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Modification of Spur Gear Using Computational Method-Involutes Profile Being Modify

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Abstract

This Dissertation is a study characteristic of involutes gear system including contact stresses, bending stresses and the transmission error of the gear in mesh. To estimate the transmission error in the actual gear system which arises because of an irregular tool geometry or imperfect geometry or mounting? A model of a spur gear is used to study the effect of intentional tooth profile modifications by using two dimensional FEM. The deflection of teeth is calculated by using the bending and shear influence function. In this paper tooth relief modification is consider for profile modification, by using computational method.

Keywords
Spur gear, transmission error, tooth relief, profile modification, computational method.

1. Introduction

Gear geometry performs a very important role in gear designing process. An abrupt change in the cross-section may acts as the stress raiser leading to stress concentration and increases the amount of localized stress, or can decrease the transmission error and smooth engaging and much more. There are some critical section where the change in magnitude of the stress concentration depends on the size and shape of the section. For gear, it depends on the radius of curvature of the fillet at the critical section of the gear tooth. The objectives of this thesis is to use a numerical approach to develop theoretical models of the behavior of spur gears in mesh, to help to predict the effect of gear tooth stresses and transmission error. Actually, the main reason of noise and vibration is transmission error between master and slave gears, but the error must exist in any manufacture process. To decrease harmful noise and vibration, the most effective method is "Tooth Profile Modification", which is by tip relief or root relief for modifying geometry profile of gear tooth to regulate transmission error. In this paper, the transmission error of original model and modified model will be compared to show the gear profile modification, which will influence the transmission error.

According to Munro [7], the idea of "Transmission Error (TE)" was introduced first by Harris (1958) and, some years later, by Niemann and Baethge. Nevertheless, before the analytic definition of TE, it was universally recognized that tooth deformation under load during mesh affects the meshing kinematics, thus causing impacts between mating teeth, which are the main source of the acoustic emission of the transmission. Profile modifications were born from this recognition and are widely applied today, but these modifications are often based on empirical rules, or they are selected according to similar successful experiences. Therefore, the practice of applying profile modifications to reduce gear noise and vibration was introduced before the definition of TE, as confirmed by papers in the 1940s by H. Walker and D.W. Dudley, who proposed to use it "to reduce noise, friction and wear" [2,8]. Several Authors [2, 3, 4, 5, and 7] studied the correlation between TE and profile modification, in order to reduce the TE. Niemann [4] proposed long and short modifications. Today there are many theoretical and computational tools which can predict the transmission error of a gear during the design stage. These tools allow the designer to introduce appropriate profile modifications to reduce TE, not empirically, but on the basis of consolidated theories.

2.1 Nomenclature of gear

P	:	Diametral Pitch	e_v	:	Gear relief amount
b	:	Face width	SAP	:	Start Active Profile roll angle
h	:	Tooth height	EAP	:	End Active Profile roll angle
s	:	Coordinate along profile	M	:	Module
TE	:	Transmission Error	α	:	Pressure angle
PPTE	:	Peak to Peak TE	ν_e	:	Total amount of material removed
θ_P	:	Pinion start relief roll angle	r_b	:	Base circle radius
θ_G	:	Gear start relief roll angle	θ	:	Roll angle

2. Transmission Error

What is Transmission Error (TE)? Transmission Error (TE) is defined as the difference between the effective and the ideal position of the output shaft with reference to the input shaft. TE can be expressed either by an angular displacement or, more conveniently, as a linear displacement measured along a line of action at base circle according to [9], the denomination is based on the start point of tip relief modification along the profile. According to experimental results, gears with long modification have the minimum Peak to Peak Transmission Error (PPTE) and the minimum noise level at the design torque. At lower torque this optimum condition was not verified and an intermediate or short modification is suggested. In these works short and long modifications are considered to be technological limits and it is not useful to extend the relief to the pitch radius, as described in [10, 11]. Now days profile modification is named as Profile correction of the tooth. The profile correction factor is also called addendum modification coefficient [1]. The transmission error and the whining noise is produce by the following:

- Changing tooth load amplitude,
  - Changing load position along the tooth profile,
    - Changing tooth load direction.

In the present paper, instead, the start tip relief modifications (θ_P. θ_G) have been considered as design variables ranging from Start Active Profile roll angle (SAP) up to the End Active Profile roll angle (EAP). Total relief ν_e (at the tooth tip) is imposed equal to the deflection of tooth pairs under the nominal torque. The topography of the tip relief is parabolic. The conditions for obtaining the minimum PPTE configuration are found and discussed.

3. Geometrical Inaccuracy

According to Z. Li, and K. Mao [13], the noise and vibration of gear set are not only caused by dynamic reasons such as transmission error, but also can be caused by some geometrical reasons. The gear design always be focus on analyzing the static performance (strength, stress, friction) and dynamic performance (inertia, noise, vibration), and especially for dynamic, the noise and vibration of gear are big problems. To decrease harmful noise and vibration, the most effective method is "Tooth Profile Modification", which is by tip relief or root relief for modifying geometry profile of gear tooth to regulate transmission error.

3.1 Tooth Profile Error

The tooth profile error is defined as the distance between the theoretical involutes profile and the real tooth profile in the normal direction. The profile error function *ei(t)* due to manufacturing can be defined as

$$ei(t)= E_i \sin (\omega_z t + \alpha) \ (17) \qquad (1)$$

where ω_z is the mesh frequency and
α is the pressure angle.

The composite error e_s is the sum of tooth errors of the pinion and gears. Tooth profile errors are added to the theoretical profile in normal directions [11].

4. Tactic

The methodology proposed by the authors [9] is applied here. Simulations of meshing gears have been carried out by means of a hybrid method, combining the finite element technique with a semi-analytical solution [20, 21]. The main assumptions for the analysis are the following:

1. Plain strain conditions suggested by the spur gear geometry (high ratio b/h). Two dimensional plane strain analysis is adequate for this kind of tooth. Moreover, the two-dimensional version of the software requires little time both for model generation and simulations, with very precise results.
2. Static analysis. Static TE was determined while neglecting rotational speed and inertia forces. This is the main assumption; undertaking dynamic analysis is too time consuming.
3. Friction neglected. It is assumed that it has little effect on TE output.
4. Space error and pitch error not considered.

No statistical consideration was included in the analysis. The quantities ν_e P (ν_e G) and θ_P (θ_G) are defined in Figure 3. The ranges for both of these two variables are the start of active profile roll angle (SAP) and the end of active profile roll angle (EAP) for each gear.

4.1 Profile Modification

Gear tooth modification is a way to fight against not conjugated contact in real gear world. Not conjugated contact may be caused by different things. It can be spacing error, tooth profile, lead, and deflections under the load or misalignments in positioning of the gears. Any of these issues can cause the edge contact in the mesh. The edge contact can take place twice during the engagement of a pair of teeth. The first time the tip of the pinion tooth impacts the root of the gear tooth. Then the edge contact takes place at the end of contact of two teeth. In both cases the edge contact causes higher contact stresses on the tooth surface. Modification of the tooth surface is the way to avoid the edge contact.

In tooth modification design, tip relief is defined as the thickness ν_e of the material removed along the tooth flank with reference to the nominal involutes profile. Profile modification is usually defined versus the roll angle coordinate (θ), shown in Figures 1 and 2, and measured in the direction of the inner normal, shown in Figure 3. The type of function $\nu = \nu(\theta)$ is usually indicated as profile modification topography. Meshing sensitivity to topography is explored throughout this article. A different shape for the tip relief profile modification is proposed with the aim of reducing noise since TE can be significantly reduced if an optimized profile modification is produced [5, 6, 7,10]. Litvin [15] agrees with this approach even though tooth contact analysis is considered only, instead of loaded tooth contact analysis (LTCA). This article shows how the new profile tip relief modification proposed here can influence TE meshing response, according to LTCA hypothesis.

Following Figures 1 and 2 will show the roll angle and tip relief which is the process done on the involutes profile to modify the profile.

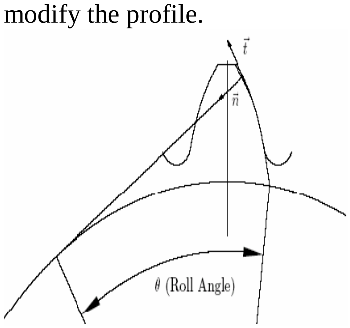

Figure 1: Roll Angle θ

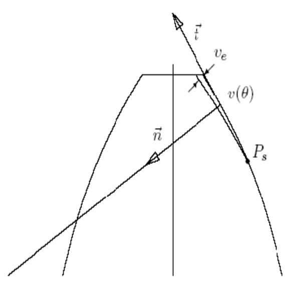

Figure 2: Total amount of material removal v_e

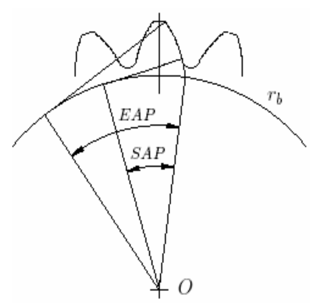

Figure 3: Symbols SAP and EAP

Profile modifications are micro-geometrical removals of material both from the tip and from the root of the tooth. The parameters that define these profile modifications are the roll angle of start and magnitude relief at the tip, the roll angles of start and magnitudes of relief at the root; this way the parameter space is 8-dimensional. Figure 3 shows the parameters defining tooth profile modifications.

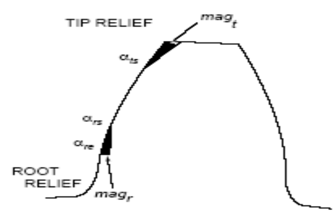

Figure 4: Representation of profile modification parameters

The type of the modifications can be linear or parabolic with respect to the roll angle. Since parabolic modifications give worst results in optimization therefore linear modifications are considered [14].

4.2 Corner contact boundary.

Corner contact is produced when the contact region includes zones of the fillet of the tooth tip [17]. As a consequence of tooth deflection, the effective contact ratio is greater than that found according to rigid geometry. Hypothesis: The contact pressure rises locally at the tip fillet, as shown in Figure 5.

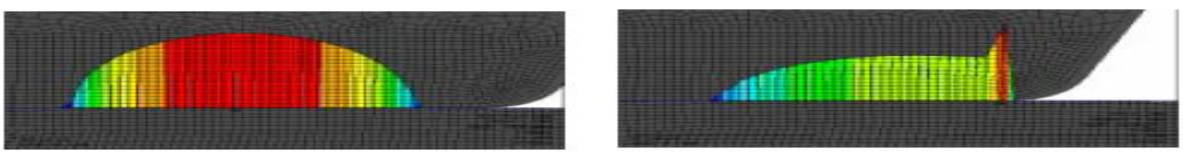

Figure 5: (a) No Corner Contact detected. (b) Corner Contact detected

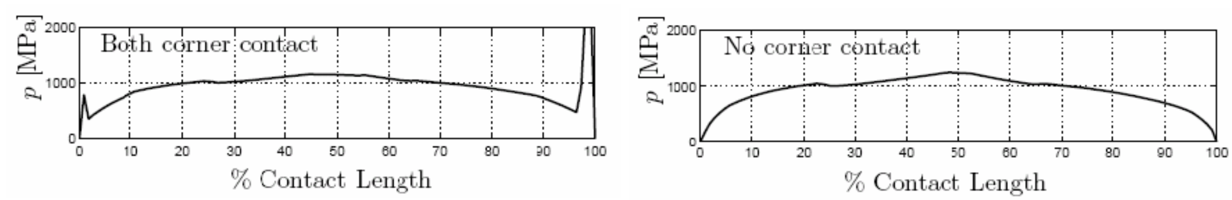

Figure 6: Graphical representations of contact detected and no corner

This definition of corner contact can be exploited if an FEM analysis is performed. When the corner contact is detected, the calculated pressure peak was not considered reliable since the maximum is strongly affected by the radius of the fillet, which is a very unpredictable quantity. Maximum pressure rises due to very high stress values if contact appears

4.3 High curvature boundary.

There is also the possibility of getting an anomalous contact condition if the tip relief is too high along the tooth profile. In this condition, high curvature occurs and then contact pressure is expected to be much higher than the Hertz model according to the nominal involutes curvature. It is worth noting that corner contact is related to torque versus tip relief, so configurations can show corner contact if the load is high enough. This is obviously detectable only if loaded tooth contact analysis (LTCA) is considered.

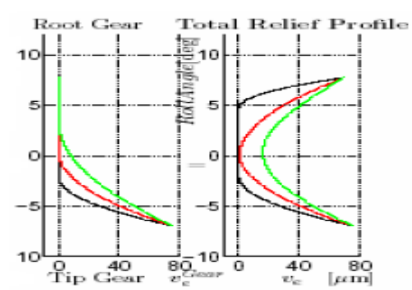

Figure 7: Total relief Profile

4.4 Contact Pressure.

Local contact pressure along the profile can change considerably due to profile modifications. Actually, by changing the Start Relief point along the profile, relative curvature considerably changes as well (Figure 7). It is known that a contact pressure rise with the curvature discontinuity inside the contact region. This numerical result has been confirmed theoretically in [12].

4.5 Computational Performances

To perform parameter sensitivity analysis, a common PC platform was used with the following characteristics: CPU 2.6 GHz and RAM 2 GB. Plane strain analysis was performed by ExtPair2DTM [18, 19]. Analyses were automatically performed in about 4.5 CPU hours, simulating 50 time steps for each meshing, for 200 different tip relief (θ_P, θ_G) configurations.

5. Results

The analysed gear parameters are listed in Table 1.

Table 1: LCR gear Design parameters

Modulus	2	Face width	11 mm
No. of teeth	60	ν_e	23.3
External diameter	143.2 mm	Load	300 N
Root diameter	135.3 mm		

The object function PPTE for the LCR gear set is shown in Figure 8. No boundaries are presented yet.
It is worth noting that:

- The PPTE minimum is unique inside the ranges for the two variables.
  - Near the minimum the Hessian matrix is positive defined.

According to these conditions the minimum could be found by adopting a classical gradient derived method. As pinion and gear have the same number of teeth, meshing properties are symmetric about the domain diagonal defined by the equation $\theta_P=\theta_G$ and the absolute minimum is on this diagonal (Figure 9). The numerical values are the following: θ_P min = 23.035 deg θ_G min = 23.035 deg PPTEmin =1.8 µm.. In order to analyze this minimum, TE functions for $\theta_P = \theta^{Pmin}$ at different θ_G are plotted. It is remarkable that the shape of the TE functions is different between configurations with $\theta_G < \theta_G^{min}$ and $\theta_G > \theta_G^{min}$. This is due to the fact that for

$\theta_G < \theta_G^{min}$, profile configurations cause a ν_e drop. In fact in the analyzed configuration, start relief roll angles couple (θ_P, θ_G) for the minimum PPTE produces overlapping modified profiles and a reduced effective total amount relief as already discussed and shown in Figure 9. This is due to an overestimation of the total relief amount calculated at the beginning. For example in this configuration the total amount is reduced from 25 µm to 23.3 µm. And this result can be an insight for better evaluating ν_e. it was found that the absolute PPTE minimum is outside the acceptance limits. As shown in Figure 9, the minimum PPTE would produce Corner Contact. This is due to the start relief roll angle overlapping observed at the end of the previous section that reduced effective total relief amount and thus reduce the TE.

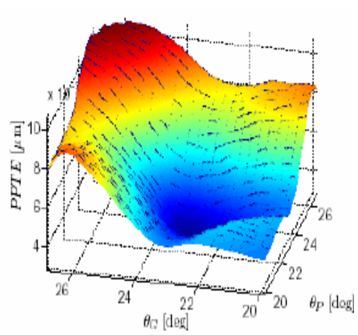

Figure 8: 3D plot of PPTE, LCR gear

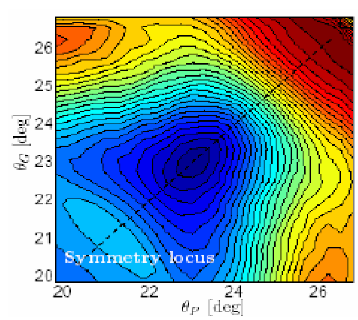

Figure 9: Symmetry locus of PPTE (θ_P, θ_G) Function for LCR gear

5.2 Bending Stress

Bending stress at tooth root is another important issue in high performance gear design. By applying profile modifications, load transfer between teeth pairs, contact points and load directions can change. As a consequence, different bending stress at tooth base can be produced. If a LCR gear were considered, bending stress variations would not be greater than 5% It can be convenient to consider bending stress as a possible penalty for the object function instead of a boundary.

Table 3: Result Table

	Module	Stress (MPa)	Deflection (* 10^{-3}mm)
Involute profile	2	90	0.0027
Modified involute profile	2	93	**0.002**

6. Conclusion

The contribution of this thesis work presented here can be summarized as follows: The aim of the work is to minimize the noise of the spur gear. A semi analytical and FEM software has been used for performing meshing simulation. LCR gears have been studied and the maximum tooth detection is calculated with and without total tip relief. Where it is found that the total tooth relief shows the better result and help to reduce the TE. Thus profile modification is done to reduce the TE and it shows the proper result during analysis.

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