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# A Robust Optimization Approach for the Milk Run Problem with Time Windows under Inventory Uncertainty - An Auto Industry Supply Chain Case Study

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## Abstract

In this paper we introduce a robust optimization approach to solve the milk run system with time window with inventory uncertainty. This approach yields routes that minimize transportation costs while satisfying all inventory in a given bounded uncertainty set. The idea of Milk Run problem has been used in the context of logistic and supply chain problems in order to manage the transportation of materials. Since the resulted problem formulation is NP-Hard, In order to solve the underlying problem, a novel algorithm entitled robust optimization has been proposed. We apply the model to solve some numerical examples to show robust solution efficiency versus deterministic. Since the resulted problem illustrate that grows up time in this method is progressively, In order to solve problem in large scale, particle swarm optimization has been proposed. We also observe that the robust solution amounts to a clever management of the remaining vehicle capacity compared to uniformly and non-uniformly distributing this slack over the vehicles.

**Keywords:** Milk Run problem with time windows, robust linear optimization, Vehicle Routing Problem, Inventory uncertainty, Particle Swarm Optimization

## 1. Introduction

Many industrial applications deal with the problem of routing a fleet of vehicles from a depot to service a set of customers that are geographically dispersed. The idea of lean production has increasingly become popular and there is a high degree of correlation between a lean production plan and a good logistic strategy. Services have experienced tremendous growth in recent years. For example, SAIPA Company in Iran has sell over $12 billion, and the cost of transport in company in years is over $12 million [1]. However, congestion and variability in demand and travel times affects these industries on four major service dimensions: (i) travel time; (ii) reliability; (iii) cost of transport; and (iv) cost of inventory [2].

The vehicle routing problem (VRP) is commonly defined as the problem of determining optimal delivery or collection routes from several depots to a set of geographically scattered customers, under a variety of side conditions [3].

The proposed method of this paper is purposely designed for one of the biggest auto maker in the world called SAIPA. According to the company's financial statement for the fiscal year ends in March 2008, the company assembles about half a million cars which is almost half of Iran's market share. The proposed method of this paper is more customized to take into account the management strategy which did not explicitly exist in previous research. We hope the proposed method of this paper could also be used for many other industries around the world.

## 2. Literature review

VRP belongs to the typical complicated combination optimization problem, as a NP difficult problem. VRP has close relation with TSP (Traveling-Salesman Problem). Some scholars consider VRP as the combination of BPP (Bin Packing Problem) and TSP [9, 10]. There are many kinds of VRP models which consider classification in structure bottom (and figure 1):

[1] Capacitated VRP (CVRP)

[2] VRP with Time Window (VRPTW)

[3] VRP with Backhauls (VRPB)

[4] VRP with Pick-Up and Deliveries (VRPPD)

[5] Multiple depots VRP (MDVRP)

[6] Periodic VRP (Deliveries in some days) (PVRP)

[7] Split Delivery VRP (SDVRP)

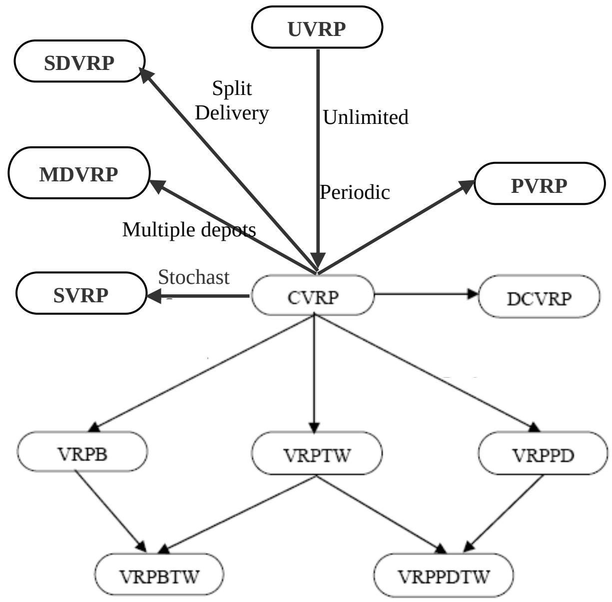


Fig. 1 Classification of VRP

Milk Run System determines the route, the time schedule, the type and the number of parts that different trucks must choose in order to receive the orders from various suppliers with the primary assumption that all trucks must return the empty pallets to the demand center (e.g. Auto maker). There are different objectives involved in this kind of modeling which need to be minimized such as inventory and transportation costs.

Robust optimization refers to the modeling of optimization problems with data uncertainty to obtain a solution that is guaranteed to be good for all or most possible realizations of the uncertain parameters [11-13]. In recent years, robust optimization has gained substantial popularity as a competing methodology for solving several types of stochastic optimization models. Robust optimization has been successful in immunizing uncertain mathematical optimization [14-17]. The first step in this direction is taken by Soyster [14] who proposes a worst case model to linear optimization.

The goal of this paper is to present such an approach, based on robust optimization which has recently become an interesting research topic. These ideas are ported to a mathematical programming context beginning with the work by Ben-Tal and Nemirovski [18, 19] where the authors formulate the robust optimization problems of linear programs, quadratic programs, and general convex programs. We utilize the approach which leads to robust counterparts while controlling the level of conservativeness of the solution. Independently El-Ghaoui et al. [16] study the same robust optimization counterpart for semi-definite programming problems. More recently, this approach is extended to integer programming problems.

## 3. Robust milk runs with time windows formulations

In this section, we first identify the deterministic Milk Run with Time Windows formulation and inventory uncertainty sets that will be used and then we present the derivation for the Robust Milk Run with Time Windows.

### 3.1 The Proposed milk run with time windows model

In this section we deal with the milk system with time windows modeling. As we already explained, the proposed method of this paper is a special case of Milk run with time windows which is purposely designed and customized for SAIPA auto maker.

#### 3.1.1 Decision variables and parameters

$x_{t_j kpj}$ : The number of shipping palates of part $p$ transported with vehicle $k$ from supplier $j$ with the due date of time $t_j$

$V_k$ : Truck capacity $k$

$$y_{t_j kij} : \begin{cases} 1 & if\ x_{t_j kpj>0} \\ 0 & if\ x_{t_j kpj=0} \end{cases}$$

$c_p^{\min}$ : The minimum inventory level of part $p$ ; $V_P^{PL}$ : The maximum capacity of palate for part $p$

$U_{tp}$ : The average consumption of part $p$ in time $t$ ; $X_{tp}^{M}$ : The remaining part $p$ at the end of time $t$

$H_P$ : The inventory cost per hour of each palate containing part $p$ in warehouse

$C_{kij}$ : The cost of the truck $k$ moving from supplier $i$ to supplier $j$

$\gamma_{pj}$ : The percentage of part $p$ allocated to supplier $j$ ; $C'$ : The fixed cost of waiting a truck at each supplier

$S_i$ : The service time required at supplier $i$ ; $T_{ij}$ : The time needed to travel from supplier $i$ to supplier $j$

$B_k$ : The departure time of vehicle $k$ at the distribution center

$r_k$ : The required returning time of vehicle $k$ to the distribution center

**3.1.2 Objective function**

Since the main goal of implementation of the Milk Run system with Time Windows is to decrease the transportation costs and reduce the level of inventory of parts at the warehouse we consider the following objective function:

$$\min \sum_t \sum_k \sum_i \sum_j [C_{kij} + C']Y_{tkij} + \sum_t \sum_p H_p \times X_{tp}^{M} + \sum_i P_i(t_i) \qquad (0)$$

The first part of the objective function explains the total costs of transportation. This function states that if a transporting vehicle moves from one supplier to another one, the transportation cost with the vehicle and the fixed cost of the loading must be considered.

The second part is used for inventory costs. In summary, the basis of the objective function is to simultaneously reduce the transportation costs as well as the inventory expenditures. The tertiary part is the penalty costs. The penalty function is formally defined by Eq. (1).

$$P_i(t_i) = \begin{cases} \phi & if\ t_i \prec e_i \\ f_i^e + U_e(a_i - t_i) & if\ e_i \le t_i \prec a_i \\ 0 & if\ a_i \le t_i \le b_i \\ f_i^l + U_l(t_i - b_i) & if\ b_i \prec t_i \le l_i \\ \phi & if\ t_i \ge l_i \end{cases} \qquad (1)$$

Where $t_i$ is the time a vehicle arrives at supplier $i$, and $f_i^e, f_i^l$ are the respect minimum (fixed) costs when waiting or lateness occurs, $U_e$ is the waiting cost per unit of time, and $U_l$ is the lateness penalty per unit of time.

$$\sum_p \sum_j X_{t_j kpj} \times V_p^{PL} \le V_k \qquad \forall (t_j, k) \qquad (2)$$

This constraint investigates that the number of palates collected from the suppliers for transportation to warehouses from a proper volume of parts for collecting and transporting to warehouses would not exceed the number of transporting vehicles.

$$\sum_{t_j}\sum_{k} X_{t_j kpj} = \gamma_{pj} \qquad \forall (p,j) \qquad (3)$$

This constraint states that every supplier is only allowed to transfer the volume of parts committed in the contract.

$$X_{tp}^{M} \geq c_p^{\min} \qquad \forall (t,p) \qquad (4)$$

Each supplier may set a minimum inventory level for its own parts which is defined according to various factors. This amount of the inventory is usually determined for some working days (2 or 3 days)

$$X_{tp}^{M} = \sum_{k}\sum_{j} X_{t_j kpj} + X_{(t-1)p}^{M} - U_{tp} \quad \forall (t,p) \qquad (5)$$

The total number of each parts is equal to the total number of parts arrive at the end of the time t, plus the inventory of the parts at the end of time t-1 minus the number of parts used in time t.

$$\sum_{k}\sum_{j} Y_{tkij} \leq 1 \qquad \forall i \geq 2, \forall t \qquad (6) \quad ; \quad \sum_{k}\sum_{i} Y_{tkij} \leq 1 \qquad \forall j \geq 2, \forall t \qquad (7)$$

These constraints use one and only one transporting vehicle for each time schedule t. Note that since the company's warehouse is also considered as a supplier we must separate these two constraints. Therefore, this issue is explained in the next two constraints.

$$\sum_{j} Y_{tk1j} \leq 1 \qquad \forall (t,k) \qquad (8) \quad ; \quad \sum_{i} Y_{tki1} \leq 1 \qquad \forall (t,k) \qquad (9)$$

Constraints (8) and (9) are similar to (6) and (7) but the difference is that these constraints are exclusively related to the warehouse of the manufacturing company.

$$\sum_{i} Y_{tkiq} = \sum_{j} Y_{tkqj} \qquad \forall (t,k), \forall q > 1 \qquad (10)$$

This constraint which is designed for sequencing the routes states that if a vehicle gets to a knot, it must exit from it. Note that the transporting vehicle only stops when it arrives at the supplier's warehouse, and this condition will be stated in the following constraint,

$$\sum_{i\in S}\sum_{j\in S} Y_{tkij} \leq |S|-1 \;\; S \subseteq \{2,3,\ldots,KT\} \;\; \forall (t,k) \;\; (11)$$

(KT is the total number of the suppliers)

This relation states that the each transporting vehicle starts from the supplier's warehouse and its destination is the same warehouse.

$$X_{t_j kpj} \leq M \times \sum_{i} Y_{t_j kij} \qquad \forall (t_j,k,p,j) \qquad (12)$$

(M is a big number)

The constraint (12) states that we can bring parts from the supplier j if the transporting vehicle is able to enter from one supplier to this supplier.

$$t_j \geq Max\{t_i, a_i\} + S_i + T_{ij} - M(1 - X_{t_j kpj}) \qquad (13)$$ (M is a big number)

The precedence relation between two successive nodes is expressed.

$$B_k + \sum_{i}\sum_{j} \left\{ X_{t_j kpj}\, Max\{t_j, a_j\} + S_j \right\} \leq r_k \quad \forall k \;\; (14)$$

The required returning time of each vehicle is constrained and Eq. (1).

### 3.2 Uncertainty in inventory

We consider that the minimum inventory level of part $p$, parameter $c_p^{\min}$ is uncertain and belongs to a bounded set $U$. We consider uncertainty sets which are constructed as deviations around an expected minimum inventory value $c_p^{\min^0}$. The possible deviation directions from these nominal values are fixed and identified by scenario vectors, $c_p^{\min^k} \in \Re^N$, where $N$ is the number of nodes. The scenario vectors are allowed to have negative deviation values [20]. For a given number of scenario vectors, $n$ , the general uncertainty set $U$ is a linear combination of the scenario vectors with weights $g \in \Re^n$ that must belong to a bounded set $g \in G$ :

$$U_C = \left\{ c_p^{\min} \middle| c_p^{\min^0} + \sum_{k=1}^{n} g_k c_p^{\min^k} \quad , \ g \in G \right\}$$

In particular, we consider the following set for $G$ [19]:

$$convex\ hull \quad G = \left\{ g \in \Re^N \middle| g \geq 0, \sum_{k=1}^{n} g_k \leq 1 \right\}$$

**3.3 Robust Milk Run with Time Windows formulation**

We now propose the robust counterpart problem milk run with minimum inventory belonging to an uncertainty set $U$ . Recall that we consider the problem only with uncertainty in constraint (4).

We can therefore state the robust milk run. This problem minimizes objective, subject to constraints (1), (2), (3), (5), (6), (7), (8), (9), (10), (11), (12),(13),(14). If we substitute in the definition of the uncertainty set $U_c$ , we can write the robust constraint (4) as the following inequality:

$$X_{tp}^{M} - c_p^{\min^0} \geq \sum_{k=1}^{n} g_k c_p^{\min^k} \quad , \forall g \in G \ , \forall (t, p) \qquad (4')$$

## 4. Experimental analysis

In order to analyze the performance of the proposed method we have first solved the resulted model using Mixed Integer software package using some real data from SAIPA Complex for thee group of variables: the number of suppliers, the number of parts and the time horizon for planning. Since there are many binary variables in our proposed model we may not be able to run the resulted model for large problems. Therefore we use a Robust Optimization to solve the problem. Table1 shows the details of the implementation of both methods. As we can observe there is no difference between the optimal solutions of the proposed method for some small scale problems. However, as the size of the applications grows the gap between the optimal solution and the Robust Optimization is getting bigger. On the other hand, the CPU time needed to solve the proposed method is relatively acceptable for some small problems but for real world case problem we may not be able to find the optimal solution very easily. All experiments were carried out with a runtime limit of an hour on a COMPAQ EVo N1020v computer with a 2.4 GHz Intel Xeon Processor and 2 GB RAM running Red Hat Windows XP.

Table 1: the Computational Results of deterministic and Robust Optimization

| The Number of | | Time Horizon | The Proposed deterministic | | The Robust Optimization | |
|---|---|---|---|---|---|---|
| Supplier | Part | | CPU | Cost | CPU | Cost |
| 2 | 5 | 5 | 0.3 | 3.6E+4 | 0.45 | 4.1E+4 |
| 2 | 5 | 10 | 0.37 | 7.2E+4 | 1.06 | 9.32E+4 |
| 3 | 10 | 15 | 2.37 | 32E+4 | 4.43 | 39.3E+4 |
| 3 | 10 | 20 | 5.47 | 41+E4 | 8.45 | 50.1E+4 |
| 4 | 15 | 30 | 10.58 | 129+E4 | 15.1 | 141E+4 |
| 4 | 15 | 40 | 17.85 | 172E+4 | 23.25 | 197E+4 |
| 5 | 20 | 60 | 37.5 | 384E+4 | 45.12 | 432E+4 |
| 5 | 20 | 80 | 68.95 | 492E+4 | 84.32 | 545E+4 |
| 6 | 25 | 150 | 157.98 | 106E+5 | 205.55 | 127E+5 |
| 6 | 25 | 200 | 453.87 | 135E+5 | 764 | 205E+5 |
| 7 | 30 | 280 | 1026.35 | 253E+5 | 1798.35 | 352E+5 |
| 7 | 30 | 360 | 2845.63 | 316E+5 | 4050.45 | 424E+5 |

As table 1, sensitivity increase when the number of suppliers and uncertainty go up. CPU time, in this method, grows up progressively and it is necessary to use a heuristic or Meta heuristic method to solve the large scale problems. Since the resulted problem illustrate that grows up time in this method is progressively, In order to solve problem in large scale, particle swarm optimization has been proposed.

Particle Swarm Optimization (PSO), a new heuristic global optimization technique motivated from the simulation of social behavior was originally designed and developed by Kennedy and Eberhart in 1995[21]. In PSO, instead of using genetic operators, each particle (individual) adjusts its “flying” according to its own flying experience and its companions' flying experience. In [22], the maximum velocity on the performance of the particle swarm optimizer was first analyzed and the guidelines for selecting parameters are provided. The impact of the inertia weight was firstly introduced in [23] and the efficiency of PSO has improved greatly ever since.

The solution representation for Milk run with time window with $n$ customers and $k$ vehicles is a $(n+2k)$-dimensional particle. Table 2 shows the details of the implementation for both methods. The execution CPU times are given in minutes while the object function is in local Iranian currencies, Rial. The results of Table 2 represent that there is no difference between the optimal solutions of the proposed method for some small-scale problems. Also, the gap between the PSO and the actual plan change from 1% to 10%.

Table 2: the Computational Results of MIP for Robust Optimization and PSO algorithm for Robust Optimization

| The Number of | | Time Horizon | The Proposed MIP Method for Robust Optimization | | The Proposed PSO algorithm for Robust Optimization | |
|---|---|---|---|---|---|---|
| Supplier | Part | | CPU | Cost | CPU | Cost |
| 2 | 5 | 5 | 0.45 | 4.1E+4 | 0 | 4.1E+4 |
| 2 | 5 | 10 | 1.06 | 9.32E+4 | 0 | 9.32E+4 |
| 3 | 10 | 15 | 4.43 | 39.3E+4 | 0 | 39.3E+4 |
| 3 | 10 | 20 | 8.45 | 50.1E+4 | 0.01 | 50.2E+4 |
| 4 | 15 | 30 | 15.1 | 141E+4 | 0.015 | 141.1E+4 |
| 4 | 15 | 40 | 23.25 | 197E+4 | 0.05 | 196.8E+4 |
| 5 | 20 | 60 | 45.12 | 432E+4 | 1.35 | 432E+4 |
| 5 | 20 | 80 | 84.32 | 545E+4 | 3.43 | 544.1E+4 |
| 6 | 25 | 150 | 205.55 | 127E+5 | 13.34 | 127.1E+5 |
| 6 | 25 | 200 | 764 | 205E+5 | 27.5 | 212E+5 |
| 7 | 30 | 280 | 1798.35 | 352E+5 | 75.75 | 382E+5 |
| 7 | 30 | 350 | 4050.45 | 424E+5 | 105.65 | 466E+5 |
| 8 | 40 | 450 | - | - | 187.45 | 601E+5 |
| 8 | 40 | 600 | - | - | 396.4 | 922E+5 |
| 10 | 50 | 720 | - | - | 845.87 | 124E+6 |

## 5. Conclusions

In this study, we have proposed a deterministic, numerically tractable methodology to address a new problem of optimal controlling supply chains, Milk run system with Time Windows, subject to uncertain inventory. We propose the use of robust optimization to obtain efficient routing solutions for problems under uncertainty. Our work has shown that robust optimization is an attractive alternative for formulating milk run problem under uncertainty since it does not require distribution assumptions on the uncertainty or a cumbersome representation through scenarios.

This method uses very little information on the uncertain inventory. We present computational results which investigate some sample test problems. Our computational results show that the robust solution can be protected from unmet inventory while incurring a small additional cost over deterministic optimal routes. Results of our investigation show that although the robust solution imposes extra cost to the transportation and inventory warehouse, it does not remain any unmet inventory in the network so it was more efficient than the deterministic solution. The method which we used is an exact and is not applicable for large scale problems but it is a proper tool to validate the heuristic algorithms. Finally, we have discussed about some affective parameters and role of them in the robust solution.

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