A Meta-Heuristic Algorithm for Solving Economic Lot Scheduling Problem (ELSP)

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Abstract

Economic lot scheduling problem has been an important topic in production planning and scheduling research for more than four decades. The problem is known to be NP-hard due to its combinatorial nature. In this paper, a meta-heuristics algorithm based on Simulated Annealing (SA) - is proposed. Besides on explaining how to use SA, the effects of changing in production frequency on sequence and also on production lot are also considered. The result shows the reduction of cost.

Keywords
Economic Lot Scheduling Problem, Simulating Annealing, Common Cycle, Independent Solution

1. Introduction
It is more than four decades that the problem of determining economic lot size and production scheduling of several products on a single machine (ELSP) is considered and examined by many researchers. This problem was explained first by Roger [1]. In this problem demand rate is fixed and no shortage is allowed. Production rate is also fixed. Setup may be needed before starting the production of each product. Holding rate of each product is fixed and setup has cost too. Criterion to find size and production scheduling is the average of setup and holding cost.

Many usages and also difficulty to solve this problem in spite of its simple description are the main reasons to keep ELSP as ongoing research topic. Its complication has arisen from NP-hard nature of the problem. Consequently most of efficient solutions of this problem in new studies are based on innovative ways that mostly do not considered the issue of feasibility. The feasibility is arising by maintaining the capacity restriction so machine load should not exceed its capacity, and with no shortage, it is required to start its production before its turn.

Most of studies are based on a cycle planning policy and the program is repeated regularly. By considering this policy, two techniques can be chosen. One is using basic period (BP), the cycle of each product is a true multiple of this basic period and if the product is produced more than once in the cycle its production size will be equal. In this field, Almaghrebi [2] has presented a complete review until 1978. He has made his dynamic planning technique based on basic period. Davis [3] has presented an enumeration technique.

The other technique is selecting a cycle time, T, as the total period of the system. The first pioneer was Maxwel [4]. Some of products may be produced several times during a cycle and also the production size may vary. We also can call on Dobson [5]. The later technique has been used in this study too. Raza and Akgunduz [6] have conducted a comparative study of heuristic algorithms on Economic Lot Scheduling Problem. They have found that SA algorithm shows a faster convergence than other Meta heuristic algorithm such as Tabu search. Toress and Rogers [7] have shown that Genetic Algorithm also can be used for solving ELSP.

This paper is arranged in seven sections. In section 2 the problem is explained and formulated. In section 3, SA technique and its conformity with combined optimization issues are examined and in section 4, suggested heuristic algorithm based on SA is presented. In section 5, a numerical example for algorithm is introduced and in section 6, the manner of determining parameters of SA technique for solving the problems is examined and by using Pascal language the problem is solved. In section 7, results and suggestions are presented.

2. Problem formulation
ELSP can be indicated as follow. There are a single facility and many products. Required information for problem is as followed:

i – Different product's index $i = 1, \dots l$

P_i – Production rate of product i,

D_i – Demand rate for product i,

h_i – Holding cost of product i,

A_i – Setup cost of product i,

S_i – Setup time for product i,

Stock shortage is not allowed. The cycle time, T, and production scheduling, $f^1,\dots,f^n$ $(\forall f^j \in \{1,\dots,l\})$ with possible repetition are the decision parameters. Production times of $t^1,t^2\dots,t^n$ and idle times between each two successive production of $u^1,\dots,u^n$ are indicated in a way that the cycle repeats unlimited times. The demand should be satisfied and the total holding costs and setup cost should be minimized.

Subscripts are used to refer to the ith part; however superscripts are used to refer to the data related to the part produced, where data related to the part produced at jth position in the sequence. For example p_i means production rate of product i and p^j means the production rate of a product that is in the jth production position.

With this kind of formulation, production schedule of f = (1, 3, 2, 1, 2) shows that product 1 is produced two times, product 2 is produced two times and product 3 is produced only once during T cycle and f represents the sequencing. F is also the limited set of all possible sequences. Let L_k represents the set containing the products that are produced in a given sequence from k to the position in the sequence where the product k is produced again, but not included in the same cycle. Let J_i is a position in the sequence, which product i is produced, in other word $J_i = \{j \mid f^j = i\}$. With these notations, ELSP can be written as follow:

$$\underset{j\in J}{Min}\frac{1}{T}\left(\sum_{j=1}^{n}\frac{1}{2}h^j(p^j-d^j)\left(\frac{p^j}{d^j}\right)(t^j)^2+\sum_{j=1}^{n}A^j\right) \quad (1)$$

Subject to:

$$\sum_{j\varepsilon L_k}(t^j+S^j+u^j)=\left(\frac{p^k}{d^k}\right)(t^k) \quad k=1,\dots,l \quad (2)$$

$$\sum_{j=1}^{n}(t^j+S^j+u^j)=T \qquad t,u,T\geq 0 \quad (3)$$

Constraints (2) indicate production size without any shortage. Constraints (3) are the limitation of capacity.
To simplify the above model, for an assumed sequence, with f and m_i as production frequency of product i, we will have $\sum_{j=1}^{n}A^j=\sum_{i=1}^{l}m_iA_i$. So, the total holding cost is equal to:

$$\frac{1}{2}\sum_{i=1}^{I}H_i\frac{T^2}{m_i} \quad \text{Or} \quad \frac{1}{2T}\sum_{k=1}^{n}H_{f(k)}\left(\frac{t^kP_{f(k)}}{d_{f(k)}}\right)^2 \quad (4)$$

Which $f(k)=(f^1,f^2,\dots f^K)$ shows the assumed sequence and $n=\sum_{i=1}^{I}m_i$ is the sum of production runs or in other word is the sum of setup times and $H_{f(k)}$ is described in a way that $H_i=h_id_i(1-\frac{d_i}{P_i})$. Therefore the average total cost is equal to simplified form $C=[\sum_{i=1}^{I}m_iA_i+\frac{1}{2}\sum_{i=1}^{I}H_i\frac{T^2}{m_i}]$ or:

$$C=\left(\frac{1}{T}\right)\left[\left(\sum_{i=1}^{I}m_iA_i\right)+\frac{1}{2}\sum_{k=1}^{n}H_{f(k)}\left(\frac{t^kP_{f(k)}}{d_{f(k)}}\right)^2\right] \quad (5)$$

Constraints (2) can be rewritten as follow:

$$t=(I-P^{-1}L)P^{-1}L(S+U) \quad (6)$$

I is the unit matrix with dimensions of k × k, P is diameter matrix with $\frac{p_{f(k)}}{d_{f(k)}}$ as its elements and L is the occurrence matrix, a matrix with elements of 0 and 1 and with dimensions of k × k and S is the vector of k of setup times and u is the vector of k of idle time. Sum of idle times (3) can be rewritten as equation (7):

$$\sum_{k=1}^{n} u^k = T - \sum_{i=1}^{I}(m_i s_i + T\frac{d_i}{p_i}) \quad \text{or} \quad T \geq (\sum_{i=1}^{I} m_i s_i)/(1-\sum_{i=1}^{I}\frac{d_i}{p_i}) \qquad (7)$$

So, ELSP problem model now have been changed to minimizing the equation (5) with new constraints (6) and (7). This model is completely equivalent to minimizing the equation (1) with constraints (2) and (3).

3. SA technique

It is a new technique that is considered recently for approximate solution of difficult combined optimizing problems. This technique is based on statistical mechanics theory and similar to the behavior of physical systems in high temperature. One of SA's important features is to find answers with high quality that do not depend on beginning answer. Pre-cooking is a physical process that, high temperature is given to a solid object in order to change it to liquid and then temperature is reduced slowly. Cooling speed is so effective in the final shape of the object. Slow cooling gives suitable form to the object's crystals. If above system is in the dynamic equilibrium temperature such as T, then the possibility that the system is in i state $P\{E = E(i)\}$ depends on the energy of E (i).

Dynamic equilibrium is the main subject in the statistical mechanics and it is a state that particles exchange energy with each other accidentally. In this energy exchanges if ΔE the difference between current situation and previous situation is negative, in this case particles go to lower energy level and if ΔE is positive, the acceptance possibility of that energy level is equal to $\exp(-\Delta E / KT)$ which K is a constant and T is the fixed temperature.

Similarity between combined optimizing problems and the problem of finding the lowest level of energy in a physical system with particles that are exchanging energy, has been observed by KrikPatrik [8] for the first time and SA technique is made on the basis of this similarity. This recent technique is recalled as one of best solutions for combined optimizing problems. SA technique can be explained as follow: At first a high amount is given to control parameter C. The state of x (an initial beginning answer) is selected and another answer such as y is selected randomly in the neighborhood of x. let that $C(x,y) = C(x) - C(y)$ is changing in the cost resulted from this selection. In this case if $C(x,y) \leq 0$ the answer is accepted with possibility of $\exp\{-C(x,y)/c\}$ but if $C(x,y) \geq 0$; the answer will be accepted with the possibility of one. Control parameter reduces gradually until the system reaches to an equilibrium that is equal to optimum answer. This process is called "cooling process".

4. Heuristic Technique based on SA

As it was indicated for solving ELSP problem, the proposed SA technique will be used. At first an initial start solution should be found and the method of generating nearness answers should be specified.

4.1. Initial sequence generating approach

A random number is assigned to each product with Uniform distribution between 1 and K; U (1, K). This is the number of repetition of products in the working cycle. For example, consider three products of A, B and C. If the repetition numbers of these three products are 3 and 4 and 2 in the step 1, C, A, and B in ascending order according to repeated number is wrote.

Sequence (from left)	Repetition number	The action
C A B	2, 3, 4	Reduce the numbers
C A B C A B	1, 2, 3	Reduce the numbers
C A B C A B A B	0, 1, 2	Reduce the numbers
C A B C A B A B B	0, 0, 1	Delete the last B
C A B C A B A B		The result

4.2. Generating of neighborhood solutions

For finding of neighborhood solutions, one of the products is randomly selected and a new repetition random number between 1 and K is selected and again with previous method, a new sequence is found.

4.3. The method of solving the model and finding the beginning answer

ELSP is introduced in section (2). Table 1 shows the steps to find the solution and also find the relative costs. To avoid any mistake for T, cycle time with temperature in SA technique, from now on CYCLE is used instead of T.

4.4. Introducing decision parameters in suggested SA technique

Eight parameters are explained in this method that by an educated selecting them they ensure the speed of reaching to final answer and also its reliability. These parameters are presented in Table 2. Between above parameters, Y affects on distribution of accessible points that by changing it, the amount of q_i ; the repetition number of ith product; changes and the length of cycle time is modifies. Parameters of T and T_0 and g are in relation to accepting bad answer. By changing them, flexibility and dynamism of the algorithm is changed too. Four last parameters, TOTAL, ACCEPT, PERCENT and COUNT affect the accessible range of the problem and by changing them, examined points will be changed. If the amount of these four parameters changes in a way that accessible points reduce, it may not reach to final answer and also in another case if the accessible points increase it may take a long time to reach to the final answer.

5. Presenting a Numerical Example

Production size algorithms are usually compared with the problems that were solved by Bamberger [9] for the first time. Three kinds of problems that are presented by Bamberger define separately by using common basic information have multiplied demand rate by one index of a_1=1 or a_2=3 or a_3=4. As higher demand rate needs more capacity and as a result its programming is more difficult, so the demand with index a_3=4 has been chosen as a benchmark in this study.

Table 1: The steps to find the solution

A: with derivative of objective function find cycle* $(cycle)^* = \left((2\sum_{i=1}^{I} m_i A_i)/(\sum_{i=1}^{I} H_i / m_i) \right)^{1/2}$	D: find U^k . Idle time is considered positive and equal for all of products except for those are at the beginning of sequence. $U^k = \begin{cases} [Cycle - \sum (m_i S_i + (Cycle) d_i / P_i)] / n \\ 0 \quad \textit{if product is at the beginnig} \end{cases}$
B: find (Cycle) $_{\min}$ from below relation: $(cycle)_{\min} = (\sum_{i=1}^{I} m_i S_i)/(1 - \sum_{i=1}^{I} d_i / P_i)$	E: find production periods of t from below relation: $t = (I - P_{-1} L) P^{-1} L (S + u)$
C: choose the cycle $(cycle) = \max[(cycle)^*, (cycle)_{\min}]$	F: find the cost for this sequence from below relation. $C = (\frac{1}{cycle})[(\sum_{i=1}^{I} m_i A_i) + \frac{1}{2} \sum_{k=1}^{n} H_{f(k)} (\frac{t^k p_{f(k)}}{d_{f(k)}})^2]$

6. Determining Decision Making Parameters

At first the problem is solved by y = 3 with different amounts for other parameters and then by try and error an approximate range has been found. Then in a situation that there is minimum restriction the problem is solved 30 times and the best answer is considered as the initial solution to the problem. Then by changing parameters it is tried to control the problem as much as possible and reach to the final answer by using the minimum search in the range. Acceptance criteria for a special amount for parameters are:

1. The maximum reached number of the final answers in 30 times of solving the problem.
2. The minimum numbers of examined points in accessible space. After that acceptance range for y =3 has been reached, cases of y = 4 and 5 and … will be examined.

Table 2: Decision parameters in suggested SA technique

T: Initial temperature T_0: Freezing temperature $T < T_0$ *then stop*	ACCEPT: Total of acceptable points in a temperature.
g: Index of temperature change n : the number of temperature change levels $T_0 = g^n T$	PERCENT: The ratio of acceptable points to the total of produced points in a temperature.
y = Maximum level of repetition number.	COUNT: The number of steps which in a temperature the ratio of $\frac{ACCEPT}{TOTAL}$ becomes less than PERCENT.
If q_i is the repetition number of i[th] product then $q_i = U(1, y)$	TOTAL: Total points generated in a temperature

6.1. Temperature changing range and a rule for changing temperature

Try and error method will be used to find the amount of T and T_0 and g.

6.2. Determining the range of examinable space

Four parameters of TOTAL, ACCEPT, PERCENT and COUNT limit the solution space. At first the acceptable amount of one of them is considered fixed and others are found and then other parameters will be selected. Here the goal is to reduce the number of examined points. If by using the minimum points in the solution space the desired answer is reached, then:

1. Determine the upper level of parameter TOTAL. This parameter counts the number of examined points in a special temperature.
2. For determining the amount of ACCEPT, the parameter TOTAL is considered equal to 20. Then the problem is solved by changing the ACCEPT.

6.3. The rule of stop

Parameters of PERCENT and COUNT are connected to each other. By increasing PERCENT the speed rate of COUNT increased. In the situation of y = 3 because the number of acceptable points are high, changing in PERCENT does not have any effect on the problem so it is selected to be equal to 25. If the amount of PERCENT becomes more than 50, algorithm will stop suddenly. This situation is equal to cooling at once. COUNT parameter along with T_0 stop the algorithm and if it becomes too small, the problem losses its efficiency. The minimum level of this parameter will be 3.

6.4. Final selection

After the parameters were selected in the case of y = 3, by using the chose range and changing it, the amount of parameters for situations of y = 4 and 5 and …. have been calculated. The result is shown in Table 3.

Table 3: Selected parameters in case of Y ≥ 3

Y	T	T_0	g	TOTAL	ACCEPT	PERCENT	COUNT	# of checked points	# of product in the cycle
3	0.05	.001	0.85	20	8	25	3	340	14
4	0.05	.001	0.9	20	10	20	15	720	20
5	0.05	.001	0.9	7	20	10	15	2870	27
6	0.05	.001	-	-	-	-	-	3100	34

Because the number of checked points in case of y = 5 and y = 6 is nearly 9 times more than case of y = 3 and cycle length is more than 2 times of case of y = 3, the duration to solve the problem is nearly 15 times more than case of y = 3. Therefore the smallest range that reaches to final answer more than 50% of cases is accepted. For choosing y, final answer of the problem in each cases of y = 3, 4, 5, 6, 7, 8 have been found and with respect to the annual average cost y = 5 is selected. In this case the final answer has the lowest cost. In Table 4 final answers for above cases are presented.

7. Conclusion

It can easily be shown that independent solution is not acceptable [2]. IS method is not acceptable because it ignores required condition for feasibility. Of course this difficulty is removed by Lagrangian indexes [5].

Table 4(a): Final answers in different situation

Y	Number of selected points	Total of searched points	Number of products in each cycle	Cycle period	Cycle cost
3	262	1079	14	13.47	1092.7
4	230	1086	20	19.48	1022.79
5*	**255**	**1077**	**27**	**26.58**	**1008.87**
6	237	1079	34	33.31	1010.34
7	296	1066	39	38.39	1019.68
8	268	1078	34	33.31	1010.34

*Selected amount

Table 4(b): Final answer in different situations

Sequence	Y
234856719102348	3
2348597161023485923488	4
2348591016723485910234859234 8	5*
234859101672348591023485910234859234 8	6
248539101672485391024853910248539248539248	7
234859101672348591023485910234859234 8	8

*Selected amount

As it was discussed before, one ways in ELSP that insures feasibility is using common cycle. In this method cycle $_1$ = cycle $_2$ =… cycle $_l$ = cycle*. Solving the problem by using common cycle method that each product is produced only once has the annual cost of $1311.08. Common cycle is also 10.63 days. As it can be seen in Table 4, the minimum cost utilizing proposed method based on SA is $1008.87 which is less than cost of common cycle method.

The combination of production size and cycle time reaches to a better answer. As it is observed in Table 4, the cost in the case of using a common cycle on which each product can be produced more than once in each cycle is less than simple situation of common cycle. In other word the combination of finding production size and the sequence results in better answers. Of course it is seen that if the "y" of maximum level times that each product can be produced in a cycle, is equal to 5, the cost will have its minimum level.

Regarding application of this method, it can be said that the larger y the longer the cycle. In a dynamic environment, the length of cycle must be shorter. In other word final selection between 2, 3, 4 and 5 depends on the dynamism of environment.

At the end the following results can be presented:
1- SA technique is a suitable method for solving the problem of ELSP.
2- Combining scheduling and the production size (cycle period) bring better results in comparison to traditional method of common cycle.
3-In the presented model, capacity restriction with no shortage is considered simultaneously and the found sequence is feasible.

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