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An Integrated Supply Chain Model for the Supply Uncertainty Problem

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Abstract

This paper considers a dyadic supply chain with uncertain supplier. Both retailer and supplier use continuous review policy. The supplier is randomly available for supplying the retailer. The retailer faces Poisson demands. In this system, lead time is a random variable consists of a constant transportation time plus a random delay occurs due to availability of stock at the supplier. This paper considers both the integrity and the uncertainty in the above supply chain. We derive a good approximation for the total cost function of described system, as weighted mean costs of the one-for-one ordering policy. Finally, using simulation studies, we show that absolute errors are significantly ignorable.

Keywords
Continuous Review, Non-zero Random Lead Time, Poisson Demand, Supply Chain Management, Supply Uncertainty

1. Introduction
This paper investigates an integrated uncertain supply chain. An uncertain supply chain models are aimed to reach mathematical models of supply chain, considering uncertainty in supply, demand, lead time and the other parameters of supply chain. Up to now, the most of existing literature on the supply chain management has some major assumptions, to simplify analysis of problems. One of these assumptions that frequently used in the literature is reliable supplier. Gurler and Parlar state that this assumption is one of the unstated assumptions in almost every inventory model [1]. This assumption indicates that supplier is continuously available at any time an order is placed. There are several reasons for considering uncertainty on the supply, as are mentioned in the literature, equipment breaks, material shortages, strikes, and political crises to name but a few. A review of relevant literature indicates that existing models in uncertain supply literature can be classified into two main category, production-storage and inventory models [2]. The model presented in this paper belongs to second category.

Some early studies in inventory models with uncertain supplier use Economic Order Quantity (*EOQ*) assumptions. These works analyze problems under various characterization probability distributions that describing the *ON/OFF* periods [2]. In general, supplier could be considered to have available state *(ON)* or unavailable state *(OFF)*. Examples of papers belong to *EOQ* category are Parlar and Berkin [3], Weiss and Rosenthal [4], Parlar and Perry [5], Parlar and Perry [6], Gurler and Parlar [1] and Parlar [7]. In these studies demand is deterministic, lead time is zero and replenishments are instantaneous. The *EOQ* assumptions provide some bases for earlier studies. In the late of 1990s and after that, some researchers try to relax *EOQ* assumptions. A number of studies including Parlar et al.[8], Arreola-Risa and DeCroix[9], and Ozekici and Parlar[10] considered the problem in the context of inventory systems with random demand, zero lead time and unreliable suppliers. In 2006, Mohebbi and Hao [11] indicate that analytical treatment of inventory systems with random supply interruption and non-zero lead time remains largely unexplored and there are just a few existing models in the inventory control literature on this area.

Parlar [12] considered an unreliable supplier. He used continuous review inventory system with stochastic demand, random lead time and backorder. He extended Hadly and Within [13] approximation for understudy problem with assumption that at any time at most one order can be outstanding. Gupta [14] presented an exact cost minimization model for a continuous review inventory system with unit-sized Poisson demand, constant lead time and lost sales in which the supplier's *ON* and *OFF* periods are exponentially distributed and never more than a single order is outstanding at any time. In 2003, Mohebbi [15] develops an exact cost-minimization model for a lost sales, continuous review inventory system with compound Poisson demand and Erlang lead time under an (*s*,*Q*)-type control policy with at most one outstanding order at any time. Later, Mohebbi [2] presents the exact treatment of a related problem, assuming that the supplier's *ON* and *OFF* periods constitute an alternating renewal process and lead times follow a hyperexponential distributions. Mohebbi and Hao [11] study former problem (random demand, random lead time and lost sale) and assume that lead time follow Erlang distribution. They state that using Erlang distribution provides an effective means of analysis for the supply interruption problem under a broad spectrum of lead time distributions whose coefficients of variation do not exceed unity [11].

As pointed earlier, although inventory models with supply interruption and non-zero lead time present the most of real world problems, the existing literature on it is scarce. Our work provides an approximation integrated solution for a dyadic supply chain with supply interruption. Our model considers a case with Poisson demand, non-zero random lead time and backorder. The non-zero random lead time is the sum of constant transportation time and a random delay occurs due to availability of stock at the supplier. Also, in the case of unavailable supplier, the lead time, which retailer experiences, increases because of supplier's *OFF* state. Numerical examples indicate that the proposed approximation cost function works with negligible errors. In the other hand, the literature on the supply interruption problems reveals another gap. In the modern global competitive market, the supplier and the retailer should be treated as strategic partners in the supply chain with a long-term cooperative relationship. But, to the best of our knowledge, there are no integrated model considers supply interruption in the literature. Previous studies only aimed to determine the optimum solutions that minimized cost from the retailer's side, so the literature on integrated uncertain supplier is remain largely unexplored. Although, there are some studies that investigate the integrated supplier systems like Sajadifar et al. [16], but they assume that the supplier is available at any time an order is placed.

This paper deals with an integrated supply chain with non-zero random lead time, Poisson demand, backorder shortages, continuous review policy and finally uncertainty in supply i.e. it is considered that the supplier has available *(ON)* or unavailable *(OFF)* state. We assume that the *ON* and the *OFF* durations are exponential random variables. As far as we know, for the first time in the literature this paper aims to derive approximation cost function of described system. Our work develops previous models in the literature by using both the integrity and the uncertainty in supply. Our study, with described assumptions, presents the real world more realistic. The rest of the paper organized as follow. In section 2, the notations and assumptions, which used in the problem formulation, are described. Section 3 presents the mathematical model that is investigated in this study. Numerical examples and simulation studies are presented in section 4. Finally, section 5 summarizes the paper and presents future researches.

2. Problem Notations and Assumptions

In this paper following notations are used:

S_r : The inventory position for the retailer in the one-for-one ordering policy;

S_s : The inventory position for the supplier in the one-for-one ordering policy;

L_r : The transportation time from the supplier to the retailer where the supplier is in the *ON* state;

L_r' : The effective transportation time from the supplier to the retailer, that is the transportation time from the supplier to the retailer plus the expected value of the interruption incurs where the supplier is in the *OFF* state;

L_s : Transportation time from the outside source to the supplier;

λ_r : Demand intensity at the retailer;

R_r : The retailer reorder point;

Q_r : The retailer batch size;

$\boldsymbol{h}_r$: The holding cost per unit per unit time at the retailer ;

R_s : The supplier reorder point (in units of the retailer batches);

Q_s : The supplier batch size (in units of the retailer batches);

$\boldsymbol{h}_s$: The holding cost per unit per unit time at the supplier;

β : The shortage cost per unit per unit time at the retailer;

$c(S_s, S_r)$: The expected total holding and shortage costs for a unit demand in an inventory system with a one-for-one ordering policy where the supplier is in the *ON* state;

$c'(S_s, S_r)$: The expected total holding and shortage costs for a unit demand in an inventory system with a one-for-one ordering policy where the supplier is in the *OFF* state;

C	:	The expected total holding and shortage costs for understudy inventory system where the supplier is in the *ON* state;
C'	:	The expected total holding and shortage costs for understudy inventory system where the supplier is in the *OFF* state;
ζ	:	The exponential distribution parameter for the *ON* duration;
ψ	:	The exponential distribution parameter for the *OFF* duration;
K	:	The expected total holding and shortage costs for a unit demand;
TC_{total}	:	The total expected cost for the inventory system.

To find $c(S_s, S_r)$ and $c'(S_s, S_r)$ we express them as a weighted mean of costs for the one-for-one ordering policies. As we shall see, with this approach we do not need to consider the parameters $L_r, L_s, \boldsymbol{h}_r, \boldsymbol{h}_s, \beta$ and $\lambda_{\mathbf{r}}$ explicitly, but these parameters will, of course, affect the costs implicitly through the one-for-one ordering policy costs. To derive the one-for-one carrying and shortage costs, we suggest the recursive method in S.äter, [17]. A summary of S. Axsäter[17], which adapted for proposed dyadic system, is presented at appendix. In 1993 using procedure introduced by S. Axsäter [17] in 1990, S. Axsäter [18], expressed exact cost function for a dyadic supply chain when supplier and retailer use continuous review policy. Equation (1) expresses such a cost function [18].

$$C = \frac{1}{Q_s \cdot Q_r} \sum_{j=R_s+1}^{R_s+Q_s} \sum_{k=R_r+1}^{R_r+Q_r} c(jQ_r, k) \tag{1}$$

Also, in this paper the following assumptions are considered:

- Each customer demands only one unit of product.
- Delayed retailer orders are satisfied on a first-come, first-served policy.

3. Problem Formulation

In the above inventory system, when the supplier is in the *ON* or in the *OFF* state the lead time that experienced by the retailer is L_r or L'_r, respectively. L'_r is the sum of constant transportation time between the supplier and the retailer and the mean of interruption time, that is $1/\psi$. The state of supplier, in introduced uncertain supplier model, is a two state continues time Markov chain. The Fig. 1 presents our model's state transition diagram.

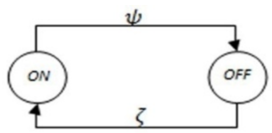

Figure 1: The state transition diagram

LEMMA 1: The steady state probability for ON state ($\mathbf{P_{ON}}$) and OFF state ($\mathbf{P_{OFF}}$) are as follow (Equations (2) and (3)):

$$P_{ON} = \frac{\zeta}{\zeta + \psi} \tag{2}$$

$$P_{OFF} = \frac{\psi}{\zeta + \psi} \tag{3}$$

Proof: The P_{ON} and P_{OFF} is easily obtain from solving flowing simultaneous equations $\zeta \times P_{ON} = \psi \times P_{OFF}$ and

$P_{ON} + P_{OFF} = 1$ [19]. As mentioned, according to continuous inventory control policy, when the retailer's inventory position reaches R_r and the supplier be in *ON* state, the retailer orders a batch with size Q_r. In this case, (1) expresses exact cost function. But, if when the retailer's inventory position reaches R_r the supplier be in *OFF* state, the effective lead time is $L'_r = L_r + {}^{1}/_{\psi}$. Using L'_r instead of L_r and in a similar way used for calculating C in (1), the cost function for *OFF* state is calculated. This cost function is named C'. It is worth mentioning that when one uses L'_r for calculating C', the related cost function can not be exact anymore. It is because of using ${}^{1}/_{\psi}$ as the mean time of being in *OFF* state from the time the retailer's inventory position reaches R_r. By using conditional probability, the expected total holding and shortage cost for a unit demand is expressed as follow (Equation (4)):

$$K = P_1 ON \times E(\blacksquare(cost\ where\ order\ to\ the\ supplier\ with\ @retailer^{\uparrow\prime}\ s\ transportation\ time, L_1 r\ )) + P_1 OFF \quad (4)$$

Using (1) to (4), we can rewrite the expected total holding and shortage costs for a unit demand as follow:

$$\rightarrow K = \frac{\zeta}{\zeta+\psi} \times \left[\frac{1}{Q_s . Q_r} \sum_{j=R_s+1}^{R_s+Q_s} \sum_{k=R_r+1}^{R_r+Q_r} c(jQ_r, k)\right] + \frac{\psi}{\zeta+\psi} \times \left[\frac{1}{Q_s . Q_r} \sum_{j=R_s+1}^{R_s+Q_s} \sum_{k=R_r+1}^{R_r+Q_r} c'(jQ_r, k)\right] \quad (5)$$

Equation (5) expresses approximation cost function for a unit demand. Since the average demand per unit of time is equal to λ_r, the total cost of the system per unit time can then be written, as in Equation (6).

$$TC_{total} = \frac{\lambda_r}{Q_s . Q_r}\left[\left(\frac{\zeta}{\zeta+\psi} \sum_{j=R_s+1}^{R_s+Q_s} \sum_{k=R_r+1}^{R_r+Q_r} c(jQ_r, k)\right) + \left(\frac{\psi}{\zeta+\psi} \sum_{j=R_s+1}^{R_s+Q_s} \sum_{k=R_r+1}^{R_r+Q_r} c'(jQ_r, k)\right)\right] \quad (6)$$

4. Numerical Results

In this section numerical examples are presented to evaluate proposed approximation cost function. The problems are constructed by taking all possible combinations of the following values of the parameters Q_r, Q_s, λ_r, and β : Q_r = 1 or 4; Q_s = 1 or 4; λ_r = 0.1 0r 1 and β = 5 or 20. Also, $L_r = 1$, L_s =1, $h_r = 1$, h_s =0.1, ψ =0.5, and ζ =0.05. Table 1 presents all constructed problems. In addition, for each problem the cost functions and the optimum values of R_r and R_s are calculated and presented in table 1. One can use recursive process suggested by Axsäter, S. [17] for calculating cost function and the optimum values of R_r and R_s. Furthermore, the cost functions are compared with related simulation studies, and the errors are calculated and presented in table 1. The Average Absolute Error (AAE), the last row of table 1, clearly indicates that our proposed approximation method works perfect.

Table 1: Sample problems and numerical examples

Problem number	Q_r	Q_s	λ_r	β	R_r^*	R_s^*	Approximation Method	Simulation	Error	Absolute Error
1	1	1	0.1	5	-1	0	1.01196	1.00436	0.00757	0.00757
2	1	1	0.1	20	0	0	1.61965	1.63369	-0.00859	0.00859
3	1	1	1	5	1	4	3.05591	3.06478	-0.00289	0.00289
4	1	1	1	20	2	5	4.37747	4.39046	-0.00296	0.00296
5	1	4	0.1	5	-1	0	1.10398	1.10659	-0.00236	0.00236
6	1	4	0.1	20	0	0	1.75261	1.75388	-0.00073	0.00073
7	1	4	1	5	0	4	3.12155	3.12227	-0.00023	0.00023
8	1	4	1	20	1	5	4.44208	4.43705	0.00113	0.00113
9	4	1	0.1	5	-1	-1	1.80263	1.79982	0.00156	0.00156
10	4	1	0.1	20	-1	0	2.50952	2.50938	0.00006	0.00006
11	4	1	1	5	0	2	3.59451	3.58381	0.00299	0.00299
12	4	1	1	20	0	4	4.97804	4.98697	-0.00179	0.00179
13	4	4	0.1	5	-1	-1	2.32036	2.32021	0.00006	0.00006
14	4	4	0.1	20	-1	0	3.04638	3.04498	0.00046	0.00046
15	4	4	1	5	-1	3	3.99662	3.99710	-0.00012	0.00012
16	4	4	1	20	-1	4	5.56957	5.56882	0.00014	0.00014

Average Absolute Error (AAE):	0.0021029

5. Conclusion

In this paper, we studied an integrated dyadic supply chain model with uncertainty in supply and Poisson demand. In addition, in proposed integrated inventory system, lead time is a random variable consists of a fix transportation time plus a random delay occurs due to availability of stock at the supplier, therefore the lead time is non-zero random variable. Furthermore, in the case of unavailable supplier, the lead time which retailer experiences increases because of supplier's *OFF* state. As mentioned, in the modern global competitive market, supplier and retailer should be treated as strategic partners in the supply chain with a long-term cooperative relationship. Therefore models need to take into account both retailer and supplier point of view. By considering both the integrity and the uncertainty, our integrated inventory system presents the real world more realistic than former studies.

Investigating an integrated supply chain with multi-supplier is a valuable potential area for future researches, because using multi suppliers can reduce the expected total cost. It is because, usually, in practice for supply chains with uncertainty in the supply, multi-sourcing strategies are used. Although this study investigates a dyadic supply chain with uncertainty in supply, but the proposed method in this paper provides a base for analyzing more complex supply chains, which going to appear in future works.

References

1. Gurler U., Parlar, M., 1997, "An Inventory Problem with Two Randomly Available Suppliers", Operations Research, 45(6), 904-918.
2. Mohebbi, E., 2004, "A replenishment model for the supply-uncertainty problem", Int. J. Production Economics, 87, 25–37.
3. Parlar, M., Berkin, D., 1991 "Future supply uncertainty in EOQ models", Naval Research Logistics, 38, 107–121.
4. Weiss, H.J., Rosenthal, E.C., 1992, "Optimal ordering policies when anticipating a disruption in supplier demand", European Journal of Operational Research, 59, 370–382.
5. Parlar, M., Perry, D., 1995, "Optimal (Q, r, T) policies in deterministic and random yield models with uncertain future supply", European Journal of Operational Research, 84, 431–443.
6. Parlar, M., Perry, D., 1996, "Inventory models of future supply uncertainty with single and multiple suppliers", Naval Research Logistics, 43, 192–210.
7. Parlar, M., 2000, "Probabilistic analysis of renewal cycles: An application to a non-Markovian inventory problem with multiple objectives", Operations Research, 48, 243–255.
8. Parlar, M., Wang, Y., Gerchak, Y., 1995, "A periodic review inventory model with Markovian supply availability", Int. J. Production Economics, 42, 131–136.
9. Arreola-Risa, A., DeCroix, G.A., 1998, "Inventory management under random supply disruptions and partial backorders", Naval Research Logistics, 46, 687–703.
10. Ozekici, S., Parlar, M., 1999, "Inventory models with unreliable suppliers in a random environment", Annals of Operations Research, 91, 123–136.
11. Mohebbi, E., Hao, D., 2006, "When supplier's availability affects the replenishment lead time—An extension of the supply interruption problem", European Journal of Operational Research, 175, 992–1008.
12. Parlar, M., 1997, "Continuous-review inventory problem with random supply interruptions", European Journal of Operational Research, 99, 366–385.
13. Hadley, G., Whitin, T.M., 1963, "Analysis of Inventory Systems", Prentice-Hall, Englewood Cliffs, N.J.
14. Gupta, D., 1996, "The (Q, r) inventory system with an unreliable supplier", INFOR, 34, 59–76.
15. Mohebbi, E. , 2003, "Supply interruptions in a lost-sales inventory system with random lead time", Computers & Operations Research, 30, 411–426.
16. Sajadifar, S.M., Hendi, A.M., Haji, R., 2008, ''Exact Evaluation of a Two Sourcing Supply Chain with Order Splitting and Information Sharing'', IEEM conference, Singapore.
17. Axsäter, S., 1990, "Simple solution procedures for a class of two-echelon inventory problems", Operations Research, 38(1), 64-69.
18. Axsäter, S., 1993, "Exact and approximate evaluation of batch-ordering policies for two-level inventory systems", Operations Research, 41(4), 777-785.
19. Cooper, R.B., 1981, " Introduction to queuing theory (second edition)", North Holland, New York, 1981

APPENDIX

This Appendix is a summary of Axsäter, S. [17]. For more details one can see [17]. We define, as in [17], the following notations [17]:

$g^{S_s}(t) =$ Density function of the Erlang (λ, S_s) , and $G^{S_s}(t) =$ Cumulative distribution function of $g^{S_s}(t)$.

Thus,

$$g^{S_s}(t) = \frac{\lambda^{S_s} t^{S_s - 1}}{(S_s - 1)!} e^{-\lambda t}, \text{ and } G^{S_s}(t) = \sum_{k=S_s}^{\infty} \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

The average warehouse holding costs per unit is $\gamma(S_s) = \frac{h_s S_s}{\lambda}(1 - G^{S_s+1}(L_s^i)) - h_s L_s (1 - G^{S_s}(L_s)),\ \ S_s > 0$. And for $S_s = 0$, $\gamma(0) = 0$. Given that the value of the random delay at the warehouse is equal to t , the conditional expected costs per unit at the retailer is:

$$\pi^{S_r}(t) = e^{-\lambda(L_i + t)} \frac{h + \beta}{\lambda} \sum_{k=0}^{S_r - 1} \frac{(S_r - k)}{k!} (L_r + t)^k \lambda^k + \beta(L_r + t - \frac{S_r}{\lambda}) \qquad \text{(A.5)}$$

The expected retailer's inventory carrying and shortage cost to fill a unit of demand is:

$$\Pi^{S_r}(S_s) = \int_0^{L_s} g_0^{S_s}(L_s - t) \pi^{S_r}(t) dt + (1 - G_0^{S_s}(L_s)) \pi^{S_r}(0)$$

and,

$$\Pi^{S_r}(0) = \pi^{S_r}(L_s) \qquad \text{(A.7)}$$

Furthermore, for large value of S_s , we have

$$\Pi^{S_r}(S_s) \approx \pi^{S_r}(0) \qquad \text{(A.8)}$$

The procedure starts by determining $\bar{S}_0$ such that

$$G^{\bar{S}_s}(L_s) < \varepsilon \qquad \text{(A.9)}$$

Where ε is a small positive number.

The recursive computational procedure is:

$$\Pi^{S_r}(S_s - 1) = \Pi^{S_r - 1}(S_s) + (1 - G^{S_s}(L_s)) \times (\pi^{S_r}(0) - \pi^{S_r - 1}(0)), \qquad \text{(A.10)}$$

$$\Pi^0(S_s) = G^{S_s}(L_s) \beta L_s - G^{S_s+1}(L_s) \beta \frac{S_s}{\lambda} + \beta L_r \qquad \text{(A.11)}$$

and,

The expected total holding and shortage cost for a unit demand in an inventory system with a one-for-one ordering policy is:

$$c(S_s, S_r) = \Pi^{S_r}(S_s) + \gamma(S_s) \qquad \text{(A.12)}$$