Application of Integer Linear Programming Model for Vendor Selection in a Two Stage Supply Chain

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Abstract

Contemporary organizations rely on outsourcing for success in today’s competitive marketplace, and selecting a vendor is an important process as developing new products. Vendor selection is one of the most important decisions of purchasing function. As organizations become more dependent on vendors, the direct and the indirect consequences of poor decision-making become more severe. Literature shows many vendor evaluation models. In this paper we have proposed a vendor selection model using Integer Linear Programming (ILP) Model for multi-product, multi-vendor environment. The contribution of this research lies in the implementation of this model as a customized decision support system according to the expectation of any company. The model is validated with a case study by implementing the model for Agricultural equipments whole sale company.

Keywords
Vendor selection, Integer Linear Programming (ILP) Model, Supply chain, Vendor Assignment.

1. Introduction

ILP is a linear programming model in which there a particular function to be maximized is or minimized subject to several constraints. As the unknown variables are all required to be integers, then the problem is called an integer programming (IP) or integer linear programming (ILP) problem. 0-1 integer programming or binary integer programming (BIP) is the special case of integer programming where variables are required to be 0 or 1 (rather than arbitrary integers). In ILP problem constraints forces the variables to take on binary values only. Much of the modeling flexibility provided by integer linear programming is due to the use of 0-1 variables. In vendor selection, 0-1 variables provide selections or choices with the value of the variable equal to 1 if a vendor is assigned to supply a product and equal to 0 if the vendor is not assigned. The decision variables defined in this model will determine the allocation of the products to the vendors. In this paper, the integer linear programming (ILP) problem is applied to develop a supplier selection model that can fulfill the requirements of the company.

Supplier selection is widely considered to be one of the most important responsibilities of the purchasing function of management. An organization's suppliers directly affect the price, quality, delivery reliability, and availability of its products--all of which have a profound impact on customer satisfaction. Determining the most suitable suppliers is an important problem to deal with when managing supply chain of a company. The main objective of supplier selection process is to reduce purchase risk, maximize overall value to the purchaser, and develop closeness and long-term relationships between buyers and suppliers. It is vital in enhancing the competitiveness of the company and has a positive impact on expanding the life span of the company. There are several supplier selection applications available in the literature. Given an appropriate decision setting, Mathematical Programming (MP) allows the decision-maker to formulate the vendor selection problem in terms of a mathematical objective function that subsequently needs to be maximized (e.g., maximize profit) or minimized (e.g., minimize costs) by varying the values of the variables in the objective function (e.g., the amount ordered with vendor X).
Some of the mathematical programming models (Chaudhry et al 1993; Rosenthal et al 1995; Sadrian and Yoon 1994; Ganeshan et al 1999) focus on the modeling of specific discounting environments. Akinc (1993) concentrates on decision support regarding the number of vendors. Current and Weber use facility location model constructs for the vendor selection problem. Das and Tyagi (1994) develop a decision support system for a wholesaler where the selection of the manufacturer is only one of the several factors that have to be optimized in order to minimize the total cost of the wholesaling service. Weber et al., (1998) combine MP and the DEA method to provide buyers with a tool for negotiations with the vendors that were not selected right away as well as to evaluate different numbers of vendors to use (Weber et al., 2000). Karpak et al., (1999) use goal programming to minimize costs and maximize quality and delivery reliability when selecting vendors and allocating orders between them. Manoj Kumar et al.,(2005) provides a decision tool using multi objective integer linear programming model that facilitates the vendor selection and their quota allocation under different degrees of information vagueness in the decision parameters of a supply chain modeling. Zhimin Guan et al., (2007) develops a multiple objective mixed integer stochastic programming (SMIP) model for the vendor selection problem (VSP) with stochastic demand under multi-products purchases. Junyan Wang et al., (2008) characterizes quality, budget, and demand as fuzzy variables in a fuzzy vendor selection expected value model and a fuzzy vendor selection chance-constrained programming model, to maximize the total quality level.

2. Problem Definition
Vendor selection problem with multi vendor and multi item is studied in agricultural equipment whole sale company and it is described as follows. This organization deals with spares and accessories of agricultural equipments such as harvesters, tractors, expellers, mini lorries and mini door vans of various brands, namely, Mahendra, HMT, TAFE, Ford, etc. All the spares and accessories are used for the equipments used in agriculture. Currently, the company has vendors all over India supplying more than 600 parts pertaining to the above equipments. From the company, the parts are sent to various customers all over Tamil Nadu.

Each equipment has a sales person, who has to give report about the sales to the senior sales executive. The sales of the products of the company depend on monsoon and weather conditions, as all the spares are used for the equipments pertaining to agriculture. The customers of this company are also dealing with agricultural equipments. The company needs to select a set of vendors for a set of products to increase its profit. The case is a two stage supply chain with the entities such as vendors, wholesaler, and customers which is shown in figure 1.

![Diagram of Two Stage Supply Chain](image)

where, \( W_s \) -Wholesaler, \( V_i \) = Vendor and \( C_j \) = Customers

Figure 1: Two Stage Supply Chain

If the number of vendors increases, then ordering cost will be high. But if the same number decreases, then serviceability may get reduced. The company likes to reduce the total number of vendors without compromising the
quality and serviceability. The aim of reducing the vendors is to enhance the partnership, so that the order processing cost and cost per item can be reduced.

3. General Vendor Assignment Problem Using ILP

Vendor selection problem is formulated using General Assignment problem. The General Assignment Problem (Fred Glover et al., 2003) can be mapped onto the current vendor assignment problem defined in this paper. The preference of each vendor of all the products is found out by Analytical hierarchy process. Since a product can be allocated to multiple vendors and a vendor can be assigned with multiple products, there is a scope for maximizing the preference weightage while doing this assignment. This is stated in the objective function of the mathematical model developed in this work. The base for this research is derived from John Rajan et al. (2007).

The required number of vendors for each item depends upon the requirement of the buying organization. This is considered as the first constraint. By considering the past performance of the vendor, the maximum number of products that can be allotted to each vendor is considered as the second constraint. The total number of vendor assignments required for a set of products is considered as the third constraint. This mathematical model is formulated as an Integer Linear Programming model and it is presented below.

Decision variable: \( X_{ij} = \begin{cases} 1, & \text{vendor } i \text{ is allocated to product } j \\ 0, & \text{otherwise} \end{cases} \)

where,

- \( i \) - Vendor index, \( i = 1, 2, \ldots, T \), \( T \) = Number of vendors in a set
- \( j \) - Product index, \( j = 1, 2, \ldots, M \), \( M \) = Number of products in a set
- \( W_{ij} \) - Preference weightage of vendor ‘i’ for product ‘j’
- \( N_j \) - Minimum requirement of vendors for product ‘j’
- \( O_i \) - Maximum number of products allocated to vendor ‘i’
- \( A \) - Total number of vendor assignments needed for ‘M’ number of products

Maximize 
\[
Z = \sum_{i=1}^{T} \sum_{j=1}^{M} W_{ij} X_{ij}
\]
(1)

The objective function represents the maximization of the preference weightage. (\( W_{ij} \) - Preference weightage of vendor ‘i’ for the product ‘j’).

Subject to 
\[
\sum_{i=1}^{T} X_{ij} \geq N_j \quad j = 1,2,\ldots,M
\]
(2)

This constraint ensures the minimum requirement of the number of vendors for each product.

\[
\sum_{j=1}^{M} X_{ij} \leq O_i \quad i = 1, 2, \ldots, T
\]
(3)

This constraint ensures that the maximum permissible number of products is allocated to each vendor. The number of products allocated to each vendor is estimated based on the ratio of total preference weightage of individual vendor and total preference weightage of all the vendors multiplied by the total number of products in a set.

\[
\sum_{i=1}^{T} \sum_{j=1}^{M} X_{ij} \leq A
\]
(4)

This constraint ensures that the total number of vendor assignments does not exceed the availability.

\( X_{ij} = \{1 \text{ or } 0\} \)
(5)

This constraint enforces binary and non-negative restrictions on the decision variables.
4. Case Study - Agricultural Equipments Wholesale Company

The data needed for the validation of the model is collected from agricultural equipment wholesale company for a period of six months. The objective of this problem is to maximize the preference weightage which will give the preference of a particular set of vendors (4) for a set of products (15). The objective function is formulated by multiplying the preference weightages obtained through Analytic Hierarchy Process with the associated decision variable. The formulation and solution details are given below.

4.1 Maximize

\[
\begin{align*}
0.00011X_{11} + 0.00365X_{12} + 0.00319X_{13} + 0.06052X_{14} + 0.25361X_{15} + 0.08854X_{16} + 0.11035X_{17} + 0.63464X_{18} + 0.24146X_{19} + 0.87229X_{110} + 2.31741X_{111} + 1.78262X_{112} + 1.83869X_{113} + 2.04561X_{114} + 0.81491X_{115} + 0.00006X_{21} + 0.0034X_{22} + 0.0104X_{23} + 0.05135X_{24} + 0.68475X_{25} + 0.02213X_{26} + 0.1594X_{27} + 0.41325X_{28} + 0.85314X_{29} + 0.50324X_{210} + 0.64373X_{211} + 3.76331X_{212} + 0.38042X_{213} + 0.83922X_{214} + 0.30182X_{215} + 0.00011X_{31} + 0.00264X_{32} + 0.00134X_{33} + 0.03301X_{34} + 0.12681X_{35} + 0.12684X_{37} + 0.62359X_{38} + 0.32194X_{39} + 0.50324X_{310} + 0.60081X_{311} + 0.39614X_{312} + 0.76084X_{313} + 1.7309X_{314} + 0.39237X_{315} + 0.00007X_{41} + 0.00289X_{42} + 0.00168X_{43} + 0.03851X_{44} + 0.21557X_{45} + 0.15863X_{46} + 0.12262X_{47} + 0.26566X_{48} + 0.19316X_{49} + 1.47618X_{410} + 0.77247X_{411} + 0.66023X_{412} + 3.29696X_{413} + 0.62942X_{414} + 1.53928X_{415}
\end{align*}
\]

(6)

4.2 Constraint-1 (At least one vendor for each product)

The decision maker (wholesaler) would like to have at least one vendor for each product. This is stated as the first constraint.

\[
\begin{align*}
X_{112} + X_{122} + X_{132} + X_{142} + X_{152} + X_{162} + X_{172} + X_{182} + X_{192} + X_{1102} + X_{1112} + X_{1122} + X_{1132} + X_{1142} + X_{1152} & \geq 1 \\
X_{212} + X_{222} + X_{232} + X_{242} + X_{252} + X_{262} + X_{272} + X_{282} + X_{292} + X_{2102} + X_{2112} + X_{2122} + X_{2132} + X_{2142} + X_{2152} & \geq 1 \\
X_{312} + X_{322} + X_{332} + X_{342} + X_{352} + X_{362} + X_{372} + X_{382} + X_{392} + X_{3102} + X_{3112} + X_{3122} + X_{3132} + X_{3142} + X_{3152} & \geq 1 \\
X_{412} + X_{422} + X_{432} + X_{442} + X_{452} + X_{462} + X_{472} + X_{482} + X_{492} + X_{4102} + X_{4112} + X_{4122} + X_{4132} + X_{4142} + X_{4152} & \geq 1
\end{align*}
\]

(7)

4.3 Constraint-2 (Maximum number of products allocated to each vendor)

This constraint number of products allocated to each vendor depends on the sum of preference weightages of that vendor.

\[
\begin{align*}
X_{112} + X_{122} + X_{132} + X_{142} + X_{152} + X_{162} + X_{172} + X_{182} + X_{192} + X_{1102} + X_{1112} + X_{1122} + X_{1132} + X_{1142} + X_{1152} & \leq 5 \\
X_{212} + X_{222} + X_{232} + X_{242} + X_{252} + X_{262} + X_{272} + X_{282} + X_{292} + X_{2102} + X_{2112} + X_{2122} + X_{2132} + X_{2142} + X_{2152} & \leq 4 \\
X_{312} + X_{322} + X_{332} + X_{342} + X_{352} + X_{362} + X_{372} + X_{382} + X_{392} + X_{3102} + X_{3112} + X_{3122} + X_{3132} + X_{3142} + X_{3152} & \leq 2 \\
X_{412} + X_{422} + X_{432} + X_{442} + X_{452} + X_{462} + X_{472} + X_{482} + X_{492} + X_{4102} + X_{4112} + X_{4122} + X_{4132} + X_{4142} + X_{4152} & \leq 4
\end{align*}
\]

(8)

4.4 Constraint-3 (Total number of vendor assignments for product set A1)

The third constraint is formulated that the total number of vendor assignments for the set of product cannot exceed sixty.

\[
\begin{align*}
X_{112} + X_{122} + X_{132} + X_{142} + X_{152} + X_{162} + X_{172} + X_{182} + X_{192} + X_{1102} + X_{1112} + X_{1122} + X_{1132} + X_{1142} + X_{1152} + X_{1222} + X_{1232} + X_{1242} + X_{1252} + X_{1262} + X_{1272} + X_{1282} + X_{1292} + X_{12102} + X_{12112} + X_{12122} + X_{12132} + X_{12142} + X_{12152} + X_{1322} + X_{1332} + X_{1342} + X_{1352} + X_{1362} + X_{1372} + X_{1382} + X_{1392} + X_{13102} + X_{13112} + X_{13122} + X_{13132} + X_{13142} + X_{13152} + X_{1422} + X_{1432} + X_{1442} + X_{1452} + X_{1462} + X_{1472} + X_{1482} + X_{1492} + X_{14102} + X_{14112} + X_{14122} + X_{14132} + X_{14142} + X_{14152} & \leq 60
\end{align*}
\]

(9)
### Table 1. Allocation of Set of Products to a Set of Vendors

<table>
<thead>
<tr>
<th>No.</th>
<th>Product Name</th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
<th>V4</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>Front brake hose Calex</td>
<td></td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>P2</td>
<td>Brake drum H/D</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P3</td>
<td>Rear bumper mounting APE</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P4</td>
<td>Engine mounting L/M W/Bolt CTS</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P5</td>
<td>Air cleaner assy.</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P6</td>
<td>Center axle pin Grinde</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P7</td>
<td>Tie rod end Rane N/M</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P8</td>
<td>Choke lever assy.</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P9</td>
<td>Air cleaner assy.</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P10</td>
<td>Cent axle pin Grind 4hole</td>
<td>✓</td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>P11</td>
<td>Clutch disc Ford</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P12</td>
<td>Air cleaner bowl</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P13</td>
<td>Die filter 404 MICO</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P14</td>
<td>Spider gear 39T</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P15</td>
<td>Clutch lever kit Massey</td>
<td></td>
<td></td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

#### 4.5 Constraint-4 (All the decision variables are binary)
Fourth constraint states that all the decision variables are binary.

\[
x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{110}, x_{111}, x_{112}, x_{113}, x_{114}, x_{21}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{210}, x_{211}, x_{212}, x_{213}, x_{214}, x_{215}, x_{216}, x_{217}, x_{218}, x_{219}, x_{310}, x_{311}, x_{312}, x_{313}, x_{314}, x_{315}, x_{316}, x_{317}, x_{318}, x_{319}, x_{320}, x_{321}, x_{322}, x_{323}, x_{324}, x_{325}, x_{326}, x_{327}, x_{328}, x_{329}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, x_{330}, x_{331}, x_{332}, x_{333}, x_{334}, x_{335}, x_{336}, x_{337}, x_{338}, x_{339}, ✓ \quad \text{and} \quad 0 \quad \text{or} \quad 1 \quad \text{(10)}
\]

#### 4.6 Solution for Vendor Allocation Problem
The problem formulated in previous sections has been solved using LINDO 6.1. The allocated products for each vendor obtained from the LINDO output are presented in Table 1.

### 5. Conclusion
Vendor selection model using ILP is developed to select the vendors for a business environment having two stage supply chain. The model is tested in an agricultural equipment wholesaler and is effectively working out. This model can also be applied in real-life cases of other domains like automobile, textiles, electronic equipments and food industries. The model can be further improved by splitting the allocation of each product among vendors and by considering the limited capacity vendors. The mathematical model used in this research work can be further extended towards multi objective optimization to minimize overall procurement cost.

### References