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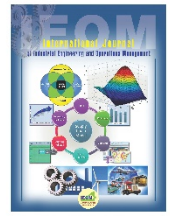

Deviation Degree between Ideal and Real Domain Transition Probabilities in Resource Circulation Considering the Production Synchronization Ratio

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ABSTRACT

The ideal state for achieving smooth resource circulation on Earth is achieved when the intake, expulsion, and natural purification rates (three-element) are equal. In this research, we propose the four-element deviation degree, which combines the three-element deviation degree with the synchronization deviation degree, by considering the deviation degree of the synchronization between production and sales in corporate activities. We use a domain transition probability model for resource circulation, and based on this model, we investigate the three-element deviation degree to evaluate the deviation degrees of these three elements in ideal and real situations quantitatively. Further, we conduct a quantitative discussion of the gap between the ideal and real situations in the four domain transitions, including corporate activities, by considering the four-element model. We considered simple numerical examples of the current state ($t = 0$) vector (initial state vector) and the domain transition probability matrix and simulated the resource circulation. Each simulation indicated the effect of slow natural purification from the waste domain to the resource domain in the natural space. Unlike previous studies based on input-output analyses, the proposed model, which can describe the circulation of natural and social spaces, considers the time course of natural purification. We believe that the proposed model is more realistic than the previous ones and is more suitable to describe the real society.

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1. Introduction

Since the industrial revolution, humans have developed a prosperous, civilized society through the exploitation of global resources and mass production, consumption, and disposal. However, the observed global environmental problems indicate that these activities have distorted the Earth's resource circulation. To create a more sustainable society, humans must induce a smooth resource circulation. To this end, Ohno et al. proposed a conceptual resource circulation model (Ohno et al. 1992). To analyze their model quantitatively, we further developed it into a probability model of resource domain transition (Yamashita and Jung 2010). This can predict the state vector at time t and the steady-state state vector at time $t \rightarrow \infty$ if the initial state vector ($t = 0$) and the domain transition probability matrix are given. Further, it can predict the future state of the global environment macroscopically.

This resource circulation model is based on the description of the resource circulation between the natural space and the social space on Earth through domain transitions between four macroscopic areas, namely, the resource, production, consumption, and waste domains. Considering smooth resource circulation on the Earth, reducing the gap between ideal and real conditions of each domain transition can be an approach to solve global environmental problems. We defined the state wherein the intake, expulsion, and natural purification rates are equal as the ideal state for achieving smooth resource circulation; further, we evaluated the deviation degree of these three ideal and real elements quantitatively. After examining the production and sales activities in a company (Yamashita et al. 2010; Yamashita 2013), we established a synchronization deviation degree index considering the synchronization of production and sales, and we proposed the four-element deviation degree index. This comprehensive index considers both social and natural spaces. Next, to confirm the validity and effectiveness of this index, we considered simple numerical examples for the initial state vector and the domain transition probability matrix. Then, the relationships among the three-element deviation degree, the four-element deviation degree, the synchronization deviation degree, the 3R matrix (where 3R indicates the reduce, reuse, and recycling activities, as proposed by Yamashita et al. 2011), the time-series change of the state vector, and the steady-state state vector were analyzed. Based on the model, we suggest a number of strategies to solve some of the global environmental problems.

2. Previous Research

In this chapter, we describe the previous research concerning natural resources and environmental issues related to our research. For example, the issue of exhaustible resources has been discussed for some time. Sraffa described the natural resources used in production, e.g., land and mineral deposits (Sraffa 1960), and several scholars have subsequently studied the theory of exhaustible resources using Sraffian or classical frameworks (Parrinello 2001, 2004; Schefold 2001; Ravagnani 2008; Kurz and Salvadori 1997, 2000, 2001, 2006). In recent years, significant efforts have been made to regulate the use of resources and the production of pollution in most industrialized countries; the stringency of pollution regulations has continued to increase all over the world. Managi, S. and Halkos, G. (Managi and Halkos 2015) investigated how recent technological advances contribute to the environment, resource, and infrastructure management.

The input-output (IO) model is used to analyze issues with resources macroscopically. Duchin, F. and Levine, S. H. (Duchin and Levine 2010) described the resource and product flows by combining a Markov chain with the IO model. Nakamura et al. have proposed the waste IO (WIO) model, which uses material flow data to incorporate the waste and waste-treatment sectors into the IO model (Nakamura et al. 2002, 2005, 2007, 2009). Many other researchers have used the WIO model (Ayres and Kneese 1969; Converse 1971). Many studies using the multi-regional WIO (MRWIO) model considering Japanese data have been published by Kagawa and colleagues (Kagawa et al. 2007). WIO models have been applied to many aspects of waste generation and management including waste metal flows (Nakamura and Nakajima 2005), the demand for landfills (Nakamura 2000), wastewater treatments (Li et al. 2013), and the recycling of home electrical appliances (Nakamura and Kondo 2006).

However, no research that describes the resource problem considering the whole cycle from production to resources has been conducted. In this research, we focus on the natural purification and the effect of the transition from the emission to the resource domains, and we describe this problem as a circulation model. We suggest that waste that is no longer needed in social spaces be naturally purified and turned into resources over a long time in natural spaces. In the next section, we explain the conceptual model of resource circulation determined in our research.

3. Research Methodology

Figure 1 shows a flowchart of our research.

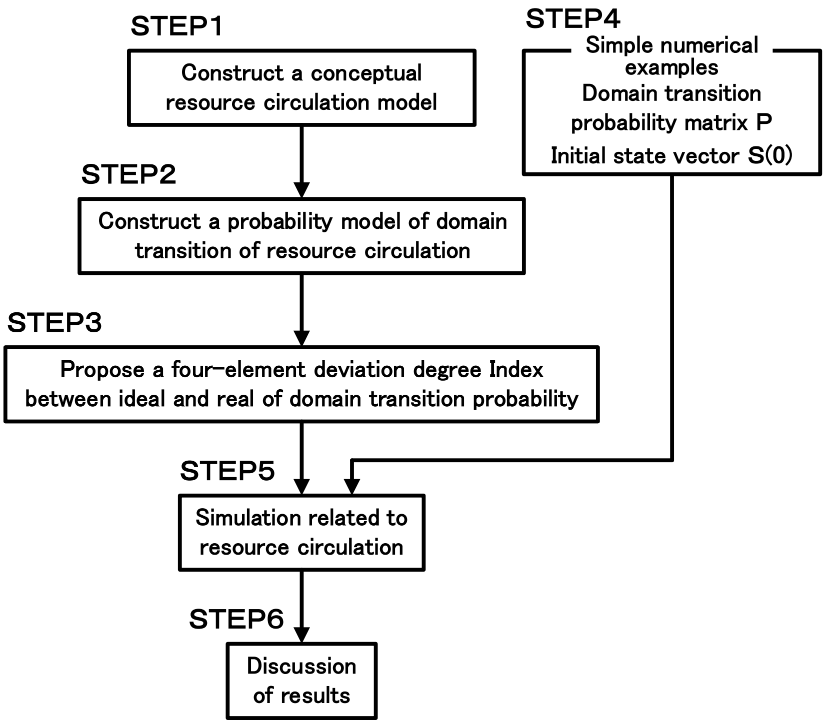

Figure 1. Flowchart of our research

Below, we describe each step.

STEP 1:
We construct a conceptual resource circulation model based on previous research to facilitate the understanding of resource issues. The details are described in Chapter 4.

STEP 2:
We convert the conceptual resource circulation model into a model of the probability of resource domain transition based on previous research to quantify resource issues. The details are described in Chapter 5.

STEP 3:
We propose a four-element deviation degree index between ideal and real domain transition probabilities to achieve a smooth resource circulation between natural and social spaces. The details are described in Chapter 6.

STEP 4:
We provide simple numerical examples considering a practical problem between natural and social spaces on Earth. The details are described in Chapter 7.

STEP 5:
We simulate the relationship between the intake, expulsion, natural purification, and synchronization between production and sales in corporate activities (four-element) by using simple numerical examples. The details are described in Chapter 7.

STEP 6:
We analyze the results of the simulation and discuss a new relationship in resource circulation. The details are described in Chapter 8.

4. Conceptual Resource Circulation Model

In this study, we consider issues with resources through a circulation model. There are various issues leading to the global resource problem, such as oil depletion (McCay et al. 2013), diminishing mineral resources (Halada et al. 2008), and water scarcity (Freire et al. 2001), and various efforts are currently being made to solve these issues. IN particular, the limits of oil extraction are being explored (Bettini and Karaliotas 2013), rare metals are mined from urban mines (Halada et al. 2009; Binnemans et al. 2013; Wang et al. 2017), and 3R (reduce, reuse, and recycle) activities are explored considering the life cycle (Arora et al. 2019; Hosoda 2007; Itoh et al. 2010; Yoshida et al. 2007). Ohno et al. proposed the macroscopic conceptual resource circulation model depicted in **Figure 2**. In this model, the global resource circulation was described as a process involving transitions between domains; for example, the resources transition from the resource domain to the production, intake, and waste domains, and then back to the resource domain. As shown in the conceptual model presented in **Figure 2**, in the societal space, producers (production domain) and consumers (intake domain) use resources from the natural space to attain value in alignment with their own goals and expel these resources back into the natural space after their value has been extracted. In contrast, the principles of purpose, including the attainment of profit and satisfaction, do not exist in the natural space, which consists of the resources that are used by the production domain in the societal space and the waste domains (Yamashita 2003).

From the conceptual model depicted in **Figure 2,** it is clear that a large portion of the environmental problems we are facing today involve the waste domain. Historically, the natural space has maintained a balance between the resource and waste domains through self-purification. However, with the advent of the industrial revolution, the natural purification process has not kept up with the speed of intake and expulsion, and the balance has been lost. The essence of the environmental problems we face today lies in the loss of balance between the resource and waste domains in the natural space, which is due to the shrinkage of the resource domain and the enlargement of the waste domain.

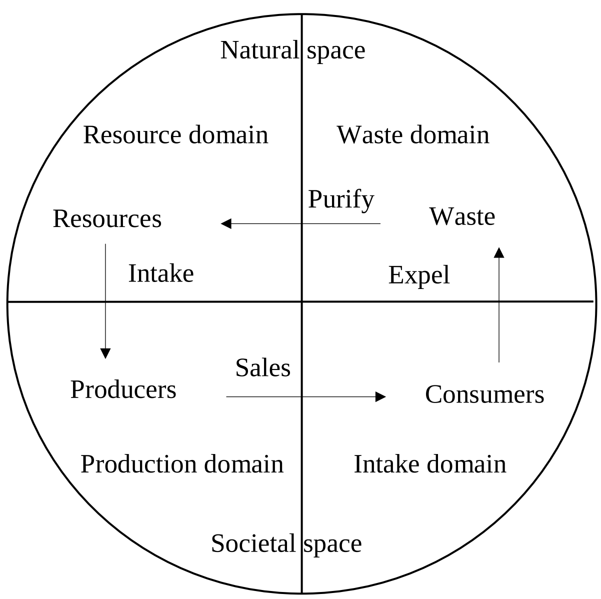

Figure 2. Conceptual illustration of the resource

Nevertheless, many efforts have been made to develop production and intake activities in societal spaces at minimal costs by expelling resources with exhausted value into natural spaces. In this way, over a long time, the waste not needed in societal spaces is purified within natural spaces (natural purification), such that the balance between the resource and waste domains is maintained, and the supply of resources needed by society is secure (Yamashita 2003). To that end, the speed of intake, expulsion, and purification (natural purification) must be equal. Considering this point, Yamashita proposed real and ideal three-element deviation degree indices, which will be discussed later.

After the industrial revolution, the rate of resource intake and expulsion has increased rapidly, and it has been increasingly difficult for the rate of natural purification to keep up with the said rate. Therefore, considering the rate of conversion from waste to natural resources, it is clear that artificial purification is required to purify resources that are insufficiently processed by natural purification alone. Artificial purification is a human activity within societal spaces, and it is a responsibility that must be addressed by both producers and consumers to reduce global environmental problems.

Furthermore, as typical efforts necessary to enhance environmental improvement, the 3Rs, i.e., reduce, reuse, and recycle, as well as artificial purification, play a significant role in reducing the resource domain and suppressing the expansion of the waste domain in natural spaces. The "reduce" activity reduces the intake from the natural space, whereas the "reuse" and "recycle" activities suppress the expulsion into the natural space.

5. Probability Model of Resources Domain Transition

To quantitatively describe the composition ratios (resource, inventory, utilization, and expulsion rates) of the four domains depicted in the conceptual model (**Figure 2**), Yamashita et al. proposed a probability model of resource domain transition.

In this model, $a(t), b(t), c(t)$ and $d(t)$ denote the resource, inventory, utilization, and expulsion rates at time t, respectively. An environmental state vector $\boldsymbol{S}(t) = (a(t), b(t), c(t), d(t))$ is constructed from each of these elements, which is an analytic model for a Markov chain in which the transition from domain i to domain j follows the domain transition probability matrix $\boldsymbol{P} = \left(P_{ij}\right)$, as represented in Equation (1). $\boldsymbol{S}(t)$ represents the state vector at time t. In the equation, p_1 is the rate of intake from the resource domain and transition into the production domain; p_2 is the rate of production sales synchronization from the production domain and transition into the intake domain; p_3 is the rate of expulsion from the intake domain and transition into the waste domain; and p_4 is the purification rate (natural purification rate) from the waste domain and transition into the natural resource domain.

$$P = \begin{pmatrix} 1-p_1 & p_1 & 0 & 0 \\ 0 & 1-p_2 & p_2 & 0 \\ 0 & 0 & 1-p_3 & p_3 \\ p_4 & 0 & 0 & 1-p_4 \end{pmatrix} \qquad (1)$$

If we assume that the domain transition probability matrix $\boldsymbol{P}$ is constant with respect to time t, then $\boldsymbol{S}(t + 1)$, which is the state vector at time $t + 1$, can be represented by the following equation:

$$\boldsymbol{S}(t+1) = \boldsymbol{S}(t) \cdot \boldsymbol{P} \qquad (2)$$

Let $\boldsymbol{S^*}$ represent the state vector $\boldsymbol{S}(\infty)$ after these domain transitions are repeated infinitely many times ($t \rightarrow \infty$). The value of $\boldsymbol{S}(t)$ will form a Markov chain and converge to the steady-state state vector $\boldsymbol{S^*}$, which satisfies Equation (3) if the state vector $\boldsymbol{S}(t)$ converges without periodicity (Morimura and Takahashi 1999). $\boldsymbol{S^*}$ can be calculated by solving the following four-dimensional first-order equation:

$$\boldsymbol{S}^* = \lim_{t\rightarrow\infty} \boldsymbol{S}(0) \cdot \boldsymbol{P}^t = \boldsymbol{S}^* \cdot \boldsymbol{P} \qquad (3)$$

Furthermore, Yamashita et al. (Yamashita et al. 2011) focused on the 3R activities, which are common measures for improving the environment, and described their effect using the matrix $\boldsymbol{R}$:

$$R = \begin{pmatrix} r_1 & -r_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & r_3 & r_2 & -r_2-r_3 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad (4)$$

where r_1, r_2, and r_3 are the reduce, reuse, and recycle rates, respectively.

If we include Equation (4) in the resource domain transition probability model described earlier, Equation (3) can be expanded to Equation (5) below (domain transition probability model using the 3R matrix). Therefore, if we replace matrix $\boldsymbol{P}$ in Equation (3) with matrix $\boldsymbol{P} + \boldsymbol{R}$, then the steady-state state vector $\boldsymbol{S}^*$ can be calculated by solving the following four-dimensional first-order equation (similar to Equation (3)):

$$\boldsymbol{S}^* = \lim_{t \to \infty} \boldsymbol{S}(0) \cdot (\boldsymbol{P} + \boldsymbol{R})^t = \boldsymbol{S}^* \cdot (\boldsymbol{P} + \boldsymbol{R}) \quad (5)$$

6. Deviation Degree between Ideal and Real Indices based on Domain Transition Probabilities

6.1 Three-Element Deviation Degree Index

It is well-known that the current global environment is in a poor state. To improve this, it is necessary to regain the smooth circulation of resources between the natural and societal spaces. Using the domain transition probability matrix (Equation (1)), the smoothness of the resource circulation can be obtained by maintaining an equilibrium state among the elements P_{12} (intake rate p_1), P_{34} (expulsion rate p_3), and P_{41} (natural purification rate p_4). In particular, if these three elements are equal, the natural domain can be balanced without enlarging the waste domain or reducing the intake domain. Therefore, the focus is on these three elements (intake, expulsion, and natural purification rates), and real and ideal three-element vectors (Equations (6) and (7), respectively) are proposed as follows:

$$\boldsymbol{c}' = (c_1, c_2, c_3) = (p_1/(p_1 + p_3 + p_4), p_3/(p_1 + p_3 + p_4), p_4/(p_1 + p_3 + p_4)) \quad (6)$$

$$\boldsymbol{d}' = (d_1, d_2, d_3) = (1/3, 1/3, 1/3) \quad (7)$$

Here, the sums of the elements of the two vectors [real three-element vector $\boldsymbol{c}$ and ideal three-element vector $\boldsymbol{d}$, (Equations (6) and (7), respectively)] is 1, and all the elements of the ideal three-element vector $\boldsymbol{d}$ are equal to $1/3$. This is because, as described before, it is considered ideal for the global environment to maintain the equilibrium between the intake rate p_1, the expulsion rate p_3, and the natural purification rate p_4.

Then, to evaluate the deviation between the three ideal and real elements quantitatively, we propose the three-element deviation degree β based on the Kullback–Leibler divergence, as represented in Equation (8). The variable β is an indicator that quantifies the deviation between the ideal and real values of those elements of the three-element domain transition probability matrix that have a particularly significant effect on the global environment, namely the intake, expulsion, and natural purification rates:

$$\begin{aligned} \beta = \sum_{i=1}^{3} d_i \cdot \log(d_i/c_i) &= -\log 3 - (1/3) \cdot \{(\log p_1 + \log p_3 + \log p_4) - \log(p_1 + p_3 + p_4)\} \\ &= -\log 3 + \{\log(p_1 + p_3 + p_4) - (1/3) \cdot \log(p_1 \cdot p_3 \cdot p_4)\} \quad (8) \end{aligned}$$

When $\boldsymbol{c}$ and $\boldsymbol{d}$ are equal, β is minimized to 0, and, as the deviation between the elements c_i and d_i increases, the value of β increases. This facilitates a quantitative understanding of the deviation of the real intake, expulsion, and natural purification rates from their corresponding ideal states.

6.2 Four-Element Deviation degree Index

Next, we focus on the synchronization of production and sales. The production and sales activities from the production domain to the consumption domain correspond to corporate activities in the societal spaces. The probability of transition from the production domain to the consumption domain is the synchronization rate between the production and sales, p_2. By aiming at synchronized production activities (smart synchronization), i.e., just-in-time production, in societal spaces, the ideal value for the production–sales synchronization rate is $p_2 = 1$. In other words, if we consume resources of limited value from the natural domain and develop production activities, products or part of them should be used by consumers ($p_2 = 1$), rather than go to inventories ($p_2 \neq 1$). Based on this idea, we calculated the deviation between the ideal and real values for synchronization.

Calculating the synchronization deviation degree between production and sales, γ, based on the Kullback–Leibler divergence, we obtain the following equation:

$$\gamma = 1 \cdot \log(1/p_2) = -\log p_2 \qquad (9)$$

When production and sales are completely synchronized, i.e., when $p_2 = 1$, γ becomes 0, i.e., the synchronization deviation degree γ is minimized.

To combine β with γ to obtain an aggregate total four-element deviation degree *T*, the two parameters are weighted by $3/4$ and $1/4$, respectively, as follows:

$$\begin{aligned} T &= (3/4) \cdot \beta + (1/4) \cdot \gamma \\ &= (3/4) \cdot [-\log 3 + \{\log(p_1 + p_3 + p_4) - (1/3) \cdot \log(p_1 \cdot p_3 \cdot p_4)\}] - (1/4) \cdot \log p_2 \\ &= (3/4) \cdot \log\{(p_1 + p_3 + p_4)/3\} - (1/4) \cdot \log\left(\prod_{i=1}^{4} p_i\right) \qquad (10) \end{aligned}$$

This is the four-element deviation degree proposed in this research. By adding the production sales synchronization ratio, which is the ratio of the state of production activity to the intake, expulsion, and natural purification rates, the deviations between the ideal and real values can be evaluated for the entire resource circulation.

7. Simulation results considering a simple numerical example

In this section, by considering a simple numerical example for the domain transition probability matrix $\boldsymbol{P}$, initial state vector $\boldsymbol{S}(0)$, and 3R matrix, we analyze the relationship between β, T, γ, and $\boldsymbol{S}(t)$. In doing so, while considering a practical problem, we set the intake rate p_1 and expulsion rate p_3 to equal values to simplify the problem (see Cases 1–5 below). Because the speed of natural purification is low, the value of p_4 is set to a smaller magnitude. Moreover, all the cases discussed have a common initial state vector $\boldsymbol{S}(0)$, set to the following values:

$$\boldsymbol{S}(0) = (0.8, 0, 0.1, 0.1) \qquad (11)$$

By modulating the real three-element vector $\boldsymbol{c}'$, the natural purification rate p_4 was set in descending order over Cases 1 to 5, as shown below. The vectors $\boldsymbol{c}'$ in Cases 4 and 5 are equal, but the values of p_1, p_3, and p_4 in Case 5 are lower than those in Case 4. This will be described in the discussion section (Chapter 8). This is because the simulation results are compared for equal deviation degree and different element values.

Case 1 : $\boldsymbol{c}' = (0.49,\ 0.49,\ 0.02)$ $(p_1 = 0.02,\ p_3 = 0.02,\ p_4 = 0.000816)$

Case 2 : $\boldsymbol{c}' = (0.495,\ 0.495,\ 0.01)$ $(p_1 = 0.02,\ p_3 = 0.02,\ p_4 = 0.000404)$

Case 3 : $\boldsymbol{c}' = (0.499,\ 0.499,\ 0.002)$ $(p_1 = 0.02,\ p_3 = 0.02,\ p_4 = 0.00008)$

Case 4 : $\boldsymbol{c}' = (0.4995,\ 0.4995,\ 0.001)$ $(p_1 = 0.02,\ p_3 = 0.02,\ p_4 = 0.00004)$

Case 5 : $\boldsymbol{c}' = (0.4995,\ 0.4995,\ 0.001)$ $(p_1 = 0.01,\ p_3 = 0.01,\ p_4 = 0.00002)$

7.1 Results of simulation (Numerical Example 1)

In this example, we examine the transition of the environment vector $\boldsymbol{S}(t)$ when the production and sales synchronization rate p_2 is set to 0.8 with respect to the real three-element vector $\boldsymbol{c}'$ in Cases 1–5; $\boldsymbol{S}(t) = (a(t), b(t), c(t), d(t))$.

Here, β, T, and γ can be represented as follows:

(1) Case 1: $\beta = 0.6810$, $T = 0.5666$, $\gamma = 0.2231$
(2) Case 2: $\beta = 0.9052$, $T = 0.7347$, $\gamma = 0.2231$
(3) Case 3: $\beta = 1.4364$, $T = 1.1336$, $\gamma = 0.2231$
(4) Case 4: $\beta = 1.6667$, $T = 1.3061$, $\gamma = 0.2231$
(5) Case 5: $\beta = 1.6667$, $T = 1.3061$, $\gamma = 0.2231$

The simulation results are shown in **Figure 3** and **Table 1**.

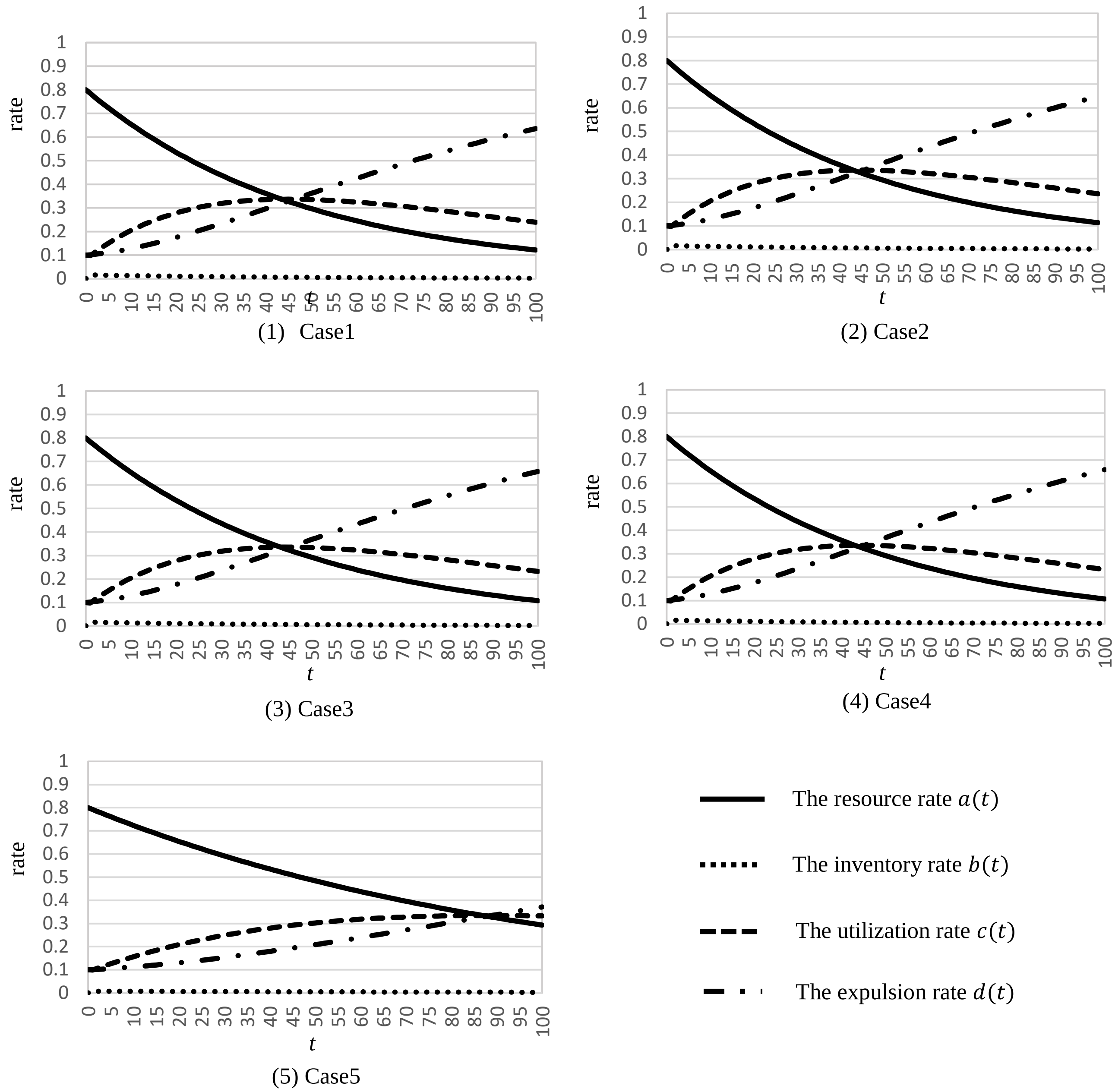

Figure 3. Transition of the environment vector $\boldsymbol{S}(t) = (a(t), b(t), c(t), d(t))$ (Cases 1–5, t = 0–100)

Table 1 Steady-state state vector $\boldsymbol{S*}$ in Cases 1–5

Steady-state vector $\boldsymbol{S*}$ / Numerical examples	Resource rate $a(\infty)$	Inventory rate $b(\infty)$	Utilization rate $c(\infty)$	Expulsion rate $d(\infty)$
Case 1	0.0377	0.0009	0.0377	0.9237
Case 2	0.0194	0.0005	0.0194	0.9607
Case 3	0.0040	0.0001	0.0040	0.9920
Case 4	0.0020	0.00004	0.0020	0.9960
Case 5	0.0020	0.00002	0.0020	0.9960

7.2 Results of simulation (Numerical Example 2)

In this example, we explore the transition of the environmental vector $\boldsymbol{S}(t)$ when the production and sales synchronization rate p_2 is changed from 0.6 to 0.8 and 1 for the real three-element vector $\boldsymbol{c}'$ in Case 1, where β, T, and γ can be represented as follows:

① Case 1 ($p_2 = 0.6$): $\beta = 0.6810, \quad T = 0.6385, \quad \gamma = 0.5108$
② Case 1 ($p_2 = 0.8$): $\beta = 0.6810, \quad T = 0.5666, \quad \gamma = 0.2231$
③ Case 1 ($p_2 = 1$) : $\beta = 0.6810, \quad T = 0.5108, \quad \gamma = 0$

Fully synchronized production is realized when the production and sales synchronization rate p_2 is 1. Companies aim at achieving a p_2 of 1, but this is difficult in practice. In this study, we simulate this by changing p_2 from 0.6 to 1. The large difference in speed between production activities in a societal space (production domain →consumption domain) and natural purification in a natural space (waste domain → resource domain) causes resource depletion. To describe this, there is a considerable difference between the values of the production and sales synchronization rate p_2 and the natural purification rate p_4.

The simulation results are shown in **Figure 4** and **Table 2**.

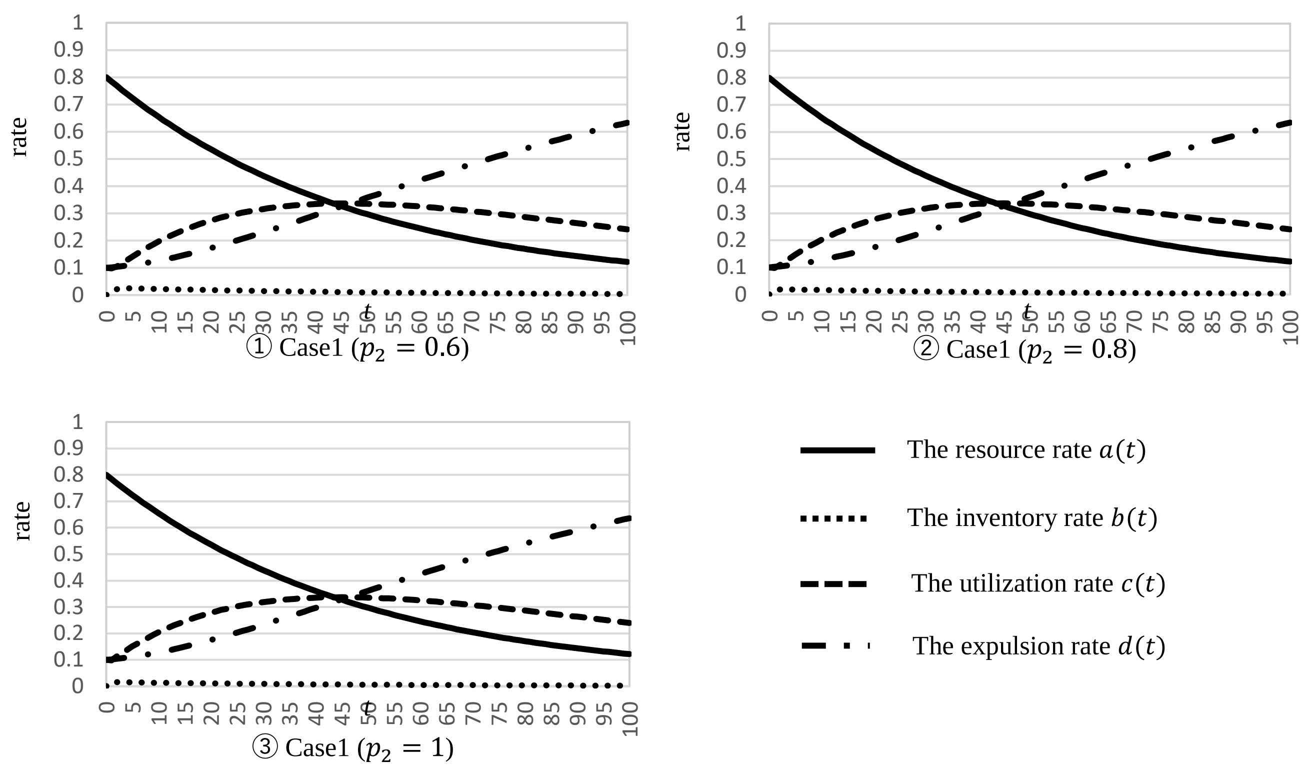

Figure 4. Transition of the environment vector $\boldsymbol{S}(t) = (a(t), b(t), c(t), d(t))$; Case 1 ($p_2 = 0.6, 0.8, 1$), t = 0–100

Table 2 Steady-state state vector $\boldsymbol{S}^*$ in Case 1 ($p_2 = 0.6, 0.8, 1$)

Steady-state vector $\boldsymbol{S}^*$ / Numerical examples	Resource rate $a(\infty)$	Inventory rate $b(\infty)$	Utilization rate $c(\infty)$	Expulsion rate $d(\infty)$
Case 1 ($p_2 = 0.6$)	0.0377	0.0013	0.0377	0.9234
Case 1 ($p_2 = 0.8$)	0.0377	0.0009	0.0377	0.9237
Case 1 ($p_2 = 1$)	0.0377	0.0008	0.0377	0.9239

7.3 Results of simulation (Numerical Example 3)

In this example, we introduce the 3Rs to the vector $\boldsymbol{c}'$ in Case 1. The production sales synchronization ratio p_2 was fixed as 0.8 and the reduce rate r_1 and re-use rate r_2 were set to 0.005 and 0.001, respectively. Then, we examined the transition of the environmental state vector $\boldsymbol{S}(t)$ when changing the recycle rate r_3 from 0.005 to 0.01 and 0.015. Here, β, T, and γ can be represented as follows:

(a) Case 1 ($r_3 = 0.005$): $\beta = 0.6810, \quad T = 0.5666, \quad \gamma = 0.2231$
(b) Case 1 ($r_3 = 0.01$) : $\beta = 0.6810, \quad T = 0.5666, \quad \gamma = 0.2231$
(c) Case 1 ($r_3 = 0.015$): $\beta = 0.6810, \quad T = 0.5666, \quad \gamma = 0.2231$

The reduction rate r_1 is set under the condition that it must be smaller than the original intake rate p_1. On the other hand, as recycling is actively performed mainly by manufacturing companies, the recycling rate r_3 is set to a value slightly higher than the reuse rate r_2. However, the reuse rate r_2 is expected to gradually increase in the future owing to the expansion of the flea market.

The simulation results are shown in **Figure 5** and **Table 3**.

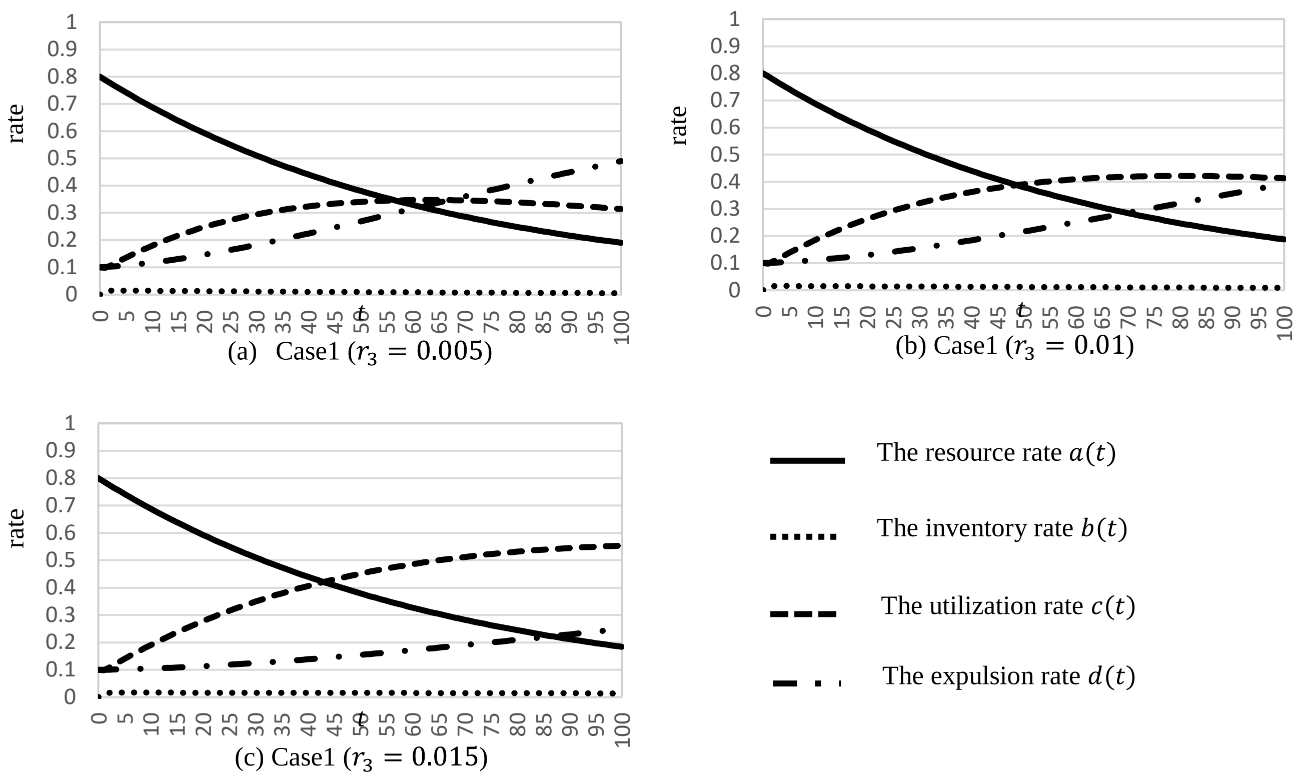

Figure 5. Transition of environment vector $\boldsymbol{S}(t) = (a(t), b(t), c(t), d(t))$; Case 1 ($r_3 = 0.005, 0.01, 0.015$), t = 0–100

Table 3 Steady-state state vector $\boldsymbol{S^*}$ in Case 1($r_3 = 0.005, 0.01, 0.015$)

Steady-state vector $\boldsymbol{S^*}$ / Numerical example	Resource rate $a(\infty)$	Inventory rate $b(\infty)$	Utilization rate $c(\infty)$	Expulsion rate $d(\infty)$
Case 1 ($r_3 = 0.005$)	0.0488	0.0012	0.0523	0.8976
Case 1 ($r_3 = 0.01$)	0.0474	0.0019	0.0790	0.8717
Case 1 ($r_3 = 0.015$)	0.0431	0.0038	0.1615	0.7916

8. Discussion

From **Figure 3** and **Table 1**, it can be seen that, in all the cases, the resource rate $a(t)$ decreases and the expulsion rate $d(t)$ increases with time *t*. Specifically, the rate of decrease in $a(t)$ and the rate of increase in $d(t)$ are particularly high in Cases 4 and 5, in which the deviation between the ideal and real values is large. This indicates a future crisis for the planet. The utilization rate $c(t)$ increases after the initial time t (where t is small) and then decreases. The speed of transition from production to intake domains in societal spaces is high, and the intake domain initially increases. However, a decrease in the rate of purification (natural purification rate p_4), probably due to the unavailability of resources, leads to a lack of necessary resources. Moreover, the value of p_4 is low compared to the inventory rate $b(t)$, and, compared to $c(t)$, it begins to decrease from an earlier stage (t = 2–4).

In Cases 4 and 5 (in **Figure 3** and **Table 1**), β is equal; however, this is because the ratio of the three elements is equal. It should be noted that the values of the three elements themselves differ in Cases 4 and 5. When compared to the elements of the steady-state state vector $\boldsymbol{S*}$, they are almost equal despite differences in the early stages. At t = 0–100, β is constant and, although the values of the three elements themselves differ, the elements of $\boldsymbol{S*}$ in both Cases 4 and 5 are similar. In these cases, the four-element deviation degree T is also equal. Accordingly, we believe that it is meaningful to investigate the degree of deviation of real values from the ideal values as an indicator.

From **Figure 4** and **Table 2** it can be seen that, as the production and sales synchronization ratio p_2 increases from 0.6 to 0.8 and 1, $b(t)$ and T decrease whereas β remains constant. As the synchronization progresses, the inventory is reduced through efficient production activity. The variable T represents the degree of deviation between ideal and real resource circulation, including production activity, and is a comprehensive index that includes both societal and natural spaces. Moreover, as p_2 increases, $d(t)$ increases slightly. This suggests that, when the rate of transition from the production domain to the consumption domain increases through efficient production activity, the waste domain expands, and the environment is negatively affected.

From **Figure 5** and **Table 3**, it can be seen that, as the recycling rate r_3 increases, $b(t)$ and $c(t)$ increase. This indicates that, when recycling is performed, the transition of materials between the production and consumption domains in the societal space becomes more active. Further, $d(t)$ is observed to decrease as r_3 increases.

Each simulation result shows the effect of slow natural purification from the waste domain to the resource domain in the natural space. This phenomenon could not be described by the model with only the IO table, as in previous studies, but it was successfully described by the cyclic model used in this research.

9. Conclusion

We developed a three-element deviation degree as an indicator to quantitatively evaluate the deviation between the ideal and real values of the intake, expulsion, and natural purification rates in the theoretical model of resource circulation. By focusing on the synchronicity of transfers between the production and intake domains in the societal space, we proposed a four-element deviation degree that facilitates the evaluation of the production and sales synchronization rate, in addition to the intake, expulsion, and natural purification rates, and the consideration of the production activity of companies in the societal space. Moreover, by adopting the 3R matrix, we also investigated the effects of the reduce, reuse, and recycle environmental activities.

To confirm the validity and effectiveness of this index, we considered simple numerical examples for the initial state vector $\boldsymbol{S}(0)$ and the domain transition probability matrix. Then, the relationship between the three-element deviation degree, the four-element deviation degree, the synchronization deviation degree, the 3R matrix (a matrix representing 3R activity—reduce, reuse, recycle), the time-series change of the state vector $\boldsymbol{S}(t)$, and the steady-state state vector $\boldsymbol{S*}$ was analyzed. Based on the results, suggestions for solving global environmental problems were proposed.

① In all numerical examples, the resource rate $a(t)$ decreased and the expulsion rate $d(t)$ increased with time t.

② As efficient production activities are realized by synchronizing production activities, the production sales synchronization ratio p_2 increases (the synchronization deviation degree reduces); thus, the inventory rate $b(t)$ decreases.

③ As the recycling rate r_3 increases, the inventory rate $b(t)$ and utilization rate $c(t)$ increase, and the production activities in the societal space become more active. Simultaneously, the expulsion rate $d(t)$ decreases; thus, increased recycling has a positive impact on the environment.

④ In Cases 4 and 5 (in **Figure 3** and **Table 1**), when the three- and four-element deviation degrees are equal, although the values of the three or four elements themselves differ, the elements of the steady-state state vector $\boldsymbol{S^*}$ remain similar.

Unlike previous studies based on IO analysis, the model proposed in this research, which can describe the circulation of resources in natural and social spaces, considers the time course of natural purification. Therefore, it is more realistic and can describe the dynamics of the actual society more accurately. In the future, we will increase the accuracy of the results by performing simulations using more realistic numerical examples.

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