeling because it gives better accuracy and it can predict the future situation for the effect of emitted smog in Dhaka city. Here,  $x(t)$  be the plants density,  $y(t)$  be the human density and  $z(t)$  be the smog density respectively at a time  $t$ . Let  $Q$  be the total increase in smog concentration into the atmosphere pollution gas and  $\delta, \delta_0, \delta_1, \delta_2$  be respectively the emitted  $CO_2$  gas coefficient rate by the human being, natural depletion coefficient rate of smog, depletion coefficient rate of plants and depletion coefficient rate denoted here. The quantity  $xy$  is the interaction rate. It causes decreasing & increasing of each species due to competition. These decreasing or increasing rates are  $\alpha xy$  and  $e\alpha xy$  respectively. Here,  $\alpha > 0$ , Let  $\alpha$  be a predation rate which its adding into interaction rate and  $e$  be conversion factor. The environmental carrying capacity of plants and human be

respectively  $K$  and  $L$ . Here the intrinsic growth rate of plants and humans be respectively  $r$  and  $s$ . When the depletion rate of highly consumed by plants and humans into their atmospheric smog that is denoted by respectively  $\delta_1 xy$  and  $\delta_2 yz$ . By considering this idea, the diagram of this model is given in Figure 4.1.

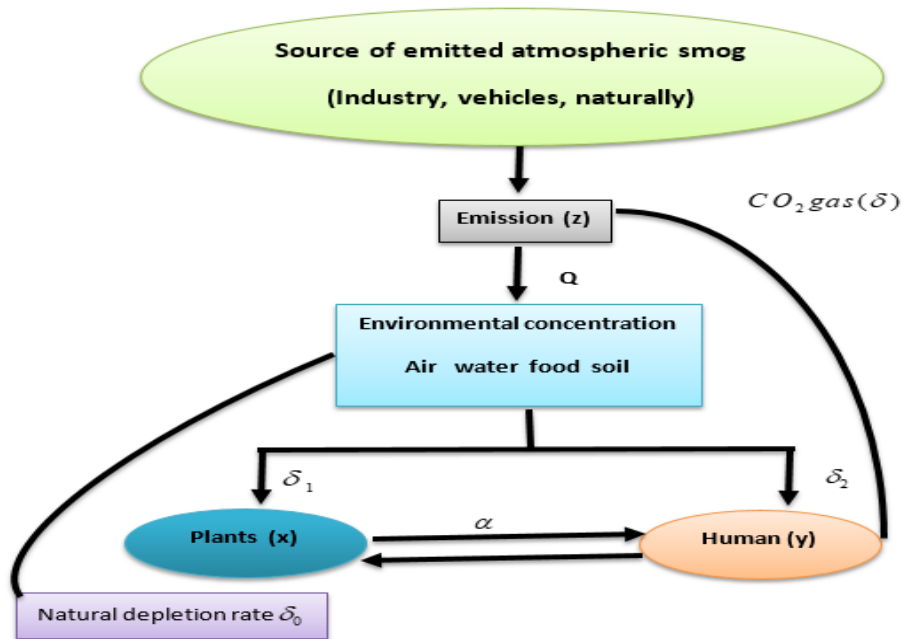


Figure 4.1: Diagram of Emitted Smog Concentration Model

From the model diagram **Figure 4.1** a nonlinear system of differential equation, we get the following dynamical system

$$\frac{dx}{dt} = rx \left( 1 - \frac{x}{K} \right) - \alpha xy - \delta_1 xz \quad (4.1)$$

$$\frac{dy}{dt} = sy \left( 1 - \frac{y}{L} \right) + e \alpha xy - \delta_2 yz \quad (4.2)$$

$$\frac{dz}{dt} = Q + \delta y - \delta_0 z - (\delta_1 x + \delta_2 y) z \quad (4.3)$$

with the initial conditions

$$x(0) = x_0, y(0) = y_0, z(0) = z_0$$

Table 4.1: Description of State Variables and Parameters

Parameters	Descriptions
$x$	Plants density
$y$	Human density
$z$	Smog density
$r$	Intrinsic growth rate of plants
$s$	Intrinsic growth rate of humans
$K$	Environmental carrying capacity of plants
$L$	Environmental carrying capacity of humans

$\alpha$	Predation rate
$e$	Conversion factor $e \in (0,1)$
$Q$	Increase in smog concentration into the atmosphere
$\delta_0$	Natural depletion coefficient rate of smog
$\delta_1$	Depletion coefficient rate of plants
$\delta_2$	Depletion coefficient rate of humans

#### 4.1 Preliminary Result

Here we will apply following lemma to establish the positivity, the boundedness of the proposed model.

#### 4.2 Positivity Analysis

**Lemma 4.1:** If  $a, b > 0$  and  $W^*(x, y, z) \leq (\geq) W(t)(a - bW(t))$  with  $W(t) > 0$  then for all  $t \geq 0$  then

$$\limsup_{t \rightarrow \infty} W(t) \leq \left( \liminf_{t \rightarrow \infty} W(t) \geq \frac{a}{b} \right)$$

**Solution:** According to the Lemma 4.1, we have from (4.1)

$$\therefore \frac{dx}{dt} = rx - \frac{rx^2}{K} \text{ here, } r = a \text{ and } \frac{r}{K} = b \quad \limsup_{t \rightarrow +\infty} x(t) \leq \left( \liminf_{t \rightarrow \infty} x(t) \geq \frac{r}{r/K} \right)$$

$$\limsup_{t \rightarrow +\infty} x(t) \leq \left( \lim_{t \rightarrow \infty} x(t) \geq K \right) \tag{4.4}$$

Similarly,

$$\frac{dy}{dt} = y(s + e\alpha K) - \frac{sy^2}{L} \text{ here, } a = s + e\alpha K \text{ and } b = \frac{s}{L}$$

$$\limsup_{t \rightarrow +\infty} x(t) \leq \left( \liminf_{t \rightarrow \infty} y(t) \geq L \right) \tag{4.5}$$

Again from equation (4.3),

$$\limsup_{t \rightarrow +\infty} z(t) \leq \left( \liminf_{t \rightarrow \infty} z(t) \geq Q \right) \tag{4.6}$$

**Lemma 4.2:** If  $a, b > 0$  and  $W^*(x, y, z) \leq (\geq) W(t)(a - bW(t))$  with  $W(t) > 0$  then for all  $t \geq 0$

$$W(t) \leq \frac{a}{b - Ce^{-at}} \text{ with } C = b - \frac{a}{W(0)}. \text{ In particular, } x(t) \leq \max \left\{ x(0), \frac{a}{b} \right\} \text{ for all } t \geq 0.$$

**Solution:**

By using the 4.2-Lemma, we have the equation (4.1),

$$\frac{dx}{dt} = rx \left( 1 - \frac{x}{K} \right) - \alpha xy - \delta_1 xz$$

$$x(t) \leq \frac{r}{\frac{r}{K} - Ce^{-at}}; C = \frac{r}{K} - \frac{r}{x(0)}$$

$$\text{so, } x(t) \leq \max \left\{ x(0), \frac{r}{K} \right\}$$

$$x(t) \leq \max \{ x(0), K \} \quad (4.7)$$

similarly, equation (4.2)

$$y(t) \leq \max \{ y(0), P \} \quad (4.8)$$

and similarly equation (4.3)

$$z(t) \leq \max \{ z(0), Q \} \quad (4.9)$$

### 4.3 Boundedness

**4.3.1 Theorem:** All the solutions of the model system (4.1)-(4.3) with initial conditions are always bounded for all  $t \geq 0$ .

**Proof:** From Lemma 4.2 equation (4.1) we have,

$$x(t) \leq \max \{ x(0), K \} \equiv M_1 \text{ for all } t \geq 0. \text{ Again from Lemma 4.2 equation (4.2), we have,}$$

$$y(t) \leq \max \left\{ y(0), \frac{L(s + e\alpha M_1)}{s} \right\} \equiv M_2 \text{ for all } t \geq 0. \text{ And again from lemma 4.2 equation (4.3),}$$

$z(t) \leq \max \{ z(0), Q \} \equiv M_3$  for all  $t \geq 0$ . the proof is complete for the boundedness of the model system and the system under.

### 4.4 Equilibrium Analysis of This Model

#### 4.4.1 Trivial Equilibrium Point

When the plants, humans and smog do not exist i.e.,  $x = y = z = 0$  thus the equilibrium point is obtained  $E_0(x_0^*, y_0^*, z_0^*) = E_0(0, 0, 0)$ .

#### 4.4.2 Boundary Equilibrium Point

When plants and humans ecological biomass exist and smog does not exist i.e.,  $x = y \neq 0$  and  $z = 0$  then from

$$\text{the system (4.1)-(4.3) we get, } E_1(x_1^*, y_1^*, z_1^*) = E_1 \left( \frac{K(r - \alpha y)}{r}, \frac{L(r + e\alpha x)}{r}, 0 \right)$$

#### 4.4.3 Axial Equilibrium Point

Where there are no trees and humans exist where i.e.,  $x = y = 0$  and smog  $z \neq 0$  then from the system (4.3) we

$$\text{get Thus the equilibrium point is obtained, } E_2(x_2^*, y_2^*, z_2^*) = E_2 \left( 0, 0, \frac{Q}{\delta_0} \right)$$

#### 4.4.4 Interior Equilibrium Point

When all the state variables of the model system are co-exists when  $x \neq 0$ ,  $y \neq 0$  and  $z \neq 0$ , then from the system have (4.1)-(4.3), we get

The interior equilibrium point

$$E_3(x_3^*, y_3^*, z_3^*) = E_2 \left( \frac{K(r - \alpha y - \delta_1 z)}{r}, \frac{L(r + e\alpha x - \delta_2 z)}{r}, \frac{Q + \delta y}{(\delta_0 + \delta_1 x + \delta_2 y)} \right)$$

#### 4.5 Dynamic Behavior of Equilibria

**4.5.1 Local Stability Analysis:** In this section we analyze the stability properties of the equilibrium points  $E_0(x, y, z)$ ,  $E_1(x, y, z)$ ,  $E_2(x, y, z)$  and  $E_3(x, y, z)$ . The local stability is established through Jacobian matrix of the system and finding the eigenvalues to evaluate at each equilibrium point. For linearized system the Jacobian matrix is given by,

$$\begin{bmatrix} \frac{df_1}{dx} & \frac{df_1}{dy} & \frac{df_1}{dz} \\ \frac{df_2}{dx} & \frac{df_2}{dy} & \frac{df_2}{dz} \\ \frac{df_3}{dx} & \frac{df_3}{dy} & \frac{df_3}{dz} \end{bmatrix} \text{ and } J(E_i) = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}$$

##### 4.5.1.1 Behavior of the system at $E_0(x_0^*, y_0^*, z_0^*) = E_0(0, 0, 0)$

The eigenvalues of the Jacobian matrix  $J_0$  at  $E_0$  are  $r, s$  and  $-\delta_0$ . Since all the Eigen values are non-negative hence the equilibrium point  $E_0(0, 0, 0)$  is asymptotically unstable.

##### 4.6.1.2 Behavior of the system at $E_2(x_2^*, y_2^*, z_2^*) = E_2\left(0, 0, \frac{Q}{\delta_0}\right)$

The eigenvalues of the Jacobian matrix  $J_2$  at  $E_2$  are  $r - \frac{\delta_1 Q}{\delta_0}, s - \frac{\delta_2 Q}{\delta_0}$  and  $-\delta_0$ . Since all the Eigen values are non-negative hence the equilibrium point  $E_0(0, 0, z)$  is asymptotically unstable.

##### 4.6.1.3 Behavior of the system at

$$E_3(x_3^*, y_3^*, z_3^*) = E_2 \left( \frac{K(r - \alpha y - \delta_1 z)}{r}, \frac{L(r + e\alpha x - \delta_2 z)}{r}, \frac{Q + \delta y}{(\delta_0 + \delta_1 x + \delta_2 y)} \right)$$

$$J(E_3) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

From Routh-Hurwitz Criterion we get,

$$\lambda^3 + A_1 \lambda^2 + A_2 \lambda + A_3 = 0$$

where,

$$A_1 = -C_{11} - C_{22} - C_{33}$$

$$A_2 = C_{22}C_{33} + C_{11}C_{22} + C_{11}C_{33} - C_{12}C_{21} - C_{23}C_{32}$$

$$A_3 = C_{12}C_{21}C_{33} + C_{11}C_{23}C_{32} - C_{11}C_{22}C_{33}$$

From Routh-Hurwitz criterion  $E_1$  is locally asymptotically stable if and only if locally asymptotically stable, if and only if

$$A_1 > 0, A_3 > 0, A_1 A_2 > A_3$$

$$\begin{aligned}
 A_1 A_2 - A_3 &= (C_{11})^2 C_{22} - (C_{22})^2 C_{33} \\
 &+ C_{11} C_{12} C_{21} - (C_{22})^2 C_{33} - (C_{22})^2 C_{11} - 2C_{11} C_{22} C_{33} \\
 &+ C_{22} C_{13} C_{21} + C_{23} C_{32} C_{22} - (C_{33})^2 C_{22} - (C_{33})^2 C_{11} + C_{23} C_{32} C_{33}
 \end{aligned}$$

∴ which satisfy the Routh-Hurwitz criterion.

#### 4.5.2 Global Stability Analysis

To show the global stability, we consider  $\left[ (x, y, z) \in \mathbb{R}_3^+, x \leq 0, y \leq 0, z \leq 0 \right]$  and the function

$J(E_3) : \mathbb{R}_3^+ \rightarrow \mathbb{R}^3$ , then construct a Lyapunov. Lasaile's function as follows,

$J(E_3) = L_1 (x - x_3 - x_3 \ln(x)) + L_2 (y - y_3 - y_3 \ln(y)) + L_3 (z - z_3 - z_3 \ln(z))$  Where  $L_1, L_2$  and  $L_3$  be positive constants to be determined.

**Proof:**

We can easily verify that the function  $J(E_3) = 0$  at  $E_3(x_3, y_3, z_3)$  and positive for all other values of  $x, y, z$ .

Then time derivation of  $L(E_3)$  along the solution of the systems given by,

$$\frac{dL}{dt} = L_1 \left( 1 - \frac{x_3}{x} \right) \frac{dx}{dt} + L_2 \left( 1 - \frac{y_3}{y} \right) \frac{dy}{dt} + L_3 \left( 1 - \frac{z_3}{z} \right) \frac{dz}{dt}$$

$$\therefore (x - x_3)(z - z_3) > 0 \text{ and } (y - y_3)(z - z_3) > 0$$

$$\therefore \frac{dL}{dt} \leq 0$$

Again  $\frac{dL}{dt} = 0$  when  $(x, y, z) = (x_3, y_3, z_3)$  which satisfy the Lyapunov. Lasaile's principle.

∴  $E_3$  globally asymptotically stable.

### 5. Numerical Simulation

We have used a set of suitable parameter values for finding numerical simulations of our model. The model system is simulated using ODE45 solvers written in MATLAB programming language. Graphical results were displayed here by using the following values of the parameter. Here we perform the numerical simulations for times  $t \in [0, 10]$ . The result of the combined classes is presented in Figure 5.1. The Figure 5.1 represents density of plants, humans and smog concentration combined for 10 years.

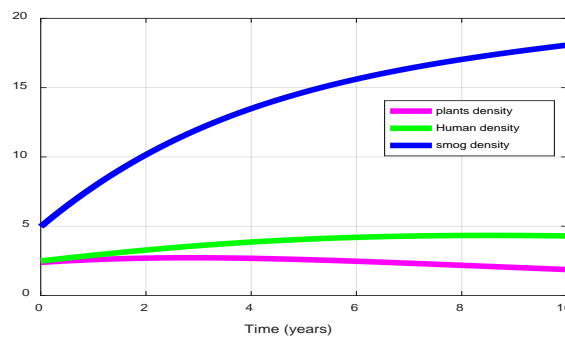


Figure 5.1: Graph of the Model (4.1)-(4.3) where Time span 0 to 10. where  $r = 0.225, s = 0.225, K = 20, L = 25,$

$$Q = 4.078, \alpha = 0.0130, \delta_1 = 0.0126, \delta_2 = 0.0123, \delta = 0.0121, \delta_0 = 0.127.$$

We observe that density of smog increasing in 10 and 15 years. Then density of plants and humans are decreasing initially up to 10 years for increasing smog concentration on the environment and results are shown in Figure 5.2 and Figure 5.3.

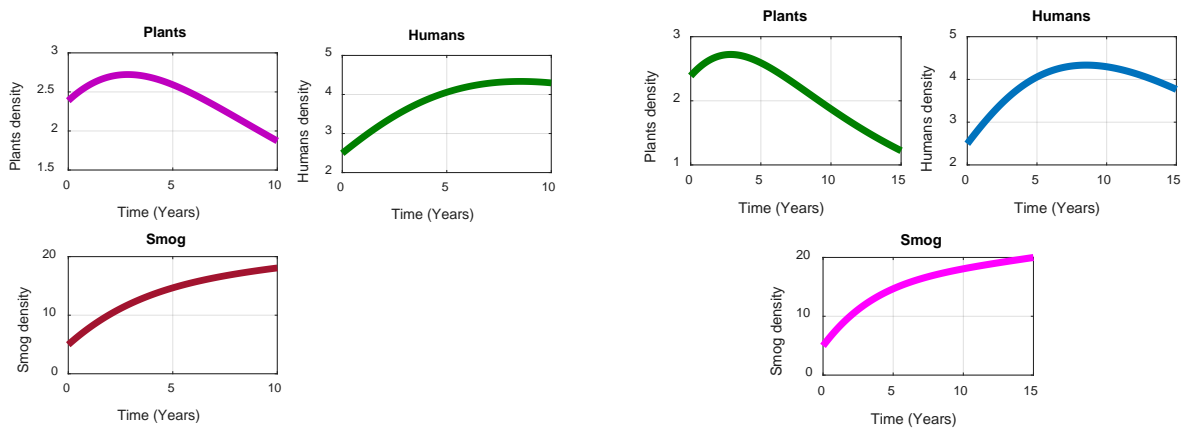


Figure 5.2: Graph of the Model (4.1)-(4.3) where Time span  $t \in (0,10)$  and  $t \in (0,15)$  .where  $r = 0.225, s = 0.225, K = 20, L = 25, Q = 4.078, \alpha = 0.0130, \delta_1 = 0.0126, \delta_2 = 0.0123, \delta = 0.0121, \delta_0 = 0.127$ .

From the Figure 5.4 we observe that the density of smog concentration is decreasing for given in 10 years. Then density of plants and humans are increasing up to 10 years.

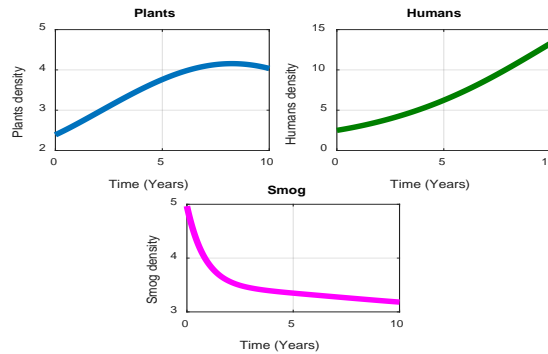


Figure 5.4: Graph of the Model (4.1)-(4.3) where Time span 0 to 10. where  $r = 0.225, s = 0.225, K = 20, L = 25, Q = 4.078, \alpha = 0.0130, \delta_1 = 0.0126, \delta_2 = 0.0123, \delta = 0.0121, \delta_0 = 1.127$ .

In Figure 5.5 represents density of plants and humans. For changing of the value of  $\delta_1$  and  $\delta_2$  the density of plants and humans is changed.



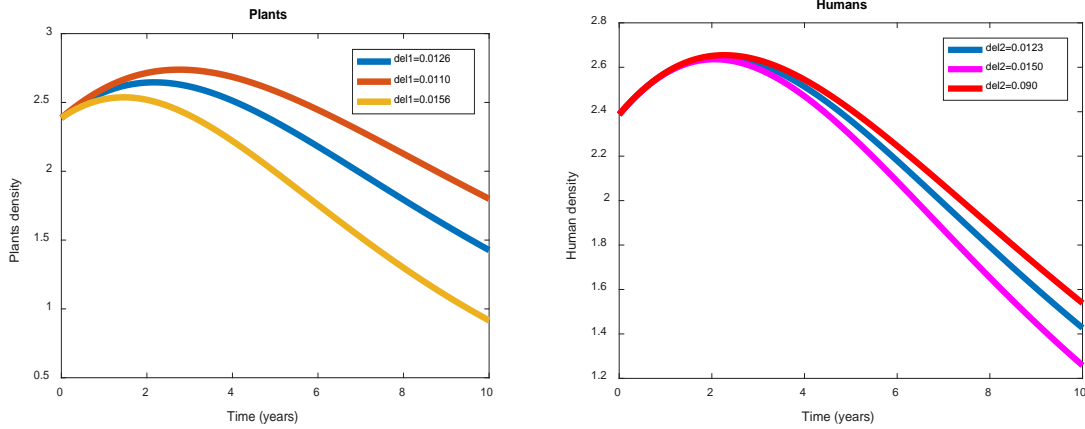


Figure 5.5: Variation of density of Plants for the parameter  $\delta_1$  is point 0.0126, 0.0110, 0.0156 and Variation of density of Humans for the parameter  $\delta_2$  are 0.0123, 0.0150, 0.090

We execute the program for different values of  $\delta_0$ . In Figure 5.6 represents density of smog concentration and the value of  $\delta_0$  that we have taken here is 0.127, 0.110, 0.227

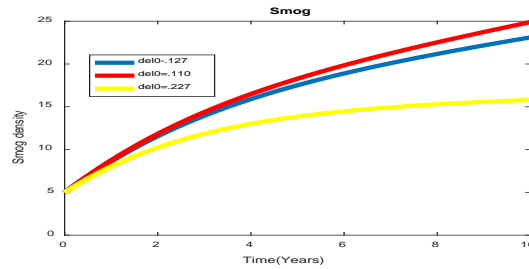


Figure 5.6: Variation of density of Smog for the parameter  $\delta_0$  are .127, .110, .227

Figure 5.7 represents phase portrait which is the geometric representation of the trajectories of a dynamical system in the phase plane. A phase plane is a visual display of certain characteristics of certain kinds of differential equations; a coordinate plane with axes being the values of the three state variables.

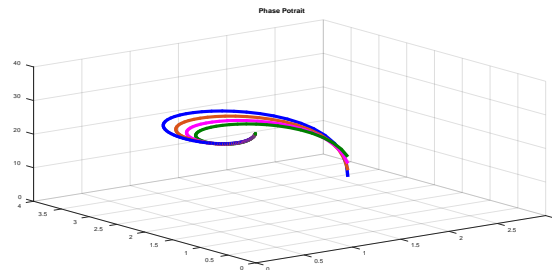


Figure 5.7: Phase Portrait Combined Picture of the Model (4.1)-(4.3) where  $r = 0.225, s = 0.225, K = 20, L = 25, Q = 4.078, \alpha = 0.0130, \delta_1 = 0.0126, \delta_2 = 0.0123, \delta = 0.0121, \delta_0 = 0.127$ .

## 6. Conclusions

In this chapter, we have presented a summary of our work. We have analyzed the behavior of the effect of smog concentration model. The mathematical model used in this project thesis is a system of nonlinear differential equations which applied to assess the environmental effect of smog concentration in Dhaka city. A dynamical model of smog concentration has been introduced at first. Then the equilibrium points of the model have been determined. The stability analysis at the interior points has been tested which ensures the validity of the model. The analytical analysis is included with positivity test, equilibrium point, stability at steady state point and stability at interior point. The model diagram, we used three compartments. Those compartments are called variables. Here variables are plants density, human density and smog concentration. All variables are interconnected relations. If smog density increases, then plants density and human density will decrease.

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## Biographies

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