Proceedings of the 2010 International Conference on Industrial Engineering and Operations Management
Dhaka, Bangladesh, January 9 – 10, 2010

An Optimal Rotational Cyclic Policy for a Supply Chain System with Imperfect Matching Inventory and JIT Delivery

Bhaba R. Sarker and Pablo Biswas
Department of Industrial Engineering
Louisiana State University, Baton Rouge, LA 70803-6409

Abstract

A supply chain system of a *Just-in-time* production facility consists of raw material suppliers, manufacturers and retailers where inventory of raw materials and finished goods are involved, respectively. This research focuses on reducing the idle time of the production facilities by assuming that the production of succeeding cycle starts immediately after the production of preceding cycle. In reality, the inventory of a supply chain system may not be completely empty. A number of products may be leftover after the deliveries are made. These leftover inventories are added to the next shipment after the production of required amount to makeup a complete batch for shipment. Therefore, it is extremely important to search for an optimal strategies for these types production facilities where leftover finished goods inventory remains after the final shipment in a production cycle. Considering these scenarios, an inventory model is developed for an imperfect matching condition where some finished goods remains after the shipments. Based on the previous observation, this research also considers a single facility that follows JIT delivery and produces multiple products to satisfy customers' demand. For this problem a rotational cycle model is developed to optimize the facility operations. Both problems are categorized as mixed integer non-linear programming problems which are to be solved to find optimum number of orders, shipments and rotational cycle policy for multiple products. Also, this solution will lead to estimate the optimum production quantity and minimum total system cost.

Keywords
Supply chain system, JIT delivery, imperfect matching, minimum idle time and rotational cycle policy.

1. Introduction

A production facility (such as refinery, aluminum conversion industry) produces multiple products from a single type raw material (crude oil, large aluminum sheets, etc) and shipped to customers according to their demand. In the supply chain system of these categories of production facilities, the raw materials are ordered from the suppliers, and process the raw material into multiple finished products and deliver to the buyers or retailers. A single facility supply chain system is represented in Figure 1. For the last few decades, *just-in-time* (*JIT*) philosophy has played an important role in supply chain systems such as the manufacturing sectors. The successful implementations of JIT phenomena are frequent shipment of high quality parts to the buyers and ordering raw materials in small batches whenever required to process finished products.

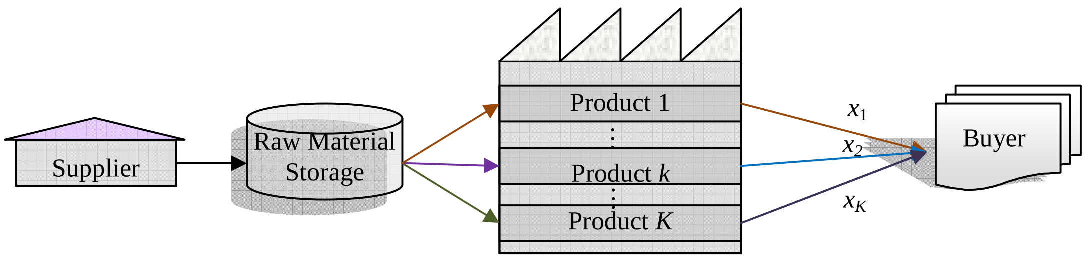

Figure 1: Single-supplier, multi-product and Single-buyer supply chain system

Hill [3] modified the ordering policy of the raw material by allowing a single order for multiple production cycles when the inventory cost for the raw material is much lower as compared to the ordering costs in each production cycle. Later, Parija and Sarker [5] extended this model to a multi-retailer system and determined the production start time and proposed a method that determines the cycle length and raw material order frequency for a long-range planning horizon. Sarker and Diponegoro [6] studied an exact analytical method to obtain an optimal policy for a more general class of problem with multiple suppliers, non-identical buyers, finite production rate and finite planning horizon. Later, Khan and Sarker [4] developed another model for a manufacturing system with a JIT raw material supply and delivery. Sarker and Khan [7] proposed a model that happened to be minimizing inactive time of the production facility. Biswas and Sarker [1] proposed a supply chain model where they considered that the production or uptime starts immediately after the production or uptime of the preceding cycle. Using this concept they formulated the inventory model with a perfect matching situation. Biswas and Sarker [2] proposed another supply chain model considering same concept with imperfect matching situations. Both papers considered a facility where a single product is being produced. In reality, large production industries (refineries, paper mills, sugar mills, Aluminum conversion etc.) do not let their production system be idle and their finished goods inventory seldom falls to zero. This research deals with single facility lot sizing supply chain system composed of production with multiple products, single supplier and single buyer operating under JIT delivery where a leftover inventory remains at the end of all possible shipments (*imperfect matching*). Also, the research considers that the production of a product starts immediately after the production of the previous product and the setup time resulting idle time minimization. The goal of this research is to minimize the total system cost of a supply chain system that consists of supplier, manufacturer and retailer by evaluating the optimum rotational cycle of the products.

2. Model Description and Formulation

In a single facility lot sizing model, multiple products are produced in a time span where completion of the production of any product can meet the customers demand during that time span. After the time span the product goes to production again to meet the next time span. This time span is referred as rotational cycle. A single production facility produces K products with a constant demand of D_{Fk} units per year for product k (where k = 1, 2, …,K), and k product is produced at a constant rate of P_k units per year to satisfy the demand D_{Fk}. All products are delivered at a fixed amount of x_k units after every L_k time units. According to the assumption, production of all k items must meet customers' demand and $\sum_{k=1}^{K}(D_{Fk}/P_k)\le 1$. Also, due to rotational cycle policy, all products with the same production cycle time, T_C, and a lot of each product is produced during this time period. Due to the rotational, the products are produced in a fixed order, which is repeated from cycle to cycle. Without permitting any shortages, it is a problem to determine the time of production and optimum number of units to produce for each item which was defined as rotational cycle by Johnson and Montgomery [8]. In their research they considered a single facility lot sizing model based on classical inventory model. In real life, the single production facility, a number of products are always left-over after the possible deliveries are made. These left-over amounts are added to the next shipment after the production of required amount to make-up a complete batch. An illustration may be observed in retail stores such as Albertson's, Target, Wal-Mart, etc. This research incorporates the inventory model with JIT delivery and imperfect matching inventory situation. The notation used to develop the model is defined when used throughout the research.

2.1 Raw Material Inventory and Cost Function

To develop the mathematical model the assumptions considered are (a) production rates are constant and finite and greater than the demand rates, $P_1 > D_{F1}$, (b) production of all K items must meet customers' demand, (c) production facility considers as just-in-time (JIT) delivery and supply of finished products and raw materials, respectively, (d) production run of a product starts immediately after the uptime or production run of previous product and setup time, (e) multiple products are produced in each rotational cycle, and (f) a fixed quantity is left-over after required shipments and carried over to the succeeding cycle.

An inventory diagram of a single facility lot sizing model is presented in Figure 2. The pattern of raw material inventory is shown in Figure 2(a) where Q_R is the raw materials required from the supplier during T_{P1} time period. These Q_R units are ordered in m_1 batches in instantaneous replenishments of Q_R/m_1 units. It is assumed that each unit of product 1 produced requires f_1 units of raw material, so that $Q'_{F1} = f_1Q_R$. Again, in this research the raw materials are ordered and converted to finished goods during the production time or uptime, T_{P1}. Thus, the time weighted inventory, $\bar{I}_R$ of raw material held in a cycle of product 1is given by

$$\bar{I}_{R1} = Q'_R T_{P1}/(2m_1) = Q'_{F1}T_{P1}/(2m_1 f_1) = Q'^2_{F1}/(2m_1 f_1 T_{P1})\,, \tag{1}$$

where $Q'_{Fk}/Q'_R = D_{Fk}/D_{Rk} = f_k$, $T_{Pk} = Q'_{Fk}/P_k$ and k = 1,…,K.

Therefore, the total cost for the raw material k can be expressed using Equation (1) as

$$TC_{Rk} = D_{Rk}K_{0k}/(Q'_R/m_k) + \bar{I}_{Rk}H_R = m_k D_{Fk}K_{0k}/(Q'_{Fk}) + Q'^2_{Fk}H_R/[2m_k f_k P_k]\,. \tag{2}$$

2.2 Finished Goods Inventory and Cost Function

According to the JIT delivery schedule, fixed amount of x_k units of product k will be delivered after every L_k time units. The lot size for product k must be equal to the demand during the rotational cycle, T_C, without permitting shortages as

$$Q'_{Fk} = (m_k x_k + I_{0k}) = T_C D_{Fk}\,. \tag{3}$$

According to Figure 2(b), at point A_1 production of product 1 starts with P_1 units/year after T_{S1} time units and produces exactly Q'_{F1} (= $n_1x_1 + I_{01}$) amount to deliver x_1 after L_1 time units. Hence, during time $L_1 - T_{S1}$ time the quantity produced is $x_1 – I_{01}$ at the rate of P_1, so that $I_{01} + (x_1 – I_{01})P_1 \geq x_1$. The first shipment of x_1 units of product 1 can be delivered at point B_1 after L_1 time units combining with the left over inventory of I_{01} from the previous cycle. Again, production continues and the inventory builds up as $P_1 > D_{F1}$ and another shipment of x_1 amount is made at point C_1 after L_1 time units.

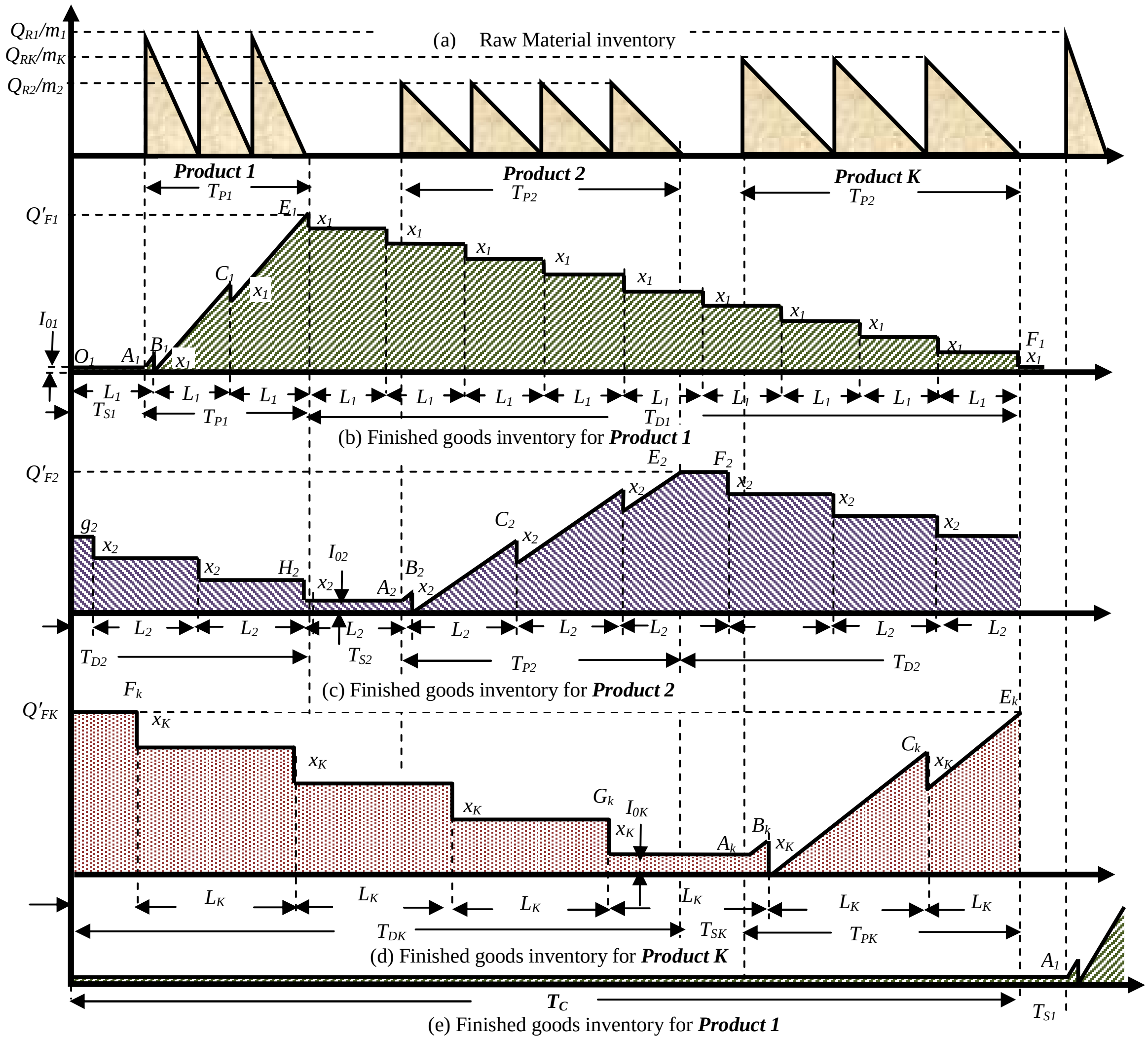

Figure 2: Rotational cycle inventory formation

Thus, after the uptime T_{P1} and point E_1, production of product 1 stops and the inventory forms a saw-tooth pattern. After point E_1, x_1 amount is shipped in every L_1 time units to the customer from built-up inventory during downtime T_{D1}. During downtime T_{D1}, the inventory forms as stair case pattern. At the end of T_{D1} time and all possible shipments I_{01} amount of inventory left out in the warehouse as $x_1 > I_{01}$, which is carried over to the next production cycle of product 1. At the end of T_{P1} and after T_{S2}, the production of product 2 starts and delivers x_2 units of product 2 after L_2 time units [Figure 2(c)]. The production of product 2 continues until T_{P2} time units followed by the downtime of T_{D2}. Thus, the process continues for product K from point A_k to E_k according to Figure 2(d). It should be noted that during T_{P2} and T_{PK}, the amount of product 2 and product K produced, respectively, must satisfy customers' demand of these products throughout rotational cycle, T_C. After T_{PK}, the production of product 1 starts again. Figure 2(b) is used to calculate the average on-hand inventory of the finished goods. I_{T1}, I_{P1}, and I_{D1} are the total inventory, uptime inventory and downtime inventory for product 1, respectively. Therefore, the total inventory can be calculated as

$$I_{T1} = I_{P1} - I_{D1} \tag{3}$$

From Figure 2(b), it can be found that

$$I_P = n_1 x_1 T_{P1}/2 + n_1 x_1 T_{D1} + I_{01} T_C \,. \tag{4}$$

Using Equation (3) upon simplification and the inventory can be found as

$$I_P = Q_{F1}'^2 [3/(2D_{F1}) - 1/P_1] - I_{01} Q_{F1}' [1/(2D_{F1}) - 1/P_1] + (I_{01} - Q_{F1}') T_{S1} \,. \tag{5}$$

Again, the total inventory shipped can be calculated from Figure 2(b) as

$$I_D = L_1 x_1 + 2L_1 x_1 + ... + (n-1) L_1 x_1 = n_1 (n_1 - 1) L_1 x_1 / 2 = n_1 x_1^2 (n_1 - 1)/(2D_{F1}) \tag{6}$$

where $L_1 = x_1 / D_{F1}$.

Hence, the average inventory for product 1, $\hat{I}_{T1}$, and time period, T_C, can be calculated by combining and simplifying Equations (3), (5) and (6) as

$$\hat{I}_{T1} = \frac{Q_{F1}'}{2D_{F1}} \left[Q_{F1}' + 4I_{01} + x_1 - 2D_{F1} T_{S1} - \frac{D_{F1}(Q_{F1}' - I_{01})}{P_1} \right] - \frac{I_{01}}{2D_{F1}} \left[I_{01} + x_1 - 2D_{F1} T_{S1} - \frac{D_{F1}(Q_{F1}' - I_{01})}{P_1} \right]. \tag{7}$$

Using Equation (7), the total cost function of product k can be expressed as

$$\begin{aligned} TC_k(Q_{Fk}') = &\left\{ D_{Fk} K_{Sk} - I_{0k} H_{Fk} \left(I_{0k} + x_k - 2D_{Fk} T_{Sk} \right)/2 \right\} / Q_{Fk}' \\ &+ Q_{Fk}' H_{Fk} \left(1 - D_{Fk}/P_k \right)/2 + H_{Fk} \left\{ 4I_{0k} + x_k + D_{Fk} \left(I_{0k}/P_k - 2T_{Sk} \right) \right\}/2 \end{aligned} \tag{8}$$

Using Equations (2) and (8), the total cost function of finished product k can be expressed as

$$\begin{aligned} TC_k(Q_{Fk}', m_k) = &\, Q_{Fk}'^2 H_R / (2 m_k f_k P_k) + \left\{ m_k D_{Fk} K_{0k} + D_{Fk} K_{Sk} - I_{0k} H_{Fk} \left(I_{0k} + x_k - 2D_{Fk} T_{Sk} \right)/2 \right\} / Q_{Fk}' \\ &+ Q_{Fk}' H_{Fk} \left(1 - D_{Fk}/P_k \right)/2 + H_{Fk} \left\{ 4I_{0k} + x_k + D_{Fk} \left(I_{0k}/P_k - 2T_{Sk} \right) \right\}/2 \,. \end{aligned} \tag{9}$$

2.3 Objective Functions and its Constraints

Combining all costs for all K products and replacing Q'_{Fk} from Equation (3), it can be written as

$$\begin{aligned} C_T(T_C, m_1, ..., m_K) = \sum_{k=1}^{K} \Big[& T_C^2 D_{Fk}^2 H_{Rk} / (2 m_k f_k P_k) + m_k K_{0k} / T_C + T_C D_{Fk} H_{Fk} (1 - D_{Fk}/P_k)/2 \\ & + [K_{Sk} - I_{0k} H_{Fk} \left(I_{0k} + x_k - 2D_{Fk} T_{Sk} \right)/(2D_{Fk})] / T_C + H_{Fk} \left\{ 4I_{0k} + x_k + D_{Fk} (I_{0k}/P_k - 2T_{Sk}) \right\}/2 \Big]. \end{aligned} \tag{10}$$

Before minimizing the problem, it is necessary to study the constraints related to the rotational cycle policy, such as the setup times the number of raw material deliveries for each product. If the setup time for product k is T_{Sk}, then the total setup time per cycle and the total production time per cycle must be smaller or equal to the rotational cycle length. Therefore, the following constraint on T_C will be

$$T_C \ge \sum_{k=1}^{K} \left[T_{Sk} + Q_{Fk}'/P_k \right]. \tag{11}$$

Replacing Q'_{Fk}, by using Equation (3), it can be re-written as

$$T_C \ge \sum_{k=1}^{K} T_{Sk} \Big/ \left(1 - \sum_{k=1}^{K} \left[D_{Fk}/P_k \right] \right) \equiv T_{\min} \ge 0 \,. \tag{12}$$

Also, the number of raw material delivery, m_k for product k cannot be less than 1 and should be an integer variable. Hence, the constraint on m_k is

$$m_k \ge 1 \text{ and is an integer,} \qquad \text{for } k = 1, 2, ..., K. \tag{13}$$

Using these two constraints defined in Equations (12) and (13), the objective function can be formulated as

Minimize: $$C_T = \sum_{k=1}^{K} \left[B_{ak} T_C^2 / m_k + B_{bk} m_k / T_C + B_{ck} T_C + B_{dk} / T_C + B_{ek} \right] \tag{14}$$

Subject to: $$T_C \ge T_{\min} \ge 0 \,, \tag{14a}$$

$m_k \geq 1$ and is an integer, for $k = 1, 2, ..., K.$ (14b)

Therefore, the problem becomes a mixed integer non-linear programming problem and the solution procedure to this problem is discussed here.

3. Solution Technique of Rotational Cycle Problem

The formulation of the single facility lot-sizing problem for imperfect matching system can be categorized as a mixed-integer non linear programming problem where m_k's are integer and T_C is a real variable and the number of variables are (K + 1). Due to formulation of the problem, it cannot be solved using derivatives and a closed form solution cannot be determined. Using the Divide and Conquer rule, the objective function is divided into two parts (a) rotational cycle for finished products, and (b) number of raw material orders. The rotational cycle for the finished products (T_C) is the same for the raw material delivery, because the raw materials are delivered from the supplier by instantaneous replenishments. Again, the raw material for a product k is ordered when the finished product k goes in production. The solution procedures are shown as follows:

(a) Rotational cycle for finished products

To solve the rotational cycle policy for the part finished product supply, the cost function from Equation (14) can be divided as

$$\text{Minimize: } C_F(T_C) = \sum_{k=1}^{K} \Big[T_C D_{Fk} H_{Fk} (1 - D_{Fk} / P_k)/2 + \{K_{Sk} - I_{0k} H_{Fk} (I_{0k} + x_k - 2D_{Fk} T_{Sk})/(2D_{Fk})\}/T_C + H_{Fk}\{4I_{0k} + x_k + D_{Fk}(I_{0k}/P_k - 2T_{Sk})\}/2 \Big] \quad (15)$$

Subject to: $T_C \geq T_{\min} \geq 0$ (15a)

It can be shown that the Equation (15) is a convex function for T_C; therefore, it can be solved by differentiation with respect to T_C and equate it to zero, which yields

$$T_C^* = \sqrt{2\sum_{k=1}^{K} [K_{Sk} - I_{0k} H_{Fk} (I_{0k} + x_k - 2D_{Fk} T_{Sk})/(2D_{Fk})] \Big/ \sum_{k=1}^{K} D_{Fk} H_{Fk} \left(1 - D_{Fk} / P_k\right)} \quad (16)$$

where k = 1, …, K. Equation (16) has to satisfy the constraint given in Equation (15a). Using the optimal rotational cycle T^*_C, the number of shipments for different finished product can be obtained from Equation (3). The optimal rotational cycle, T^*_C is used to solve the optimal number of orders for raw materials.

(b) Number of raw material orders

As the raw materials order policy is instantaneous, the production rate for the raw material is ∞; therefore, this also satisfies the condition for rotational cycle. Now, applying the value of T_C^* from Equation (17), the total cost/objective function for raw material k can be written as [from Equation (14)]

Minimize: $C_R(m_k) = m_k K_{0k} / T_C^* + T_C^{*2} D_{Fk}^2 H_{Rk} / (2 m_k f_k P_k)$ (17)

Subject to: $m_k \geq 1$ and integer, $\forall k = 1, ..., K.$ (17a)

This objective function [Equation (17)] is convex in m_k and the objective function is a discrete function, which cannot be solved using differentiation. Hence, the induction method is used to solve m_k. Using the induction method in Equation (17), the boundary condition for m^*_k is can be evaluated as

$$\left\lceil [\sqrt{1 + 4\Delta_k} - 1]/2 \right\rceil \leq m_k^* \leq \left\lfloor [\sqrt{1 + 4\Delta_k} + 1]/2 \right\rfloor \quad (18)$$

where $\Delta_k = T_C^{*3} D_{Fk}^2 H_{Rk} / (2 f_k P_k K_{0k}),$ and $k = 1, 2, ..., K.$. In addition, Equation (19) has to satisfy the constraint given in Equation (17a). Applying the boundary condition in Equation (18) the optimal objective function can be evaluated as well as the optimum number of orders m^*_k for raw material k, where k = 1, …, K. Hence, optimum total cost for *all* raw materials can be expressed as

$$C_R(m_1^*, ..., m_K^*) = \sum_{k=1}^{K} [m_k^* K_{0k} / T_C^* + T_C^{*2} D_{Fk}^2 H_{Rk} / (2 m_k^* f_k P_k)]. \quad (19)$$

As discussed before, both m_k^* and T_C^* is may not be globally optimal. Therefore, another forward search is conducted using Equation (14), starting from the constraints for T^*_C and m^*_k [given in Equations (14a) and (14b)] and with step sizes 0.01 and 1, respectively, to evaluate the optimal T_C^{opt} and m_k^{opt} that will minimize the $C_T(T_C^{opt}, m_1^{opt}, ..., m_6^{opt})$.

4. Numerical Computations of Optimum Rotational Cycle

Six products, presented in Table 1, are being produced in a single facility manufacturing system with JIT delivery. Using these data from Table 1 and Equation (16), the T^*_C can be found as T^*_C = 0.56 years. Now using the value of T^*_C in Equation (18) the boundary conditions for m^*_k can be found as $m^*_1 = 1$, $m^*_2 = 1$, $m^*_3 = 1$, $m^*_4 = 1$, $m^*_5 = 1$, and $m^*_6 = 1$ Using these values the total costs can be found as $C_T(T^*_C, m^*_1,\ldots, m^*_6)$ = (0.56, 1, 1, 1, 1, 1, 1) =$33,928.26 per year, and this is local optimum solution. Therefore, a forward search is conducted starting from T^*_C= 0.21 (with step size 0.01), and m^*_k = 1 (with step size 1) and the optimum solution is obtained in $C_T(T_C^{opt}, m_1^{opt}, m_2^{opt}, m_3^{opt}, m_4^{opt}, m_5^{opt}, m_6^{opt}) = (0.32, 1, 1, 1, 1, 1, 1) = \$32{,}373.85$ per year. The detailed results of rotational cycle policy are presented with numerical values in Table 2. In this case, it is considered that all six products are produced in a single facility in a sequence and they will be delivered using just-in-time (JIT) policy. Also, the raw materials for each product will be ordered following multiple ordering policies. According to the constraint given in Equations (14a) it can be determined by using the data given in Table 1 that $T_C \geq 0.019/0.09$ = 0.21. Also, it is observed that $\sum_{k=1}^{6}\left(D_{Fk}/P_k\right) = 0.91 \leq 1$ which satisfies the assumption for rotational cycle policy. The results for the single facility lot sizing models for imperfect matching case presented in Table 2.

Table 1: Data set for single facility lot-sizing model

Parameters	***Product 1***	***Product 2***	***Product 3***	***Product 4***	***Product 5***	***Product 6***
P (units/year)	14,000	10,500	15,000	10,000	9,000	20,000
D_F (units/year)	2,000	1,500	3,000	1,800	1,200	2,200
K_0 ($/order)	150	100	150	200	200	300
K_S ($/setup)	50	100	120	130	200	150
H_R ($/unit/year)	1	10	3.5	4	4	10.5
H_F ($/unit/year)	2	10	5	15	25	45
f	2	3	3	2.5	3	4
x (units)	100	100	150	200	300	350
I_0 (units)	25	30	50	40	60	55
T_s (years)	0.001	0.002	0.002	0.003	0.005	0.006

Table 2: Optimum results for raw materials of imperfect matching case

Parameters	***Product 1***	***Product 2***	***Product 3***	***Product 4***	***Product 5***	***Product 6***
T_C^*, years	0.32					
m_k^*	1	2	2	1	1	1
n^*	6	4	6	2	1	1
$Q_{Fk}^{\prime *}$, units/year	600	400	900	400	300	350
Q_{Rk}^{*}, units/year	300	133	300	160	100	88
$C_T^*(T_C^*, m_1^*, \ldots, m_k^*)$	$32,373.85					

5. Conclusions

This research presents an operation policy of a supply chain of a single facility lot-sizing model with just-in-time (JIT) deliveries with imperfect matching situations. Also, the current research considered a supply chain system that operates under a reduced idle time, where the production of a cycle of one product starts immediately after the end of production cycle of previous product. A set of problems are categorized as a serial system with a fixed quantity and a fixed delivery interval. The problem is solved for the optimum rotational cycle, optimum number of orders, optimum batch sizes, and optimum numbers of shipment evaluated to minimize the total system cost. The operation policies prescribe the number of orders and the ordered quantities of raw materials from suppliers, production quantities, and number of shipments to the customers for an infinite planning horizon. Prospective research issues that can be pursued further concerning the supply chain system addressed in this research by incorporating time varying demand, variable production capacity and rate, transportation costs, and multi-stage systems.

References

1. Biswas, P., and Sarker, B. R., 2005, “Economic order quantity for a JIT production system with minimal downtime,” *The Proceedings of the IERC* (CD-ROM), Atlanta, GA, May 14-18.
2. Biswas, P., and Sarker, B. R., 2006, “An Improved Production Plan to Minimize Machine Idle Time,” *The Proceedings of the IERC* (CD-ROM), Orlando, FL, May 20-24.
3. Hill, R. M., 1996, “Optimizing a production system with a fixed delivery schedule,” *Journal of Operational Research Society*, 47(7): 954-960.
4. Khan, L. R., and Sarker, R. A., 2002, “An optimal Batch size for a JIT manufacturing system,” *Computers and Industrial Engineering*, 42(2-4): 127-138.
5. Sarker, B. R., and Parija, G. R., 1994, “An optimal batch size for a production system operating under a fixed-quantity, periodic delivery policy,” *Journal of Operational Research Society*, 45(8), 891-900.
6. Sarker, B. R., and Diponegoro, A., 2006, “Finite horizon planning with fixed interval of deliveries and no shortage,” *OPSEARCH*, 43(4): 404-424.
7. Sarker, R. A., and Khan, L. R., 1999, “An optimal batch size for a production system operating under periodic delivery policy,” Computers and Industrial Engineering, 37(4): 711-718.
8. Johnson, L. A., and Montgomery, D. C., 1974, *Operations Research in Production Planning, Scheduling and Inventory Control*, First Edition, John Wiley and Sons, Inc., New York, U.S.A.