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# A Pair of Large-Incidence-Angle Cylinders in Cross-Flow with the Upstream One Subjected to a Transverse Harmonic Oscillation

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## Abstract

Cross-flow past a pair of circular cylinders, with pitch ratio 2.0 and stagger angle $45^0$, and the upstream one subjected to forced harmonic oscillation in the transverse direction is investigated experimentally for Reynolds numbers, Re = 515-730. Flow-visualization and hot-film measurements of the wake formation region are reported. Results show that when the cylinders are stationary an integral relationship exists between two distinct Strouhal numbers. Oscillation of the upstream cylinder causes considerable modification of the wake vis-à-vis when the cylinders are stationary. In particular, there are distinct regions of synchronization between the dominant wake periodicities and the cylinder oscillation.

## Keywords
Staggered cylinder pair, flow visualizations, flow patterns, fundamental synchronization, sub- and superharmonic synchronizations.

## 1. Introduction

The cross-flow around a single, isolated cylinder has been studied for years, and may now be considered to be reasonably well understood. Although the flow around small group of cylinders has only recently received significant attention, and is not so well understood, it has been shown that there are significant new complexities vis-à-vis the flow around an isolated cylinder. These include interactions between the shear layers, vortices and Kármán vortex streets shed by the individual cylinders.

The flow around a pair of staggered cylinders was first classified by Zdravkovich [1], who, based on the relative position of the cylinders, identified three different flow regimes: (i) *no interference*, where the flow around each cylinder is effectively identical to that around a single cylinder; (ii) *wake interference*, where one cylinder is partially or completely submerged in the wake of the other; and (iii) *proximity interference*, where the cylinders are close to each other, but neither is submerged in the wake of the other. However, Sumner et al. [2] proposed that the flow around two cylinders is much more complex than this. They identified nine different flow patterns, which fall into three main categories: *single bluff-body* flows where there is almost no flow between the cylinders, *small-incidence* flows where there is a small gap flow between the cylinders and *large-incidence* flows where there is sufficient gap flow for both cylinders to shed vortices.

Considering the additional complexities arising from the oscillation of a single cylinder compared with when it is stationary it is expected that the flow around a pair of cylinders will also become complex when one or both are subjected to forced oscillation. For a single cylinder one of the most signification phenomena associated with forced oscillation is the synchronization of the shedding frequency, $f_0$, with the excitation frequency, $f_e$. This synchronization can take the form of a fundamental lock-in ($f_0 = f_e$), although both sub- and superharmonic relationships between the frequencies are also observed [3, 4].

The aim of the present study is to investigate the near wake structure and synchronization phenomena for a pair of staggered circular cylinders at a large-incidence-angle when the upstream one is subjected to forced harmonic oscillation transverse to the flow direction. The configuration chosen has a centre-to-centre pitch ratio $P/D$ = 2.0 and stagger angle $\alpha = 45^0$ at the mean position of the cylinders (See Figure 1 for the definition of $P/D$ and $\alpha$). The cylinder oscillation employed has a peak-to-peak magnitude of 0.44*D*, which results in the relative transverse separation between cylinders being in the range $1.20 \leq T/D \leq 1.63$, while the longitudinal separation between cylinders remains constant at $L/D = 1.41$ (the transverse, *T*, and longitudinal , *L*, separation between cylinders are defined in Figure 1). According to Sumner et al. [2], when the cylinders are stationary this configuration belongs to the *large-incidence* regime, with the specific flow patterns being what was referred to as vortex pairing, splitting and enveloping (VPSE) for $T/D < 1.35$ and synchronized vortex shedding (SVS) for $T/D > 1.35$. For a VPSE flow the two shear layers shed in the gap between the cylinders, which have opposing circulation, initially pair up and are then partially enveloped by the outer shear layer shed from the upstream cylinder. The gap shear layer shed by the upstream cylinder then splits into two, one portion of which is enveloped, along with the gap shear layer shed by the downstream cylinder, by the shear layer shed from the outer surface of the upstream cylinder. The SVS flow pattern is characterized by a strong synchronization of the shear layers shed on either side of the gap between the cylinders as well as the resulting Kármán vortices. Within the combined wake, these two opposite-sign vortices pair up, resulting in two adjacent Kármán vortex streets that exhibit anti-phase synchronization. The most important feature and commonality of these two flow patterns is that the gap flow between the cylinders results in two different-width near-wakes, a narrow wake behind the upstream cylinder and a wide one behind the downstream cylinder. Consequently, two distinct Strouhal numbers exist which are associated with the vortex shedding frequencies of the two differently sized near-wake regions.

## 2. Experimental Apparatus and Procedure

The experiments were performed in a closed-circuit Kempf and Remmers water tunnel which has a test section 110 cm long and 254 mm × 254 mm in cross-section with a streamwise turbulence intensity of 0.5% and a velocity profile uniformity of better than 95%. The two cylinders were mounted vertically in the test section, and a scotch-yoke mechanism was employed to oscillate the upstream one with a peak-to-peak amplitude of 7 mm or 0.44*D*. The cylinders, made of plexiglas, have the diameter of 16 mm with aspect ratio 16 and blockage ratio 6.3%. A two-fold experimental approach, consisting of dye-injection flow visualization and hot-film anemometer measurements, was employed. Flow visualization was carried out using Rhodamine dye which was injected at the mid-span of each cylinder from two ports of 0.8 mm diameter and $120^0$ apart. The flow images revealed by the dye were recorded by a professional S-VHS video camera and then digitized by using Pinnacle Studio Movie Box. Spectral measurements of the flow in the wake were conducted using a constant temperature hot-film anemometer. The analog output of the anemometer system was acquired by a PMD 1208LS system from Measurement Computing. The raw voltage data were stored digitally and velocity spectra were calculated with a dimensionless frequency resolution of ±0.04.

Initially experiments were conducted with the cylinders stationary at the $T_{min}$, $T_{nom}$ and $T_{max}$ (minimum, nominal and maximum transverse pitch) positions. In the second phase of the experiments the upstream cylinder was forced to oscillate transverse to the flow. The frequency of oscillation $f_e$ was incremented such that the frequencies of the various synchronization regimes were spanned. Based on the cylinder diameter, *D*, and the mean flow velocity, *U*, the experimental results presented here correspond to Reynolds number in the range $515 \leq \mathrm{Re} \leq 730$.

To investigate all of the wake periodicities the hot-film probe was placed at two locations, *a* and *b*, see Figure 1. Measurements at position *a* revealed the periodicities resulting from the interaction of the two gap shear layers and the outer shear layer shed from the upstream cylinder, whereas the measurements at position *b* revealed those shed from the outer shear layer of the downstream cylinder.

## 3. Results

### 3.1 Stationary Cylinders

Two sample wake spectra measured at the $T_{min}$ position for probe locations *a* and *b* are shown in Figure 2. From these and other results the Strouhal numbers obtained for different orientations of the cylinders are summarized in Table 1.

The results presented in Table 1 show the existence of two distinct Strouhal numbers $\mathrm{St}_{CW} = 0.15$ and $\mathrm{St}_{SL} = 0.45$ over the complete range of *T/D*. The spectral measurements conducted at position *a* yielded the higher Strouhal number, $\mathrm{St}_{SL}$, which as ascertained from the flow visualization (to be discussed later) is associated with the shear layer periodicities; the measurements at position *b* yielded the lower Strouhal number, $\mathrm{St}_{CW}$, which is associated with a dominant periodicity in the combined wake of the cylinders. It is evident that for the whole range of *T/D*, $St_{CW} \approx St_{SL}/3$. Alam and Sakamoto [5] and Sumner and Richards [6] also obtained two distinct Strouhal numbers for similar cylinder configurations. Alam and Sakamoto [5] obtained St = 0.14 and 0.34 at $Re = 5.5 \times 10^4$ and Sumner and Richards [6] obtained St = 0.15 and 0.34 at $Re = 3.2 \times 10^4$. Although the lower Strouhal number is almost the same for all three sets of experiments, the higher Strouhal number obtained by either Alam and Sakamoto [5] or Sumner and Richards [6] is smaller than that obtained in the present case. Also of significance is that the present experiments yielded an integral relationship between the two Strouhal numbers ($St_{CW} \approx St_{SL}/3$), whereas no such integral relationship was found by the other authors. An integral relationship of 3 between the two Strouhal numbers was also observed by Hayder and Price [7] for $P/D = 2.5$ and $\alpha = 21^0$ at Re = 525-750.

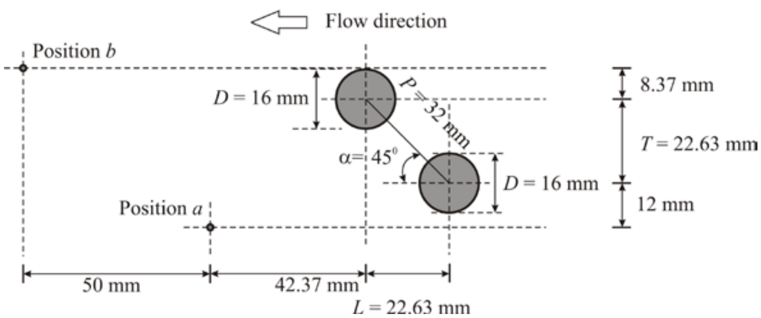



Figure 1: Schematic of cylinder position and hot-film locations

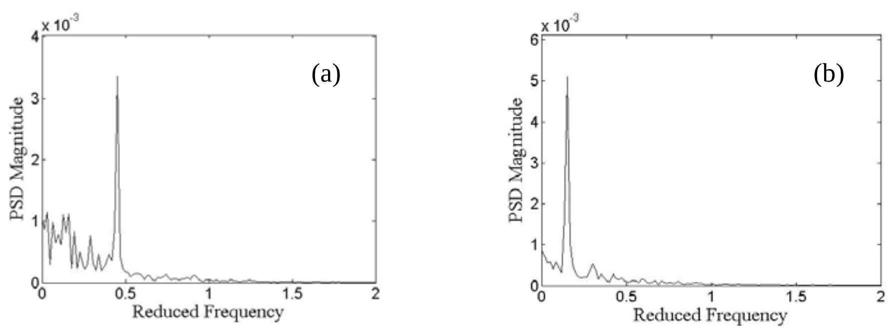


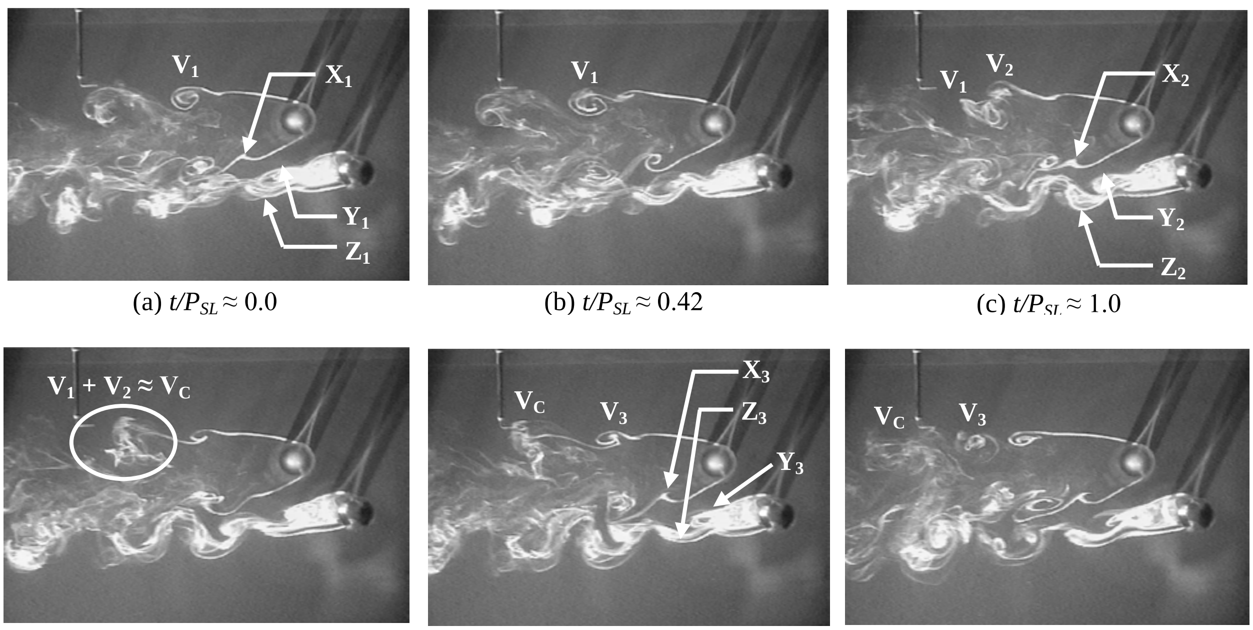

Figure 2: Wake spectra measured at (a) location *a* and (b) location *b* with both cylinders held stationary at the $T_{min}$ position. The magnitudes of the PSDs are in arbitrary units.

Table 1: Strouhal numbers measured in the wake of the stationary cylinders.

| | Probe position *a* | Probe position *b* |
|---|---|---|
| $T/D = 1.20$ ($T_{min}$ position) | $St_{SL} = 0.45$ | $St_{CW} = 0.15$ |
| $T/D = 1.41$ ($T_{nom}$ position) | $St_{SL} = 0.46$ & $St_{CW} = 0.15$ | $St_{CW} = 0.14$ |
| $T/D = 1.63$ ($T_{max}$ position) | $St_{SL} = 0.45$ | $St_{CW} = 0.15$ |

To help interpret the spectral measurements, flow visualization was carried out at all three positions, $T_{max}$, $T_{nom}$, and $T_{min}$. Figure 3 shows the flow visualization images for two stationary cylinders at the $T_{max}$ position for three periods of the flow periodicity with St = 0.45. Since the two Strouhal numbers for each of the three positions of the cylinders are associated with the same types of periodicities, the images at the $T_{max}$ position, which gave better flow visualization, are taken as the reference to interpret the spectral measurements for the whole range of *T/D*.

The series of images in Figure 3 shows three successive periods of shear layer separation from both sides of the cylinders. Vortices $V_1$, $V_2$ and $V_3$ are shed from the outer surface of the downstream cylinder in frames (a), (c) and (e), respectively. Three cycles of vortex shedding also occur from the other three surfaces during each period. Vortices $X_i$, $Y_i$ and $Z_i$ are shed from the gap side of the downstream cylinder, gap side of the upstream cylinder and outer side of the upstream cylinder, respectively, where subscript *i* indicates whether they were shed in the first, second or third cycles. Therefore, all four vortices from the two cylinders are shed at the same frequency, giving the Strouhal number $St_{SL}$ = 0.45. Further analysis suggests that the lower Strouhal number, $St_{CW}$ = 0.15, is associated with the flow on the outer side of the downstream cylinder wake. Frames (a) - (d) show that the two vortices $V_1$ and $V_2$, shed during the first and second periods as observed in frames (a) and (c), respectively, combine with each other in frame (d), and then propagate as one composite vortex $V_C$ on the outer side of the downstream cylinder. However, the next vortex $V_3$, generated during the third period of shear layer separation as shown in frame (e), propagates as an independent vortex in a street behind the composite vortex $V_C$. Therefore, from the three successive cycles of shear layer separation from the outer side of the downstream cylinder, the first two coalesce to form a composite vortex and the next one propagates as an independent vortex. Observation of the flow visualization video suggests that after the shedding of the independent vortex the next two coalesce again to form another composite vortex. Thus, during the period while three shear layers are separated from the cylinder surfaces, one independent vortex is generated from the outer shear layer of downstream cylinder which does not take part in any coalescence process. This vortex is responsible for producing the combined wake with a dominant frequency 1/3 of that for the shear layer separation, yielding the Strouhal number, $St_{CW}$ = 0.15 and leading to an integral relationship of 3 between the two Strouhal numbers. This is contradictory to the conclusion of Sumner and Richards [6] where it was suggested that the two distinct Strouhal numbers were due to the four shear layers being separated at two different frequencies.




(a) $t/P_{SL} \approx 0.0$ (b) $t/P_{SL} \approx 0.42$ (c) $t/P_{SL} \approx 1.0$

(d) $t/P_{SL} \approx 1.58$ (e) $t/P_{SL} \approx 2.0$ (f) $t/P_{SL} \approx 3.0$

Figure 3: Flow visualization images with both cylinders statically held at $T/D$ = 1.63 ($T_{max}$ position), the time as a fraction of one period, $P_{SL}$ of the flow periodicity assuming $St_{SL}$ = 0.45 ($P_{SL}$ = period of one shear layer separation).

### 3.2 Upstream Cylinder Oscillation

In the second stage of the experiments, a transverse oscillation was imposed on the upstream cylinder within the frequency range $0.07 \leq f_eD/U \leq 1.20$ with the downstream one being held stationary. Flow visualization experiments suggest that for $f_eD/U \leq 0.10$, oscillation of the cylinder causes the wake structure to shift from a VPSE- to a SVS-type flow as the cylinder separation changes from $T_{min}$ to $T_{max}$. Also, the frequencies of the dominant wake periodicities at these lower oscillation frequencies are approximately equal to those measured when the cylinders are stationary. The wake starts undergoing modification caused by the increased cylinder oscillation for $f_eD/U > 0.10$. The shear layers shed from the mean-flow surface of the downstream cylinder synchronize with the cylinder oscillation. On the other side of the combined wake the flow is synchronized with the cylinder oscillation via either a complete enveloping of the two “paired-up” gap shear layers by the outer shear layer shed from the upstream cylinder, referred to as a vortex pairing and enveloping (VPE) process, or by a VPSE process.

The wake spectral measurements obtained at positions *a* and *b* are summarized in Figure 4, where the nondimensional frequencies of the wake periodicities, $f_0D/U$, are presented as a function of the nondimensional cylinder excitation frequency, $f_eD/U$; the figure also shows which was the most dominant of the frequencies in the wake spectrum. Based on the results shown in Figure 4 and the flow visualization it is concluded that there are a number of distinct frequency regimes where the wake periodicities resonate with the cylinder oscillation. A 1/2-subharmonic synchronization ($f_0/f_e = 2$) occurs for the periodicities shed on the mean-flow side of the downstream cylinder at $f_eD/U = 0.08$. As $f_eD/U$ is increased, this is followed by a fundamental synchronization ($f_0/f_e = 1$) for $0.13 \leq f_eD/U \leq 0.57$, where the periodicities shed on the mean-flow side of the upstream cylinder resonate with the cylinder oscillation, see Figure 4(a). However, as seen from Figure 4(b), the fundamental synchronization of periodicities on the mean-flow side of the downstream cylinder exists up to $f_eD/U = 0.24$ only. Moreover, a 2-superharmonic synchronization ($f_0/f_e = 1/2$) of the periodicities on the mean-flow side of the downstream cylinder occurs for $0.32 \leq f_eD/U \leq 0.43$. Figure 4(a) indicates that the wake switches to a 2-superharmonic synchronization for $0.61 \leq f_eD/U \leq 0.79$, where only the periodicities on the mean-flow side of the upstream cylinder resonate with the cylinder motion.

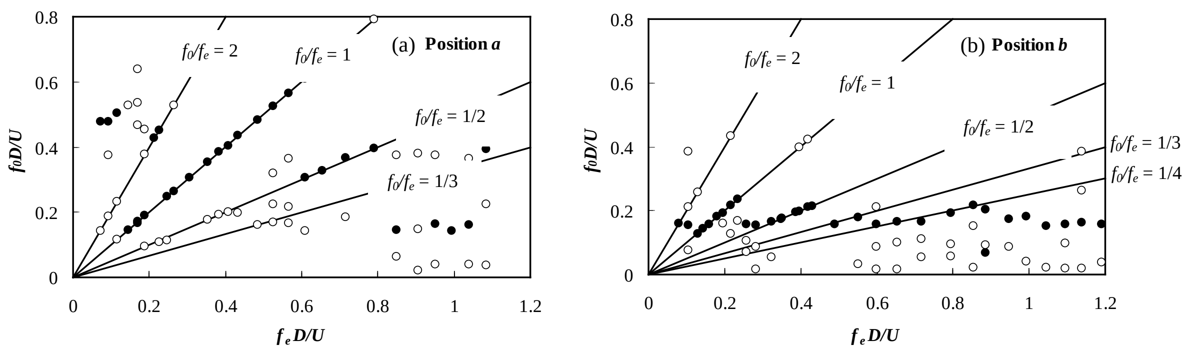



Figure 4: Distinct frequency peaks, $f_0D/U$, obtained from the wake spectra measured at positions (a) *a* and (b) *b* for upstream cylinder oscillation $f_eD/U$: ●, dominant wake periodicity; ○, other wake periodicity; —, lines showing the integral relationships between the wake and excitation frequencies.

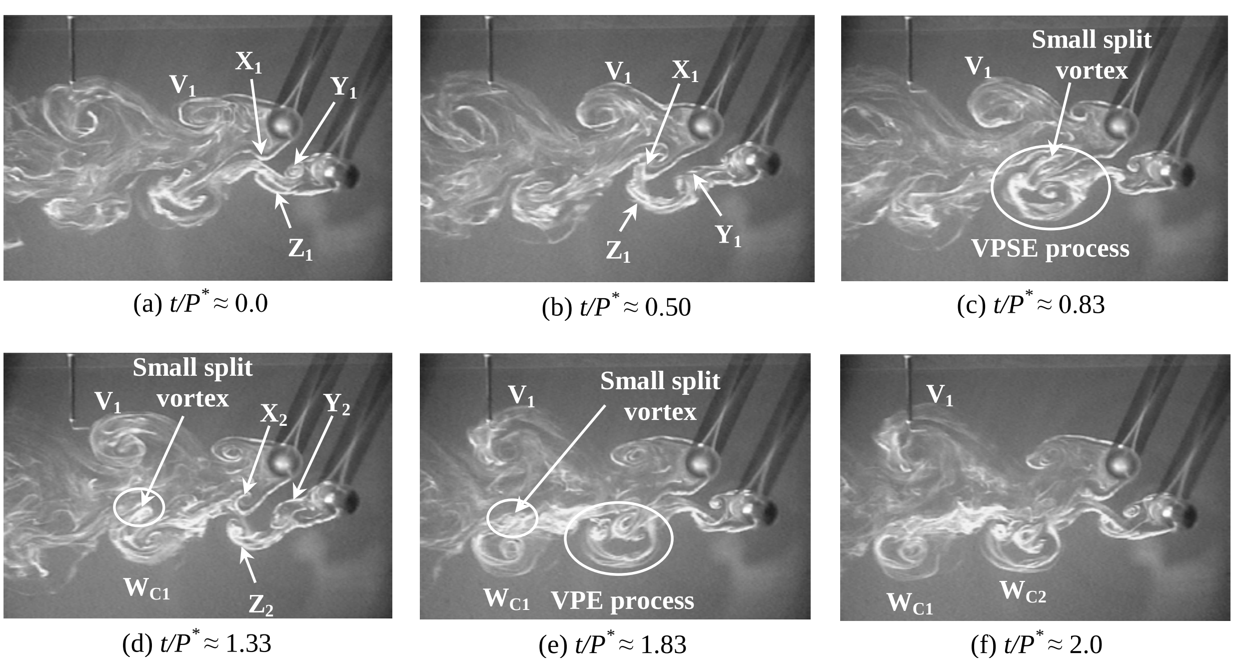



Figure 5: Flow visualization images with upstream cylinder oscillating at $f_eD/U \approx 0.42$. Two complete cycles of oscillation are shown, time as a fraction of one period of the cylinder oscillation ($P^*$).

A sample set of flow visualization images for two complete cycles of upstream cylinder oscillation at $f_eD/U \approx 0.42$ is presented in Figure 5. At this $f_eD/U$ a fundamental synchronization of the periodicities on the mean-flow side of the upstream cylinder exists along with a 2-superharmonic synchronization of those on the mean-flow side of the downstream cylinder. Vortices $X_1$ and $X_2$ are shed from the gap surface of the downstream cylinder during the first and second cycles of oscillation, respectively. The upstream cylinder also sheds one vortex from both of its two surfaces during each cycle of oscillation. Shear layers $Y_1$ and $Z_1$ and $Y_2$ and $Z_2$ separate from the gap and outer surfaces of the upstream cylinder while it completes the first and second cycles of oscillation, respectively. Frame (c) shows that during the first cycle of oscillation the gap-vortex-pair, consisting of vortices $X_1$ and $Y_1$, is partially enveloped by the outer vortex shed from the upstream cylinder, $Z_1$. Vortex $X_1$ then splits into two concentrations of vorticity, one of which propagates as a small vortex in the middle of the combined wake [frames (d) and (e)] whereas the other, along with vortex $Y_1$, is enveloped by the vortex $Z_1$ and forms a large composite vortex $W_{C1}$. Thus, the three vortices shed on the mean-flow side of the upstream cylinder during its first cycle of oscillation yield the composite vortex $W_{C1}$ via a VPSE process. During the second cycle of oscillation vortices $X_2$, $Y_2$ and $Z_2$ take part in a VPE process where the gap-vortex-pair, $X_2$ and $Y_2$, is enveloped completely by the outer vortex shed from the upstream cylinder, $Z_2$, yielding the second composite vortex, $W_{C2}$ [frames (e) and (f)]. Therefore, during two cycles of upstream cylinder oscillation two composite vortices are formed from the three vortices shed on its the mean-flow side. Figure 5 also shows one cycle of vortex formation on the mean-flow side of the downstream cylinder, $V_1$. Thus, every two cycles of the cylinder oscillation causes the propagation of two vortices on the mean-flow side of the upstream cylinder along with another one on the mean-flow side of the downstream cylinder—resulting in both a fundamental and 2-superharmonic synchronization in the combined wake.

## 4. Conclusions

An experimental investigation of the flow around a pair of circular cylinders with $P/D$ = 2.0 and $\alpha = 45^0$ and the upstream one subjected to forced harmonic oscillation transverse to the flow direction has been carried out for Reynolds numbers within the range $515 \leq Re \leq 730$. Results with cylinder excitation frequencies in the range $0.07 \leq f_eD/U \leq 1.20$ at a constant oscillation amplitude of 0.44*D* peak-to-peak are reported. For this amplitude of oscillation, the transverse pitch ratio between the cylinders varies within $1.20 \leq T/D \leq 1.63$ with a constant longitudinal pitch ratio of $L/D$ = 1.41 causing the cylinder geometry to traverse within the VPSE and SVS type flows. For stationary cylinders two distinct Strouhal numbers are observed in the wake; the higher of which, $St_{SL}$ = 0.45, is associated with the shedding frequency of the four shear layers while the lower, $St_{CW}$ = 0.15, is associated with the combined wake of the cylinder pair. An integral relationship between the two Strouhal numbers was obtained with $St_{SL} \approx 3\ St_{CW}$. When the upstream cylinder is forced to oscillate at low oscillation frequencies ($f_eD/U \leq$ 0.10), the flow around the cylinders remains essentially the same as that for the corresponding static case. However, at higher oscillation frequencies ($f_eD/U$ > 0.10) the flow changes considerably vis-à-vis the static case. More specifically, distinct regions of synchronization between the dominant wake periodicities and the cylinder oscillation are obtained, these synchronization regions involve fundamental as well as sub- and superharmonic resonances.

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