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An approach to obsolescence forecasting based on Hidden Markov Model and Compound Poisson Process

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ABSTRACT

The popularity of electronic devices has sparked research to implement components that can achieve better performance and scalability. However, companies face significant challenges when they use systems with a long-life cycle, such as in avionics, which leads to obsolescence problems. Obsolescence can be driven by many factors, primary among which could be the rapid development of technologies that lead to a short life cycle of parts. Moreover, obsolescence problems can prove costly in terms of intermittent stock availability and unmet demand. Therefore, obsolescence forecasting appears to be one of the most efficient solutions. This paper presents a review of gaps in the actual approaches and proposes a method that can better forecast the product life cycle. The proposed approach will help companies to improve obsolescence forecasting and reduce its impact in the supply chain. The method introduces a stochastic approach to estimate the obsolescence life cycle through simulation of demand data using Markov chain and homogeneous compound Poisson process. This approach uses multiple states of the life cycle curve based on the change in demand rate and introduces hidden Markov theory to estimate the model parameters. Numerical results are provided to validate the proposed method. To examine the accuracy of this approach, the standard deviation (STD) of obsolescence time is calculated. The results showed that the life cycle curves of parts can be predicted with high accuracy.

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1. Introduction

Parts obsolescence has long been perceived as a significant challenge in designing and sustaining long-lived systems. This phenomenon is referred to as diminishing manufacturing sources and material shortages (DMSMS), and electronic parts in particular frequently entail serious problems of sustainability of the whole systems, due to their short life cycles. Obsolescence issues arise in systems that have a longer life cycle than their components, such as automotive, avionics, military, etc. These sectors have had to face several challenges on this account. Figure 1 illustrates the large gap between the life cycle of a component and the system. This gap could have an impact on overall costs (Jenab, Noori, Weinsier, & Khoury, 2014).

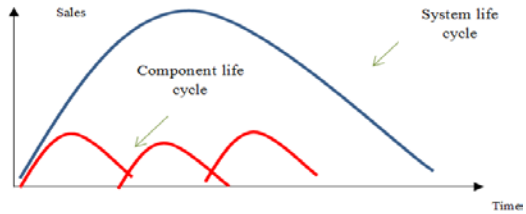


Figure 1. The life cycle of a component versus system (Fouché, 2013)

Obsolescence is a major problem caused by the rapid evolution of technologies. Researchers have defined obsolescence in various ways. (F. R. Rojo, Roy, & Shehab, 2010) define obsolescence as a part becoming obsolete when the technology used to manufacture it is no longer available, supported or produced by the supplier (F. R. Rojo, Roy, & Kelly, 2012; Shen & Willems, 2014). In other words, obsolescence can be characterized by the loss of suppliers or raw materials (Peter Sandborn, 2013), causing delays and extra cost. The negative effects of technological obsolescence on production performance have been studied in the literature, and represent a major challenge in the long term. Technological obsolescence causes problems in the supply chain and in the management of electronic systems (P. Sandborn, Prabhakar, & Ahmad, 2011). In other words, the incessant progress of technology is one of the factors that increase the rates of obsolescence. Electronic components are the parts most affected by technological obsolescence. In fact, the electronics industry has emerged as the fastest growing sector and has spread worldwide. Based on Moore's law, the evolution of technologies mainly comprised of electronic parts will continue to grow rapidly, and it is estimated that in the same way, semiconductor density can double every year (Homchalee & Sessomboon, 2014; Peter Sandborn, 2008; Tomczykowski, 2003). Consequently, many of the electronic parts that constitute a product have individual life cycles which are significantly shorter than the life cycle of the product.

Growth of the electronics industry has spurred dramatic changes in the electronic parts, with new technologies being introduced in the market at an increasing rate. Today, short technological life cycles and the lack of forecasting represent a challenge for several companies, especially for the electronics industry which represents one of the most dynamic sectors of the world economy (Solomon, Sandborn, & Pecht, 2000). From this perspective, obsolescence forecasting appears to be one of the most efficient solutions to managing obsolescence, as it assists manufacturers in identifying obsolete parts. Through obsolescence forecasting, companies can ensure support for parts in service and mitigate any negative impact by identifying parts that are likely to become obsolete. Moreover, obsolescence forecasting enables engineers to more effectively manage the introduction and ongoing use of long-field-life products based on the projected life cycle of the parts. Obsolescence prediction methodology is a critical element within risk-informed parts selection and management processes.

To overcome the problems caused by obsolescence, several studies have been conducted to create models that can effectively forecast obsolescence. Statistical methods such as regression, partial least square regression (PLS), time series and Gaussian method have been previously employed in several works (Gao, Liu, & Wang, 2011; Jungmok & Namhun, 2017; Solomon et al., 2000). However, there is a need to modify these methods to facilitate life cycle forecasting for thousands of components that affect our day-to-day lives. In this context, there are precise and stable forecasting approaches whose performance has been justified in the literature. Moreover, results have shown that these approaches could be used for nonlinear predictions with high accuracy and without human manipulation (Zurada, 1992). In the past few decades, machine learning has attracted the attention of many researchers in various disciplines and has been applied in many areas. According to (Wu et al., 2008), among the top ten algorithms identified by the Institute of Electrical and Electronics Engineers (IEEE) are AdaBoost, SVM, K-Means, decision tree, and naïve Bayes. These algorithms have been proven to be good predictors. In addition, applying machine learning in forecasting obsolescence risk and life cycle has received a lot of attention in the last three years (Yosra Grichi, Beauregard, & Dao, 2018; Y Grichi, Beauregard, & Dao, 2017; Jennings, Wu, & Terpenny, 2016). Moreover, Jaarsveld introduced a forecasting obsolescence model (van Jaarsveld & Dekker, 2011) that uses Markov chain with only two states, where the transition between states is characterized by a decrease in demand order.

1.1 Scope and contribution

In previous work, benchmarking studies have shown an approach to forecast obsolescence risk using Markov chain with a two-state model, with only one state in which the part is moving and in the other state the demand for the part has dropped dead. This last state is assumed to be an absorbing state (van Jaarsveld & Dekker, 2011). However, this approach is limited and not adequate to forecast the whole life cycle, since it considers only one transition between two states, so that the estimation of obsolescence is not accurate. In general, the life cycle of a product goes through multiple stages as

presented in Figure 2, which are: introduction–growth–maturity–decline– obsolescence. It is important to consider the whole of the life cycle in order to improve the accuracy of forecasting.
This paper addresses a new approach to forecasting the life cycle of electronic parts obsolescence. Markov chain performed with compound Poisson process (CPP) is proposed to build a predictive model for the parts' life cycle. The proposed method takes into account five stages of a part's life cycle to forecast the obsolescence risk time.
In fact, multiple states will greatly complicate the estimation of the model parameters; therefore, the theory of hidden Markov models is used to estimate the model parameters. To the best of our knowledge, methods to estimate the life cycle using Markov chain and compound Poisson process are not available in the literature.
Table 1 shows the difference between the proposed method (Markov chain performed with compound Poisson process (CPP) and homogeneous hidden Markov theory to estimate model parameters) and Jaarsveld's model (simple Markov model with only two states).

Table 1. Jaarsveld's method Vs. Proposed method

	Jaarsveld's method	**Proposed method**
Model	Markov chain with two states	Markov chain with multiple states
Advantages	- Method very simple and easy to implement - Estimation of parameters is simple - Simple optimization of model	- Powerful method - Provides for a possibility of sudden changes in the order rate through the states. - The model uses multiples states with a positive demand rate as well as negative demand.
Disadvantages	- The method described in the above model provides for decreases in demand but not increases. In practice, the parts go through several stages of an entire life cycle, with possibility of positive demand rate.	- The estimation of parameters using HMM is quite complicated.

In this paper, a Markov driven homogeneous Poisson process is introduced based on specific features. These key features are as follows:

- There are five Markov states with only forward transition, in which the last one is considered as an absorbing state.
- The transitions between different states occur randomly in time, and the duration in one particular state follows an exponential distribution.
- In each state, the demand follows a homogeneous Poisson counting process with constant intensity λ_i., i.e., constant rate of arrival of orders.
- Transitions occur when there is a sudden change in the intensity of order arrivals.
- The sequence of observed states s_i can be characterized by the corresponding λs_i, and they are in descending order, $\lambda s_1 > \lambda s_2 > ,\ldots, > \lambda s_n$

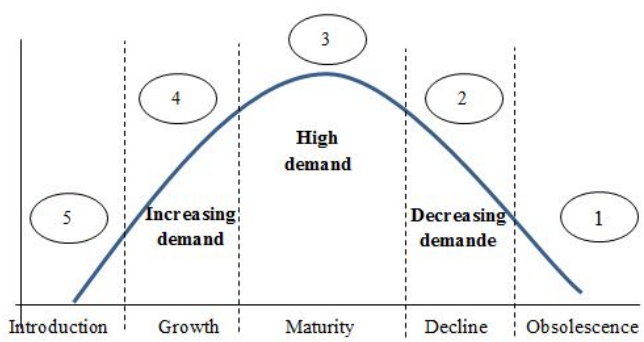


Figure 2. Life cycle stage (Solomon et al., 2000)

The paper is structured as follows. Section 2 reviews the pertinent literature on obsolescence forecasting, while Section 3 presents the proposed framework for obsolescence forecasting. A numerical case study is presented in section 4 followed by a discussion in section 5. Finally, Section 6 summarizes the paper with recommendations for future research.

2. Potential obsolescence forecasting strategies: Background

The nature of obsolescence forecasting so far has been the management of the problem once it occurs. In practice, most firms do not have effective methods for predicting obsolescence and therefore are forced into over reliance on reactive strategies. The most famous reactive approach used by companies is last time buy (LTB) and existing stock (F. R. Rojo et al., 2010). However, these strategies are only temporary and can fail if the organization runs out of ways to procure the required parts, and it can be very costly.
Obsolescence forecasting can be divided into long-term and short-term methodologies: anything lasting more than one year is named as long-term forecasting, which is represented by proactive and strategic management and life cycle planning to support a system. The second type is short-term forecasting, which is represented by a reactive management, such as reducing inventories to avoid the dormant parts, and forecasting the economic order quantity (EOQ) to cover a specific period.

2.1 Obsolescence management

The effective management of obsolescence requires three mitigation strategies that may be defined as: reactive, proactive, and strategic, as shown in Figure 3.

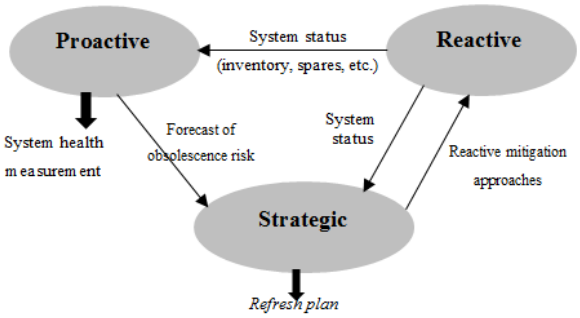


Figure 3. Obsolescence management levels (Peter Sandborn, 2008)

Reactive management involves immediately executing solutions when parts become obsolete. Possible solutions in this level include four categories currently adopted according to (F. R. Rojo et al., 2010), which are : 1-Existing stock—this represents the least expensive approach, where the only cost is the functional test of the components. It is a stock held by the supply chain that contains the original spare components. The existing stock should be sufficient to ensure supply throughout the remaining life cycle of the system (Bartels, Ermel, Pecht, & Sandborn, 2012; Pinçe & Dekker, 2011; F. J. R. Rojo, Roy, Shehab, & Cheruvu, 2012; F. R. Rojo et al., 2010); 2- Last Time Buy—when a supplier informs the end of life of a product, the manufacturer decides to purchase a quantity (last purchase) sufficient to support production until redesign. It is a short-term strategy frequently used by companies; 3-Alternate Part—consists of replacing the part with another one with the same performance, dimensions and mechanical characteristics; 4-Redesign—consists of redesign of an obsolete component.

Proactive management approach is based on the proactive monitoring of life cycle information on parts in order to prevent the risk of obsolescence leading to production shutdowns and costly design. Obsolescence forecasting, the focus of this paper, is essential to the proactive management level.
Finally, strategic management represents long-term strategic planning for part obsolescence in order to satisfy the system's requirements. In other words, it is a combination of the proactive and reactive approaches, in order to minimize the cost of the life cycle. Part obsolescence forecasting can be the input for the strategic management level.

2.2 Obsolescence forecasting

Forecasting part obsolescence plays a key role in proactive and strategic management levels. Obsolescence forecasting can be broken down into two groups. The first is based on obsolescence risk estimation risk and the second method is based on the life cycle forecasting.

Obsolescence risk forecasting is used to predict the probability that a component will become obsolete, by creating a scale to indicate the levels of the likelihood of a part becoming obsolete (Josias, Terpenny, & McLean, 2004; F. R. Rojo et al., 2012; van Jaarsveld & Dekker, 2011). In this context, (F. R. Rojo et al., 2012) conducted a Delphi study to analyze the risk of obsolescence. They developed a risk index using some indicators, which are: years to end of life, the number of sources available, and the consumption rate versus the availability of stock. Another approach developed by (Josias et al., 2004) aims to create a risk index by measuring the manufacturers' market share, number of sources available for each part, and the company's risk level. Furthermore, the Markov model was used to estimate and manage the obsolescence risk (van Jaarsveld & Dekker, 2011), wherein the authors used product demand data history to estimate the risk of obsolescence using the Markov chain and Poisson process. However, (Hu & Bidanda, 2009) focus on the decision making through the life cycle of a product using the Markov decision process (MDP) to estimate the obsolescence risk. (Yosra Grichi et al., 2018; Jennings et al., 2016) created models to estimate the obsolescence risk using machine learning algorithms as well as a meta-heuristic approach to improve the classification by random forest algorithm. Finally, (Zolghadri, Addouche, Boissie, & Richard, 2018) propose a method to estimate the obsolescence risk using a Bayesian model.

Alternatively, Life cycle forecasting refers to a process that predicts the length of time during which the product will be in production. (Solomon et al., 2000) were the first to introduce the life cycle forecasting method. They conducted a study to predict the obsolescence date from the life cycle curve. During the lifecycle, most products go through five stages that refer to the demand changes, which are named as introduction, growth, maturity, decline and phase out (obsolescence). This period represents the product availability in the market when it can be purchased by customers. Everything before the introduction stage represents the product development period. Another method based on data mining was developed by (Peter Sandborn, 2017; P. A. Sandborn, Mauro, & Knox, 2007), where the obsolescence date was obtained by applying Gaussian method to predict future sales over time. Moreover, other researchers have introduced regression analysis and time series method to predict the date of obsolescence (Gao et al., 2011; Jungmok & Namhun, 2017). Croston method and neural network approach were used to forecast intermittent demand in order to minimize the impact of obsolete and dormant stock (Babai, Dallery, Boubaker, & Kalai, 2019; Kourentzes, 2013). Lastly, (Y Grichi et al., 2017; Jennings et al., 2016) have used a data-driven method by creating machine learning algorithms to forecast the obsolescence date.

This paper mainly discusses the life cycle curve approach to forecasting. Two main papers are critically discussed in this work, which are (Song & Zipkin, 1996; van Jaarsveld & Dekker, 2011). Jaarsveld used a continuous Markov model for spare parts demand with two states, where state one represents the part which is moving (having demand), while the second state represents the part which is dropped (dead). The two states of the Markov process are driven by a compound Poisson process. On the other hand, Song used Markov chain with the Poisson process to model the demand behavior. Both reference papers used the same basic model: compound Poisson process, to model arrival order. They assumed that the transition from high demand states to lower demand states culminates in an obsolescence state. However, these methods are not sufficient to forecast the life cycle curve, and therefore it is necessary to introduce a new obsolescence forecasting approach because of the lack of scalability and accuracy in these current methods. The novel approach will take into account all the stages of the life cycle. Nevertheless, the extensions of calculation for more than two states will complicate the estimation of the model parameters. In this aspect, hidden Markov theory is used to estimate significantly the parameters of the model (van Jaarsveld & Dekker, 2011).

2.3 HMM: Baum-Welch

The hidden Markov model is useful for modeling sequential data with a few parameters using discrete hidden states, and the estimation procedures are usually based on the EM algorithms (Viterbi algorithm).
One main feature of this algorithm is the joint usage of forward probabilities ($\alpha_t(i) = P(O_0, \ldots, O_t, q_t = S_i|M)$) and backward probabilities,$\beta_t(i) = P(O_{t+1}, \ldots, O_T, q_t = S_i|M)$ used for evaluation problems, to compute the probability of transition from state *i* to state *j*, $\xi_t(i,j) = P\big(q_t = S_i; q_{t+1} = S_j \big| O, M\big)$.
For HMM training, we use time series data of sample $\hat{\lambda}_a(t)$ as observed Markov emissions (O). This data came from repeatedly observed (or simulated) flow demand sequences.

The main functioning of this algorithm is presented as: given a set O of observed emissions (data of sample$\hat{\lambda}_a(t)$) and an initial HMM, M_0, we can compute a re-estimated model, M_1, with the main property:

$$P(O|M_1) \geq P(O|M_0) \tag{1}$$

Baum-Welch algorithm implies repeating the estimation procedure until some precision condition is reached.

$$\left|\frac{P(O|M_{i+1}) - P(O|M_i)}{P(O|M_i)}\right| < c \tag{2}$$

The output of Baum-Welch algorithm is the estimated model $\widehat{M}$.

3. Proposed framework

Throughout this research, the literature has shown an evaluation of the most promising approaches to forecast obsolescence. In fact, Markov process comprises an extraordinarily rich and flexible class of models. We suggest how this technique can be adopted to model obsolescence using the formalism of state transition diagrams. In this section, the obsolescence forecasting approach based on Markov model is presented, and a flowchart of the proposed method is illustrated in Figure 4. The main objective is to develop a Markov Poisson process model to forecast the obsolescence risk time performed with HMM. The hidden Markov will be applied in this case to estimate the model parameters. In order to estimate the transition probability, five Markov states are used. The transient state is forward and the duration in one particular state follows an exponential distribution. However, the transition from state to state may occur when there is a change in demand order.

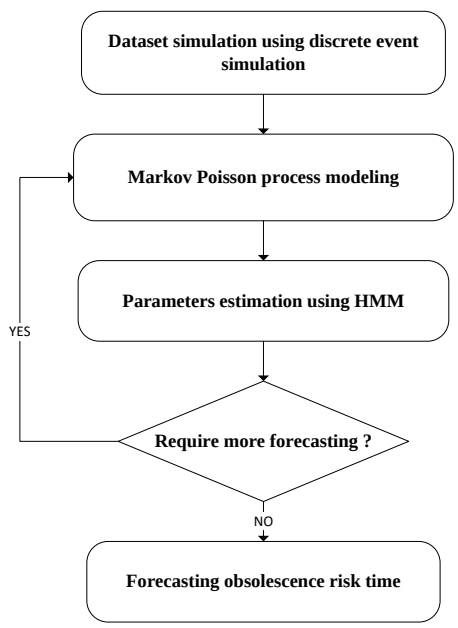


Figure 4. Flowchart of the proposed method

In each state, the demand follows a homogeneous Poisson process with constant intensity λ_i., i.e., constant rate of arrival of orders and the sequence of observed states s_i can be characterized by the corresponding intensity λs_i. Thus, the transition probability is estimated with Poisson distribution with changing order rate. Moreover, we represent the world as a finite-state.

We consider five states leading to obsolescence, as illustrated in Figure 5.
State 5: new product comes into the market (demand is slow but increasing)
State 4: increase in demand order.
State 3: demand is high and price low.
State 2: demand decrease: competitors announce a new product.
State 1: demand dead or obsolescence, which presents the last state or absorbing state.

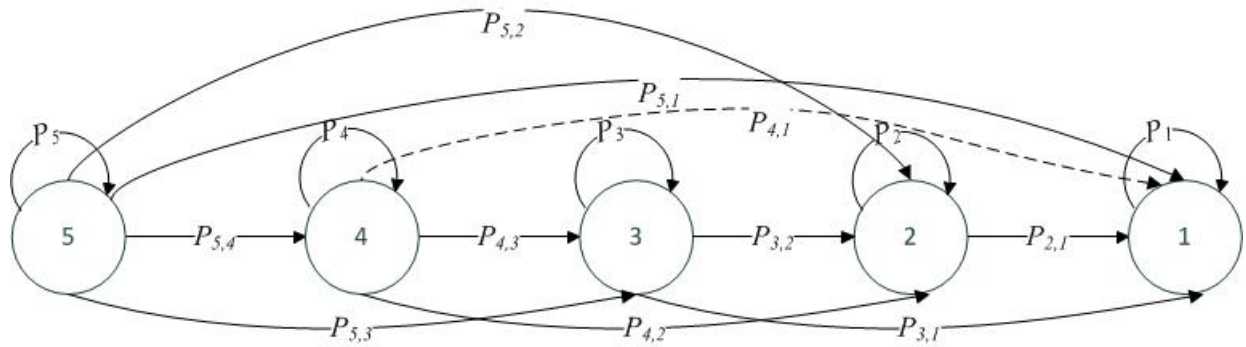


Figure 5. Probability transition between states

The transition between states may jump over several stages. For example, the transition from state 5 to 1 may be caused by a competitor's market integration. It can be also be caused by several independent future events, such as the introduction of several competing products that can cause a decline in the demand.

The proposed methodology is as follows: **Step 1** presents the data simulation from Markov hidden Poisson process as a Markov modulated Poisson process. To do this, we set up a transition rate matrix (Q), initial distribution (delta), and the set of Poisson intensities (lambda). The transition rate matrix is cyclic, so we can restart a new obsolescence cycle once we are in obsolescence state, in order to simulate many obsolescence cases. Moreover, we assume that there is a greater chance of transitioning to the nearest state than to others, i.e., if we are in state 5 there is more chance of transition to state 4 rather than to states 3, 2, or 1. Once the data is simulated, **step 2** presents the development of Markov hidden Poisson process; we estimate sample intensity function of the observed order arrivals. The next step is using the sample data. In fact, we assume that we do not know the HMM that generates the data (in real world cases, we only see the order arrival data), and we perform parameter estimation of a Poisson HMM. The estimation is done by EM (Baum-Welch) algorithm of HHM. Once we have the estimated model, obsolescence risk time is estimated from repeated simulations using estimated Poisson HMM. With simulated data from fitted Poisson HMM, we can identify the sojourn time of each state and the time until we reach obsolescence of each cycle. Therefore, we can develop the distribution histogram of obsolescence risk time. Finally, the obsolescence risk time is estimated as the mean of all obsolescence cycles in the simulated data.

3.1 Model

The obsolescence forecasting can be performed by the following steps given as a Markov chain model. We assume that the order rate depends on the state of a continuous time Markov process. For the reasons given below, our demand Markov state starts in a fixed state, namely state 5.
The probability of transition from i=5 to j=1 in time t is given by equation 3:

$$P(X_t = 1|X_0 = 5) = \left(P^t\right)_{5,1} \tag{3}$$

In this case, we cannot directly observe the sequence of Markov state of the demand flow. However, we can observe the following two main features:

(1) Time of order arrivals: they conform to a discrete time ordered set of points on [0,S) and as a whole they are a point process realization. We can count the number of orders until time "t" by a counting variable N_t, which presents the sum of order arrivals until time t. This variable is discrete, non-decreasing and has jumps of size 1 (due to the fact that two different orders cannot arrive at the same time).
(2) Each arrival order has a random positive and discrete size, which is related to the amount of parts that are required on one specific order.

The combination of points 1 and 2 conforms a parts order arrival process with arrival time epochs given by the point process in (1), and jumps of positive and discrete random size of (2).
The modeling problem implies a double stochastic structure:
First level: At any time "t" we observe a compound Poisson process realization Z_t, where:

$$Z_t = \sum_{i=1}^{N_t} S_i \tag{4}$$

Where:
Z_t : Is the cumulation of parts requirement till time *t*.
N_t : Is the cumulation of orders requirement till time *t*.
S_i : Is the size (amount of parts) of ith order (size order), and represents the amount of spare parts that are included in an individual order.
Based on Equation (4), N_t is a homogeneous Poisson process (HPP) with intensity parameterλ. N_t is a random variable with probability mass function given by:

$$P\{N_t = k\} = \frac{e^{-\lambda t}(\lambda t)^k}{k!} \tag{5}$$

With the probability of a cumulation of k orders in the time interval [0,t]. The HPP drives the sum of Z_t process, i.e., N_t fixes the number of cumulated orders at any time "*t*". The sequence of order sizes S_i is positive independent and identically distributed random variables.
The intensity parameter "λ" discussed in level 1 is not unique. We have a set of five λ´s, each one corresponding to a demand state:

$$\lambda \in \{\lambda_1, \dots, \lambda_5\} \text{ and } \lambda_1 > \cdots > \lambda_5 \tag{6}$$

At any time the demand process can be in a particular state $i \in (1, . ., 5)$ following a Markov chain structure where X_t=*i*, transition matrix = P_{5x5} and with trivial initial distribution $\Pi = (X0 = 5) = 1$ (we suppose that all demand cycles began at state 5 and finished on state 1 according to the equation (3)).
X_i : is an HMM. This means that we cannot observe the Markov states; we can only observe the order arrivals.
The transition matrix is characterized by the following features:

1) $P_{1,1}$=1 and $P_{1,j}$=0 for *j*=2,..,5 , i.e., once it is in state 1 the sequence will continue in this state forever, because it is an absorbing state. No backward transitions can be done.

2) $P_{i,j} = \begin{bmatrix} P_{i,j} > 0 & if \;\; i \geq j \\ P_{i,j} < 0 & if \;\; i < j \end{bmatrix}$

By combining Markov model and compound, we can obtain the equation (4) as:

$$Z_t = \sum_{i=1}^{N_{X_t}} S_i \tag{7}$$

Where N_{X_t} is an HPP with intensityλ_{X_t}. The intensity is driven by the Markov chain.

3.2 Parameters estimation

As shown in the previous discussion, it is not possible to observe the sequence of Markov process; only the sequence of orders can be observed. To discover the Markov sequence, it is necessary to make some data transformations: First, the time of arrival orders should be taken into account, and the sample-counting variable$\widehat{N}_t$.Second developed, estimating the sample intensity function $\hat{\lambda}_a(t)$ for a time window *a*, given by the question below:

$$\hat{\lambda}_a(t) = \frac{\widehat{N}_{t+a} - \widehat{N}_t}{a} \tag{8}$$

For a regular partition of time windows [0,*S*] of demand cycle, i.e., a sequence of disjointed intervals : [0,*a*],(*a*,2*a*],….,((*n*-1)*a*,*S*].

The$\hat{\lambda}_a(t)$, can be viewed as a discrete sequence of λ at epochs *a*, 2a, 3a… *S*. The $\hat{\lambda}_a(t)$ conforms to a discrete time series of sample intensities.
Each discrete time can be associated to Markov, and these times considered as transitions points. Some transitions will be to the same state (P_{ii}) and the sequence of sample $\hat{\lambda}_a(t)$ will have similar values. The duration in a particular state *i*, ΔT_{Si} , is a random variable with the following distribution:

$$P(\Delta T_{Si} = n) = (p_{ii})^n \tag{9}$$

The probability of *n* identical sequences of Markov states, i.e., *n* transitions to itself.
The probability of a transition to another state at duration *n* and *n*-1 duration in state *i* is:

$$\sum_{j \neq i} P\left(\Delta T_{Si} = n - 1; S_j\right) = (P_{ii})^{n-1}(1 - P_{ii}) \tag{10}$$

The former approach is the basis for a hidden Markov model. The time series of sample $\hat{\lambda}_a(t)$ are emissions from a Markov state. Due the random nature of $\widehat{N}_t$ we will not observe identical values of $\hat{\lambda}_a(t)$ during the time duration of a particular state. These fluctuations conform to a probability density of emission.

$$b_i(O_k) = P\{O_k(t) | X_t = i\} \tag{11}$$

Where $b_i(O_k)$ is the probability of emission O_k given the Markov Chain with state *i*. The emission is $\hat{\lambda}_a(t)$.
The estimation procedure consists of applying the Baum-Welch algorithm under the previous model restrictions to the sequence $\hat{\lambda}_a(t)$. This algorithm is the HMM version of an EM algorithm.

4. Numerical example

This section presents a numerical example to forecast life cycle curves and estimate the obsolescence risk time. R programming is adopted to develop the model.

In the first phase, the data is generated by discrete event simulation with repeated simulations of sequences of Markov chain. The transition matrix is first given by Figure 6. This transition matrix is fixed for simulation purposes. The main assumptions are: 1) the process will be in a specific state for a relatively long time, then the *P_ii* values are close to 1. 2) The transitions to others states occur only in forward direction starting from state 5.

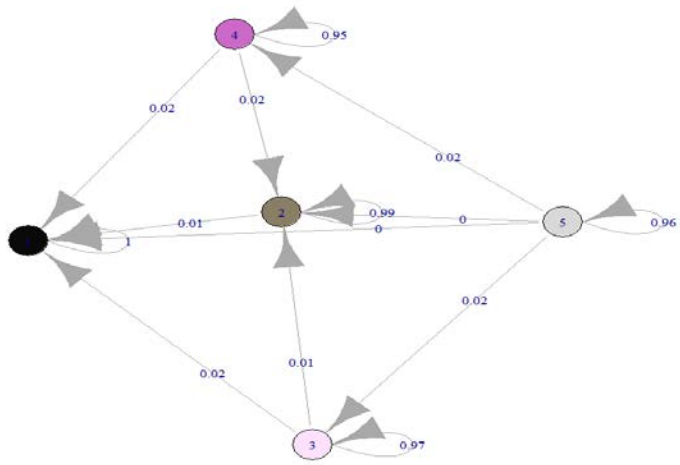


Figure 6. Transition matrix of Markov chain

The second phase is focused on the visualization of the sample process. For each Markov epoch in the simulation stage, the HPP is generated with lambda related to Markov states. This HPP has a duration given by the time window's length "a".

Figure 7 presents a plot of the first cumulated order arrival sequence generated from data simulation. Then, we present the empirical intensity in a reduced time window. Figure 8 shows the obsolescence risk time, where the first window is set to 200 time units (in days) in which we can see only one obsolescence cycle. On the other hand, the second plot is set to 2000 time units' length, where we can see a sequence of nine obsolescence cycles. This figure shows the forecast results fitted by the Markov Poisson process model with the uncertainties of forecasting.

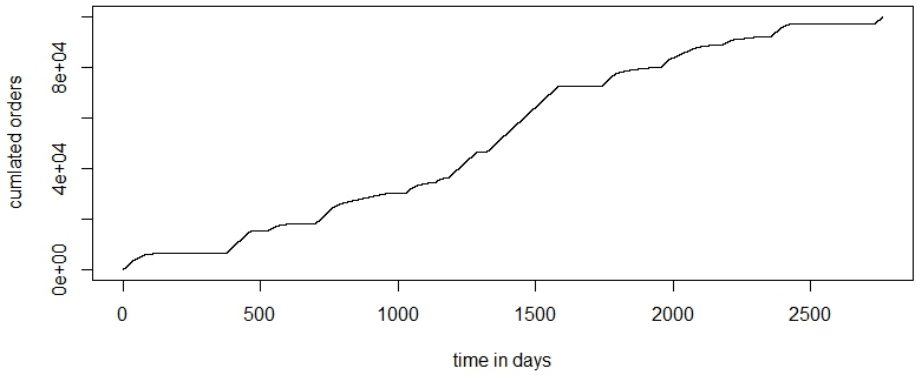
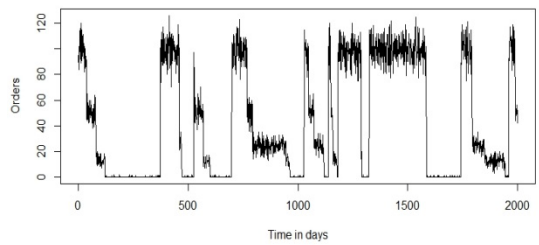


Figure 7. Cumulation of arrival orders



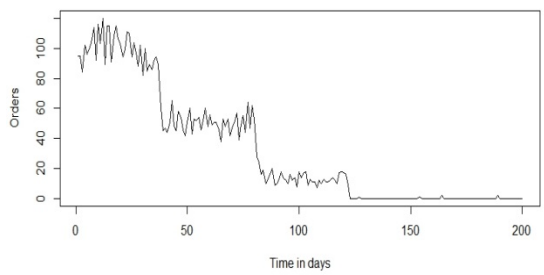


Figure 8. Order Vs Time in days

The third step is the estimation of model parameters using hidden Markov Poisson process from simulated data. The estimation is done by EM algorithm for HMM. At this point, we assume that we do not know the model, and we only know the order arrival data.

The obsolescence forecasting is estimated from order arrivals data, which is presented by the estimation of expected time for obsolescence transition, from state 5 at t=0, given by Markov model. As already discussed, the last stages of the life cycle curves are the obsolescence stages. Obsolescence risk time is given by Figure 9. The expected obsolescence time is estimated at 320 days in this case.

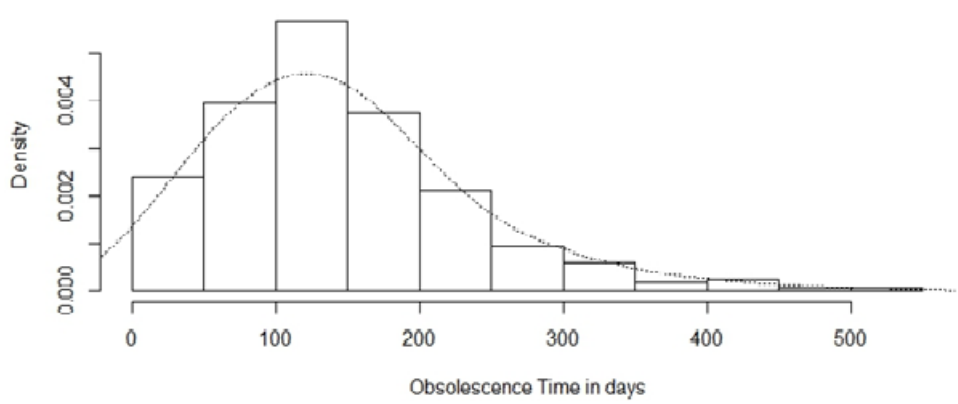


Figure 9. Distribution of obsolescence risk time

For the accuracy of life cycle curve forecasting, the variance and standard deviation (STD) of obsolescence time are calculated. To calculate the accuracy of the model, the standard deviation is compared to the mean based on this equation:

$$\frac{Coefficient\ of\ Variation\ (CV)\ std}{Mean\ value} * 100 = 6\%$$

5. Discussion

In the previous section, the obsolescence forecasting based on Markov chain and homogeneous compound Poisson process method was presented and applied using data simulation. The Paper introduced as well the hidden Markov theory to estimate the model parameters.

The first part of the model was the generation of data; therefore, a transition matrix and the Poisson intensities were set up. Once the data was simulated, the intensity function of the observed order arrival was estimated. Next, the estimation of the parameters of the Poisson HMM was performed. From the simulated data fitted from Poisson HMM, the sojourn time was identified for each state. After obtaining the full life curve from the proposed predictive method, the expected obsolescence time was estimated. On the other hand, the estimation of the life cycle curve proved complicated because the transitions between phases may jump over stages that can be caused by several independent future events as discussed by (Song & Zipkin, 1996; van Jaarsveld & Dekker, 2011).

The forecast result in Figure 9 presents the predicted life cycle curve, which indicates an increase in the demand represented by the availabilities of sales data, while the right-hand side of the curve represents a decrease in the demand. The expected obsolescence time is estimated at 320 days in this case.

For the accuracy of the life cycle curve forecasting, the method was evaluated using standard deviation (STD) to measure the amount of variability of the obsolescence zone, which was presented by 6%.

As discussed in previous sections, the proposed method can overcome the big limitation of Jaarsveld's method which presented by an approach to forecast obsolescence risk performed by Markov chain with only two states. This approach was limited and not adequate to forecast the whole life cycle since it considers only one transition between two states. The gap and limitation in the current approach were examined and improved in the proposed method. This approach is proven more accurate to forecast the obsolescence risk time.

6. Conclusion and future work

Research on obsolescence is growing due to the serious challenges that complex systems face every day. However, firms which are facing to manage these problems must react. Forecasting obsolescence is the key enabler for all proactive and strategic management approaches to addressing obsolescence. Through an accurate obsolescence forecasting, firms can reduce significantly the cost and save millions of dollars.

This paper introduced an approach to estimate obsolescence time based on simulation of demand data using Markov homogeneous Poisson process performed with hidden Markov to estimate the model parameters. The results of forecasting are presented in figures 7 and 8. The proposed method consists of three phases: data simulation, Markov homogeneous Poisson process modeling to forecast the life cycle curves, and hidden Markov to estimate the parameters of the model. As discussed regarding Table 1, the proposed method can forecast obsolescence risk time taking into account the whole life cycle curve performed by the demand rate. A numerical example was provided to demonstrate the capability of this method.

The main theoretical contribution of this paper was introducing Markov model and hidden Markov theory for obsolescence life cycle forecasting after an in-depth analysis of gaps in the actual approaches (Jungmok & Namhun, 2017; van Jaarsveld & Dekker, 2011). The method was examined for its ability to forecast the obsolescence risk time using sales data as a requirement with high accuracy.

This paper provides an approach enabling firms that are facing this kind of problems to enhance their forecasting and to ensure support to the long-life systems. The proposed method can be applied by any company using a sufficient data. The life cycle forecasting could also be useful to help make life-time buy or last buy orders more accurate.

Along with these strengths, the proposed method's limitations are also identified. Like all obsolescence forecasting frameworks, this approach has some limitations that may compromise the validity of the estimations. First, the paper considered data simulation; however, if the data used to build the model does not represent the current real world, the model will not be very effective. In obsolescence, there is an extremely high chance of this occurring due to rapid innovation. Other datasets and real-world problems for obsolescence forecasting can be tested using this method. Second, the proposed method is more complicated than Jaarsveld's method and more sensitive to data because no fixed distribution is assumed.

Moreover, there are some other market factors for which manufacturers should consider carrying out an additional risk assessment, which can cause a lot of variations in the life cycle. Another limitation is based on the Markov property, i.e., the next state depends only on the current state; this will not influence the past state sequence.
It would be interesting to extend the method by taking into account other factors to improve the accuracy of the forecasting. Real data application might be useful to test the feasibility of the model. Another direction for future research is to perform some sensitivity analysis to test output variations with changes in the model parameters.

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