

## **A Fuzzy Cost-based FMEA Model**

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### **Abstract**

Nowadays, improving and upgrading quality of products and services is one the main reason for outgoing the competitors and penetration into markets. This article is about the deficiency of the traditional FMEA, and introduces a new method based on estimated costs fuzzy FMEA and utility values. In the proposed method a new fuzzy RPN is defined. A pair wise comparison among Severity, Occurrence and Detection by the AHP method has been done to obtain a new fuzzy membership function. In the proposed method the limited budget of company for improving activities is also considered. Finally, the case study shows this new membership function calculate actual costs due to failures, can better prioritize failure modes, can improve more potential failures than Dong method and this is the most important superiority our new method.

### **Keywords**

FMEA, Fuzzy Logic, Utility Theory, Risk Priority Index, AHP.

### **1. Introduction**

FMEA is a systematically method for identifying and preventing the occurrence of potential failures in the product design, product and production process. More than 40 years of FMEA method in the world passes and this method first time was used in Aerospace Industries NASA. Despite the abundant use of FMEA, this method is faced with restrictions. First; determine the exact probability of failure is difficult or impossible. Many of the information in this way are ordinal. Second; in FMEA method influence parameters (occurrence, severity, detection) often consider with a same weight. Third; in FMEA method abutment between scores isn't clear. For example a person may considers a severity score 5, and other one 6 or 7. Fourth; In FMEA method in prioritized fails about costs and profitability does not speak and be ignored. Fifth; Some times by different O, D, S the same RPNs are obtained that will cause confusion in priority. Sixth; Most of fuzzy FMEA methods, apply if-then methods but this method needs expert knowledge. Seventh; Customer discussion in FMEA is a very pale and partly opinions, demands and customer security is considered that after multiplying the two quantities of occurrence and detection its effect reduce. Very important efforts in FMEA literature for overcome the inadequacies of the traditional RPN is done. For example fuzzy logic widely is used for FMEA. Chang and Lee (1999) used fuzzy linguistic terms such as very low, low, to evaluate O, S, D and utilized grey relational analysis to determine the risk priorities of potential causes [11]. Bowles, Pelaez (1995)described a fuzzy logic based approach for prioritizing failures in a system FMEA, which uses fuzzy linguistic terms to describe O, S, D and the risks of failures[12]. Based on the above described fuzzy logic approach, Xu and Zhu (2002) developed a fuzzy FMEA assessment expert system for diesel engine's gas turbocharger [4] and Chin, Chan, and Yang(in press) developed a fuzzy FMEA based product design system called EPDS-1[3]. Building a fuzzy if-then rule base is thought to be tedious and critical to fuzzy FMEA. Braglia et al. (2003) proposed a risk function which allows fuzzy if-then rules to be generated in an automatic way [6]. Garcia and Melo(2005) a data envelopment analysis method (DEA) for FMEA suggested[8] and according to Fuzzy probability model developed by Lertworasirikul and Nuttle (2003) to determine the ranking indicators between fail modes used[13]. Kai and Lim(2006) argued that it might be not true to assume fuzzy if-then rules to be certain and of equal importance [7]. They therefore proposed the use of weighted fuzzy production rules in fuzzy inference system of FMEA, which allows a global weight to be attached to each if-then rule.

Recently also Yang and Chin (2009) have presented a paper that in this paper they treat the risk factors O, S and D as fuzzy variables and evaluate them using fuzzy linguistic terms and fuzzy ratings [5]. In view of fact this subject that one of the major disadvantages of FMEA is not considering the cost in calculating RPN, but there are few research published since 2003. Seun and Ishii (2003) some of disadvantage the traditional FMEA

identified and new method for FMEA based on cost life course introduced that risks in terms of cost to measure[9]. Dong (2003) used utility theory and fuzzy membership functions for the assessment of severity, occurrence, and detection [1] and Carmignani (2008) presented a new integrated approach, named priority-cost FMECA (PC-FMECA), in order to exceed limits of FMEA[10]. So this paper is about solving disadvantages of FMEA method, presenting a new fuzzy method based on cost, considering firm budget as a limit and assessing weight for each RPN factors and each expert.

## 2. UT and FUT-based FMEA and New approach

Since the traditional FMEA uses ordinal numbers to rank the severity, occurrence, and detection of failure modes, it cannot provide an estimation of the cost due to failure since the cost of a failure mode ranked 10 is not always ten times of a failure mode ranked 1. Since the ultimate goal of FMEA is to reduce the cost due to failure, the cost due to failure modes should be the objective for decision-making. The expected cost  $E(C)$  due to a failure mode can be expressed as:

$$E(C) = C_{fm}P_{fm}(1 - P_d) \quad (1)$$

where  $C_{fm}$  is the cost due to a failure mode,  $p_{fm}$  is the probability of this failure mode and  $p_d$  is the probability that this failure will be detected.

Since the severity, occurrence and detection of a failure mode determines the failure cost, they can be regarded as cost drivers in the utility theory (UT). Utility theory is an attempt to infer subjective value, or utility, from choices. In this case, each cost driver is ranked from 1 to 10. Cost values are converted into utility values by dividing the cost value of the highest level for each cost driver, i.e.[1]

$$U_i = \frac{C_i}{C_{10}} \quad (2)$$

In FUT, the utility values are expressed by membership functions instead of real numbers. Consider severity, it is ranked from 1 to 10. The cost value for level  $i$  given by engineer  $j$  is denoted as  $C_{sij}$ ,  $i = 1 \dots 10$ ,  $j = 1 \dots n$ , where  $n$  is the number of engineers.

$$U_{sij} = \frac{C_{sij}}{C_{s10,j}} \quad (3)$$

After this transformation, utility values are between 0 and 1.

The cost and utility values for detection can be derived in the same approach as severity. The evaluation of occurrence is different from that of severity and detection, since the probability of failure is given, as shown in Table I. These probability values are converted to the utility values as:

$$U_o = \frac{-1}{\log p_o} \quad (4)$$

where  $p_o$  is the probability that a failure mode occurs. After this conversion, the utility values for occurrence are between 0 and 1. In FUT method the resulting RPI is fuzzy and is expressed by the membership function instead of a general utility value as:

$$RPI = \sqrt[3]{U_s U_o U_d} \quad \Longrightarrow \quad \mu(RPI) = [\mu(U_s) \mu(U_o) \mu(U_d)]^{\frac{1}{3}} \quad (5)$$

In this paper, triangular membership functions are used. For a triangular membership function, the minimum and maximum utility values given by the engineers form the two bottom points, and the average of the utility values form the top point, i.e.

$$U_u = \max(U_j) \quad U_m = \sum_{j=1}^n \frac{U_j}{n} \quad U_l = \min(U_j) \quad (6)$$

The membership grades are 0 for utility values  $U_l$  and  $U_u$  and 1 for utility value  $U_m$ . This is based on the assumption that among the utility values given by the engineers, the average of the values is more likely to denote the actual failure cost than the minimum value or maximum value[6,12]. For a specific failure mode, each engineer determines the cost values  $C_{si}$  for the severity and the ranking of severity. The utility values are obtained using Eqn. (3) and the membership function for severity is derived using Eqn. (6). In the same approach, the membership functions for the occurrence and detection can be derived.

But the process has many disadvantages. Such as it takes long time for performance and requires too much calculation for getting to utility values. Also each three RPN factors have the same importance. For example if 5 engineers take part in FMEA team, since for each RPN factors, there are 10 rating and for RPN 3 factors(S,O,D), therefore 100 calculation are needed to achievement for cost values and 150 calculation for utility values. In order to solve this disadvantage, we should describe  $U_s$ ,  $U_o$ ,  $U_d$  in away that the FMEA method can be performed easily by users and reached to desirable results. In this study in order to provide a new FMEA model in fuzzy mode and considering cost index, to reach a completely new fuzzy membership function  $U_o$ ,  $U_s$  and  $U_d$  are defined and modified as follows.

$$U_o = \frac{-1}{\log P_o} \quad U_d = \frac{C_{dij}}{\sum_{j=1}^n C_{dij}} \quad U_s = \frac{C_{sij}}{\sum_{j=1}^n C_{sij}} \quad (7)$$

$i= 1, 2, 3, \dots ; n=$  Engineers number;  $j= A, B, C, \dots$  (Failure modes);  $P_o=$  The failure event probability

### 3. Benefits and Advantages of the New Functions to the Previous Dong Functions

1. In functions proposed, by using relations (7), we will get normalized numbers. Because these relations exactly are the same with relation (8) for normalization in tables comparing numbers in AHP method.

$$r_{ij} = \frac{a_{ij}}{\sum_{i=1}^m a_{ij}} \quad (8)$$

2. Dong process method is very time consuming and needs long computing for achieving utility values. For example if 5 engineers exist in FMEA team, by considering for each RPN factors, there is 10 rank and cost values is needed for detection and severity factors. Therefore need to be calculated 100 numbers ( $5 \times 10 \times 2 = 100$ ) and for utility values because of for each RPN factors, there is 10 rank and utility values is needed for detection and severity and occurrence factors, so need to calculate 110 numbers for determining utility values ( $5 \times 10 \times 2 = 100$  for detection and severity factors and 10 numbers for occurrence factor). In Dong method we have to calculate 210 number, while using relation 7 only need to calculate 110 numbers (100 numbers for detection and severity cost values and 10 numbers for calculating occurrence utility values).

3. Work with proposed new functions is very easy and there is no need to draw detection and severity utility values diagram. As a result, using  $U_s$  and  $U_o$  and  $U_d$  proposed in this article, new fuzzy membership function will be follows form.

$$\mu(RPI) = \sqrt[3]{\mu\left(\frac{C_{sij}}{\sum_{i=1}^n C_{sij}}\right)\mu\left(\frac{-1}{\log P_o}\right)\mu\left(\frac{C_{dij}}{\sum_{i=1}^n C_{dij}}\right)} \quad 9$$

It should be mentioned that the engineers based on their experience and knowledge, we can get particular weight which this weight should be between zero and one and total weights for all engineers, should be one. For example if 5 engineers exist in FMEA team, their weights can be 0.15, 0.20, 0.30, 0.25, 0.1 respectively that total weights for them is one. According to weight ( $W$ ) for each engineer and considering their knowledge level in the cost calculation for RPN factors new fuzzy function (10) can to be defined as follows:

$$\mu(RPI) = \sqrt[3]{W_{ij} \left( \mu\left(\frac{C_{sij}}{\sum_{i=1}^n C_{sij}}\right)\mu\left(\frac{-1}{\log P_o}\right)\mu\left(\frac{C_{dij}}{\sum_{i=1}^n C_{dij}}\right) \right)} \quad (10)$$

Case study shows that work with the new fuzzy membership function is very simple and requires no long and complex calculation. As in part of the traditional FMEA disadvantages mentioned, one of the major disadvantages of the traditional FMEA is allocate equal weight to each indicator of the RPN that it has criticized by a lot of authors such as a Gilchrist [10]. Now in order to overcome this limitation in traditional FMEA method, the AHP process in order to obtain a more reliable indicator in this research in fuzzy mode is used. Therefore considering cost and time criteria a pair of comparison between indices (detection, occurrence, severity) is carried out. So by using a questionnaire we did these comparisons in Tractor Company Tabriz-Iran in quality control section to related AHP matrix derive. The results of the questionnaire are summarized in Table 3.

Table 1: Ranking of occurrence

Probability	Likely Failure Rates	Ranking
Very High: Persistent Failures	$\geq 100$ per thousand pieces	10
	50 per thousand pieces	9
High: Frequent Failures	20 per thousand pieces	8
	10 per thousand pieces	7
	5 per thousand pieces	6
Moderate: Occasional Failures	2 per thousand pieces	5
	1 per thousand pieces	4
	0.5 per thousand pieces	3
Low: Relatively Few Failures	0.1 per thousand pieces	2
	$\leq 0.01$ per thousand pieces	1

Table 2: Assigned weights

Index	Weight
Detection	0.4809
Severity	0.3538
Occurrence	0.1652

Now after finding related weights to each RPN factors and by considering (10) formula, we can define new RPI as follows:

$$\mu(RPI) = \sqrt[3]{W_{ij} \left( 0.3538 \mu\left(\frac{C_{ij}}{\sum_{i=1}^n C_{ij}}\right) + 0.1652 \mu\left(\frac{1}{\log Po}\right) + 0.4809 \mu\left(\frac{C_{dij}}{\sum_{i=1}^n C_{dij}}\right) \right)} \quad (11)$$

#### 4. Case study

As a case study new created fuzzy membership function is expanded based on Dong example to clear its effects in prioritization. In Dong method 3 failure mode by traditional FMEA and FMEA based on FUT have evaluated respectively.

Table 3: Traditional FMEA for 3 failure mode [1]

Failure mode	Severity	Occurrence	Detection	RPN
A	9	4	3	108
B	3	9	4	108
C	4	3	9	108

Table 3 shows that for these three failure modes, the traditional FMEA gives the equal RPN while the FUT-based FMEA gives different RPI's. Thus, the failure modes can be better prioritized using the FUT-based FMEA. When the FUT-based FMEA is used, the cost due to failure can be assigned. Cost and utility values for severity and detection of failure mode by 5 engineers in figure (1), (2), (3), (4) is shown.

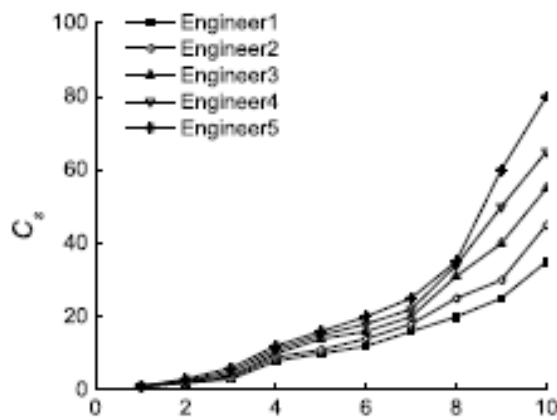


Figure 1: Cost value for severity

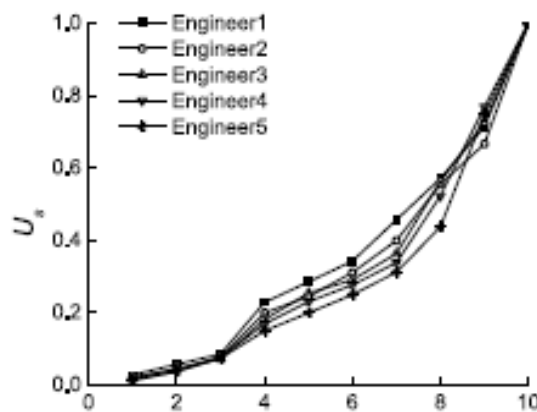


Figure 2: Utility value for severity

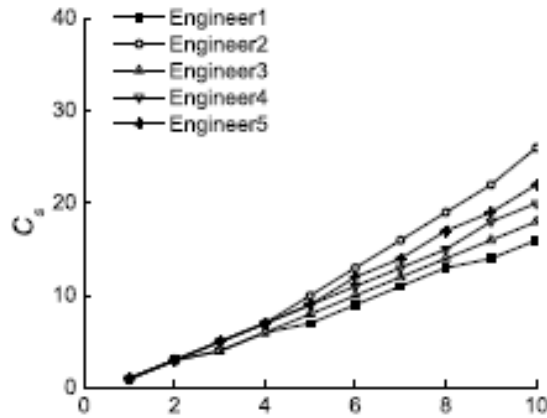


Figure 3: Cost value for Detection

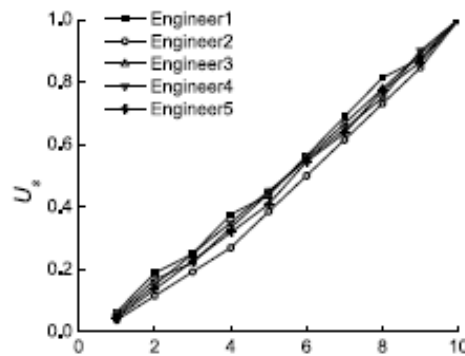


Figure 4: Utility value for Detection

Utility values for the occurrence index through Table (1) are obtained. It should be mentioned that these values are the same for each engineer. Detection, Occurrence, Severity ranking are shown in Table4.

Table 4: Ranking and utility values for severity, occurrence, and detection

Failure mode	Engineer	Severity		Occurrence		Detection	
		Ranking	$U_s$	Ranking	$U_o$	Ranking	$U_d$
A	Engineer 1	9	0.714	4	0.333	3	0.250
	Engineer 2	8	0.556	5	0.371	3	0.192
	Engineer 3	8	0.564	4	0.333	2	0.167
	Engineer 4	9	0.769	4	0.333	3	0.250
	Engineer 5	9	0.750	5	0.371	3	0.227
B	Engineer 1	3	0.086	9	0.769	4	0.375
	Engineer 2	3	0.078	8	0.589	5	0.385
	Engineer 3	2	0.045	8	0.589	4	0.333
	Engineer 4	3	0.077	9	0.769	4	0.350
	Engineer 5	3	0.075	9	0.769	5	0.409
C	Engineer 1	4	0.229	3	0.303	9	0.875
	Engineer 2	5	0.244	3	0.303	8	0.731
	Engineer 3	4	0.182	2	0.250	8	0.778
	Engineer 4	4	0.169	3	0.303	9	0.900
	Engineer 5	5	0.200	3	0.303	9	0.864

Table 5: Risk indices after defuzzification

Failure mode	(RPI) risk priority index (COM method)
A	0.362
B	0.254
C	0.360

After defuzzification, the RPI's of failure modes A-C are shown in Table 5. Now new created fuzzy membership function is expanded based on previous example to clear its effects in prioritization. Then for

testing the proposed method the numbers related to the previous example and just three Figures (1), (3) and table (1) are used.

Table 6: Ranking of severity, occurrence, and detection

Failure mode	engineer	$W_{ij}$	ranking	cost (C <sub>ij</sub> )	$U_s = \frac{C_{s_{ij}}}{\sum_{n=1}^n C_{s_{ij}}}$	$W_{ij} \times U_s$	ranking	$U_o = \frac{-1}{\log P_o}$	$W_{ij} \times U_o$	ranking	Cost(C <sub>d</sub> )	$U_d = \frac{C_{d_{ij}}}{\sum_{n=1}^n C_{d_{ij}}}$	$W_{ij} \times U_d$
A	1	0.15	9	22	0.1506	0.0226	4	0.333	0.04995	3	3	0.1428	0.0214
	2	0.20	8	23	0.1575	0.0315	5	0.371	0.0742	3	5	0.238	0.476
	3	0.30	8	32	0.2191	0.0658	4	0.333	0.0999	2	3	0.1428	0.0428
	4	0.25	9	35	0.2397	0.0599	5	0.333	0.08325	3	5	0.238	0.0595
	5	0.10	9	36	0.2465	0.0246	9	0.371	0.0371	3	5	0.238	0.0238
					$\sum_{i=1}^n C_{s_{ij}} = \sum_{i=1}^n U_s = 1$							$\sum_{i=1}^n C_{d_{ij}} = \sum_{i=1}^n U_d = 1$	
				146							21		
B	1	0.15	3	3	0.1666	0.0249	9	0.769	0.11535	4	7	0.1666	0.0249
	2	0.20	3	4	0.2222	0.0444	8	0.589	0.1178	5	11	0.2619	0.0523
	3	0.30	2	2	0.1111	0.0333	8	0.589	0.1767	4	7	0.1666	0.0499
	4	0.25	3	4	0.2222	0.0555	9	0.769	0.19225	4	8	0.1904	0.0476
	5	0.10	3	5	0.2777	0.0277	9	0.769	0.0769	5	9	0.2142	0.0214
					$\sum_{i=1}^n C_{s_{ij}} = \sum_{i=1}^n U_s = 1$							$\sum_{i=1}^n C_{d_{ij}} = \sum_{i=1}^n U_d = 1$	
				18							42		
C	1	0.15	4	6	0.1034	0.0155	3	0.303	0.04545	9	13	0.1585	0.0237
	2	0.20	5	12	0.2068	0.0413	3	0.303	0.0606	8	19	0.2317	0.0463
	3	0.30	4	10	0.1724	0.0517	2	0.250	0.075	8	13	0.1585	0.0475
	4	0.25	4	12	0.2068	0.0517	3	0.303	0.07575	9	18	0.2195	0.0548
	5	0.10	5	18	0.3104	0.0310	3	0.303	0.0303	9	19	0.2317	0.0231
					$\sum_{i=1}^n C_{s_{ij}} = \sum_{i=1}^n U_s = 1$							$\sum_{i=1}^n C_{d_{ij}} = \sum_{i=1}^n U_d = 1$	
				58							82		

Table 6 compares the new fuzzy membership function calculations with those of Dong method. In the third column ( $W_{ij}$ ) for each engineers as a innovation, a particular weight between zero and one is considered that the total value of them is equal to one. Then each of these weights in the calculation of each fuzzy membership function (detection, severity, and occurrence) are used. In the fourth column (rank), considered ranks for each failure mode by each engineers, for the severity factor is written. On the base of these ranks, using 1, 3 figures and table 1, related cost (in the fifth column) are obtained. According to C<sub>ij</sub>s obtained, fuzzy utility values by new method in the sixth column is presented that convey Superiority of this method in comparison with Dong method in terms of comfort calculations. To calculate fuzzy utility values for severity factor, it only needs to divide related cost for each engineer to the total costs obtained from five engineers. The calculations in column 7 up to last column are similar to those of columns one to six. Based on the relationship (11) and fuzzy membership Function obtained through the table (6), RPI of each failure mode is obtained (Figure 5).

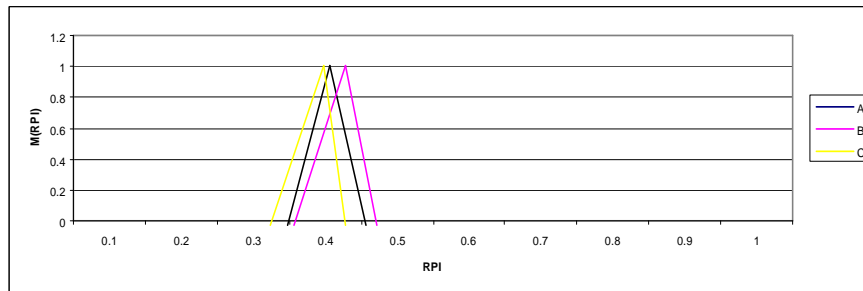


Figure 5: The membership function for RPI

Table 7: Risk indices after defuzzification

Failure mode	(RPI)
A	0.3495
B	0.37065
C	0.3127

The membership function for the RPI needs to be defuzzified to obtain the RPI value. We have used COM method in this study. In the COM method, the average of the minimum utility value and the maximum utility value is considered to be the expected RPI. The results in Table (7) shows that priority failures, will be respectively B and A and C. While priorities in the first example were A and B and C. The advantage of the new priority will be discussed at the end of section (5-1) (We will show that in new priority we can improve more failure modes than Dongs method).

### 5- Optimal mix of failure selection

Not all failures can always be repaired or avoided due to corrective actions' high costs. Only a specific mix of failure modes can be modified according to a specific budget provided by the firm. Therefore we need an algorithm able to find the optimal mix of faults to be repaired as well as to obtain the highest sum of their C.I., imposing bound on the cost that the firm is incline to invest in these problems. Using the simple simplex algorithm we can obtain the following relations [10]:

$$\max \sum_{j=1}^n X_j C.I._j$$

$$\sum_{j=1}^n X_j \times \text{cost}_j \leq \text{budget} \quad X_j \in \{0,1\}$$

with j is a generic fault, xj is a variable that can assume the value 0 if the action is not developed and 1 if the action is developed, C.I. is the Critical Index of j-th fault. The budget is the available budget for the corrective actions. For example if firm budget to be considered 50 currency, First the cost of each failure modes should be calculated using the Table (6). Then based on obtained costs for each failure modes and considering limited budget for improving potential failure modes, Final decision on the selection of failure modes for corrective action occurs. Estimated cost of each failure modes are as follows:

#### 5-1 The cost of failure mode A:

By considering that in COM method (defuzzification method) RPN values of each failure modes obtain through  $U_l$  and  $U_u$ , So costs of failure modes based on COM method can be estimated. By considering the relations obtained in the previous chapter and Table (6) we have:

Table 8: Risk indices after defuzzification and Cost of action

Failure mode	(RPI)	Cost of action(currency)
A	0.3495	31
B	0.37065	13.5
C	0.3127	26.5

$$U_{lA} = \sqrt[3]{0.3538 \times 0.0226 + 0.1652 \times 0.0371 + 0.4809 \times 0.0214} = 0.2901$$

$$U_{uA} = \sqrt[3]{0.3538 \times 0.0658 + 0.1652 \times 0.0999 + 0.4809 \times 0.0595} = 0.4089$$

$$C_{d_{3,4}} = 5 \quad C_{s_{8,3}} = 32$$

$C_{s_{9,1}} = 22$  and  $C_{d_{3,1}} = 3$  are the Costs related to the minimum of  $W_{ij} \times U_s$  in the table (6) and  $C_{s_{8,3}} = 32$  and  $C_{d_{3,4}} = 5$  are the Costs related to the minimum of  $W_{ij} \times U_d$  in the same table (Costs for the occurrence factor is not

considered, because in calculating the utility value for this index, there is no need to calculate costs for occurrence factor and utility values for this index directly obtain through  $U_o = \frac{-1}{\log p_o}$ ). Therefore, the total cost for  $U_i$  will be equal 25(22+3) and the total cost for  $U_u$  will be equal 37(32+5). Now based on COM method we should calculate mean of these cost ie 37 and 25 that it will be 31. For other failure modes, costs are calculated in the same form. Now based on obtained costs for each failure mode (Table 8) and considering limited budget (50 currency), final decision on the selection of failure modes for corrective action occurs. We will choose B and A failure modes for corrective action. While without considering limited budget, all three failures were amendable. Therefore, considering limited budget is one of the most important factors in choosing failure modes for improve. Also new priority shows that in new method B and A failure modes are selected for corrective actions, while in Dong method only failure mode A can be chosen for corrective action. It means that in new method we can improve more potential failures than Dong method and this is most important Superiority our new method.

## 6. Conclusion

The same way that was expressed in introduction, Traditional FMEA uses RPN to prioritize failure modes. Since the three indices used for RPN calculation are ordinal variables, thus producing these three descriptive variables can not be define real costs due to failures. Therefore, in this research a new fuzzy membership function for RPN with regard to cost Criteria were defined that advantage of the new fuzzy membership function is that the cost criteria is included and also the level of experience and comments on each engineers has been considered and each RPN factors have an specific important to each other based on cost and time criteria. In addition, this method includes expert knowledge and if this people not be available, this way can use their knowledge. Therefore, it doesn't have limitation related to the availability of a strong team in the old method. Finally, the case study shows this new membership function calculate actual costs due to failures and can better prioritize failure modes. Thus this method provides effective and convenient tool for failure analysis and improves FMEA implementation in failure and risk analysis for design and manufacturing production and assembly lines.

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