$O_{kj} \geq z_{kj} - (1 - x_j) * MM, \forall\, k \text{ and } j$ (B.6)

3. The variable should be less than or equal to the upper bound that is a location k is covered by fire station j.

$O_{kj} \leq z_{kj}\ , \forall\, k \text{ and } j$ (B.7)

4. The total coverage of each TCR location should be more than one. That is to ensure each fire station is only opened if it covers a TCR location.

$\sum_j^M\ O_{kj} \geq 1,\ \forall\, k$ (B.8)

5. The total coverage of each fire station should be chosen to cover the most of TCR locations

$\sum_k^R\ O_{kj} \leq x_j * MM\ ,\ \forall\, j$ (B.9)

6. The normal binary constraint applies here since O_{kj} is bounded by a linear combination of two binary variables. Therefore, any realization should be one or zero.

$O_{kj} = \{0,1\}\ \ , \forall\, k \text{ and } j$ (B.10)

Subsequently, the linear formulation is shown below:

$Maximize\ T = \sum_{j=1}^{M}\sum_{i=1}^{N} w_i * qp_{ij} * y_{ij} + \sum_{j=1}^{M}\sum_{k=1}^{R} w_k * qs_{kj} * z_{kj}$ (B.11)

Subject to

$\sum_{j=1}^{M} a_{ij} * x_j - \sum_{j=1}^{M} y_{ij} \geq 0, \forall\, i$ (B.12)

$\sum_{j=1}^{M} b_{kj} * x_j - \sum_{j=1}^{M} z_{kj} \geq 0,\ \forall\, k$ (B.13)

$\sum_{j=1}^{M} x_j \leq P$ (B.14)

$t_{kj} \geq y_{kj} - (1 - x_j) * MM,\ \forall\, k \text{ and } j$ (B.15)

$t_{kj} \leq y_{kj}\ , \forall\, k \text{ and } j$ (B.16)

$\sum_{j=1}^{M}\ t_{kj} \geq 1,\ \forall\, k$ (B.17)

$\sum_{k=1}^{R}\ t_{kj} \leq x_j * MM\ ,\ \forall\, j$ (B.18)

$O_{kj} \geq z_{kj} - (1 - x_j) * MM,\ \forall\, k \text{ and } j$ (B.19)

$O_{kj} \leq z_{kj}\ , \forall\, k \text{ and } j$ (B.20)

$\sum_{j=1}^{M}\ O_{kj} \geq 2,\ \forall\, k$ (B.21)

$\sum_{k=1}^{R}\ O_{kj} \leq x_j * MM\ ,\ \forall\, j$ (B.22)

$x_j,\ y_{ij}, z_{kj}, t_{kj} O_{kj} = \{0,1\}\ \ , \forall\, i, j, k$ (B.23)

The constraints specified by (Eq. B.15), (Eq. B.16), (Eq. B.17), and (Eq. B.18) are the linearized constraints to cater for TCR locations being covered as primary coverage. The constraints specified by (Eq. B.19), (Eq. B.20), (Eq. B.21), and (Eq. B.22) are the linearized constraints to ensure the number of TCR locations covered twice. The last constraints are the usual binary constraints. The rest of the constraints and variables are unchanged from the model presented in section 3.1.

Biographies

Abdulaziz S. Alzahrani is currently working as Organization Performance Advisor at Saudi Aramco and has more than 9 years of total professional experience in various fields from Strategy and Organization design, Business Process Improvement, and Management Consulting. Abdulaziz earned his bachelor's degree in Industrial & Systems Engineering from King Fahad University of Petroleum & Minerals (KFUPM) in January 2013 and completed his master's degree in December 2019 from the same institution. He has also won the "Excellence in Academic Research" award in 2018 and 2019 at the Annual KFUPM Student Forum. Abdulaziz highlights his Professional Interests as Data Analytics, Workforce Optimization and Modelling, and Operations Research.

Dr. Ahmad Al Hanbali received his Phd in 2006 from University of Nice, France. Between 2007 and 2017, he was faculty in the department of Industrial engineering at Univeristy of Twente, The Netherlands. He is a full member of the International Society of Inventory Research (ISIR), Production and Operations Management Society (POMS). He is a former member of the Beta research school The Netherlands, Twente is Maintenance Excellence (TIME), Service Logistics Forum (SLF), and Dutch Network on the Mathematics of Operations Research (LNMB). He is a member of many technical program committees of international conferences. He served as a reviewer for several international journals.