FFT Based Spectrum Analysis of Three Phase Signals in Park (d-q) Plane

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Abstract

In this paper, spectrum produced by a balanced three phase signals and their consequent form in Park (d-q) plane has been analyzed. The spectrums show that the peaks of the different orders of spectrums have changed after Park Transformation. Thus well renowned Park transform technique is introducing the new spectrum. The spectrum characteristics of signals from a balanced system in Park domain have been analyzed. Observation reveals the widening interrelation among the spectrums.

Keywords

FFT based spectrum, FFT, Park Domain, Park Transformation

1. Introduction

Modern machine and power system designing technologies have introduced varieties of new rotating machines and different designing methods. Different mathematical models have been introduced to study their performance. A lot of research works are going on to study the system performance with respect to different reference frame. Some of them are stationary some others are rotating. Such a reference frame is Park Frame consisting of d, q and 0 axes [1]. Different models have been developed with respect to this reference frame. Normal poly phase signals are transformed into this frame with the help of Park transformation matrix. This matrix and the frame have been widely used in the study of system performance and power quality assessment. Harmonic assessment in quality study is done with the help of spectrum analyses of a signal. In this regard different mathematical tools have been introduced [3]-[6]. One of the commonly used techniques is FFT [4]-[8]. In some recent research work, Park transformation has been used along with the FFT in power quality assessment. The use of Park transformation in power quality assessment has raised a question whether the transformation method is itself introducing some quality related problems [1], [2]. Here an attempt has been taken to study the changes of spectrums obtained from Fast Fourier Transform, in Park domain and to compare the result with the spectrums obtained in phase plane.

2. Theoretical Background

Park plane is used to assess three phase signal with respect to a non stationary reference signals rotating at synchronous speed. Signals in Park planes are obtained from three phase signals multiplied by Park transformation matrix as pictorially shown in Figure 1.

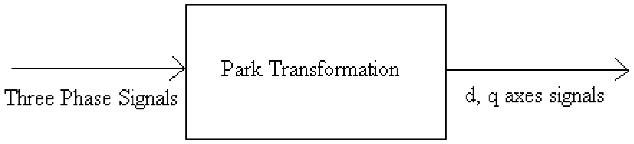


Figure 1: Park Transformation

The method is well used in performance analysis of both three phase power system and three phase rotating machine. Voltages and currents in Park Domain can be derived from phase voltage multiplying by Park transformation matrix as follows

$$\begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{4\pi}{3}\right) \\ -\sin\theta & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta - \frac{4\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \times \begin{bmatrix} v_R \\ v_Y \\ v_B \end{bmatrix} \tag{1}$$

Similarly, currents in d-q plane can be written as

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{4\pi}{3}\right) \\ -\sin\theta & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta - \frac{4\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \times \begin{bmatrix} i_R \\ i_Y \\ i_B \end{bmatrix} \tag{2}$$

In general,

$$[Signals\ in Park Plane] = [Park\ Transformation\ Matrix] \times [Three\ Phase\ Signals]$$

3. Computer Simulation

In computer simulation, as shown in Figure 2, a discrete 3- phase programmable source has been considered. Three phase signals have been take and their spectrums have been obtained with the help of Fast Fourier Transform. Then the signals are transformed into Park domain using Park transformation matrix, wherefrom d and q axes signals are obtained. Then FFT based spectrums are obtained using these signals. A comparative study has been made among the spectrums of d-axis, q-axis and other three – phase signals.

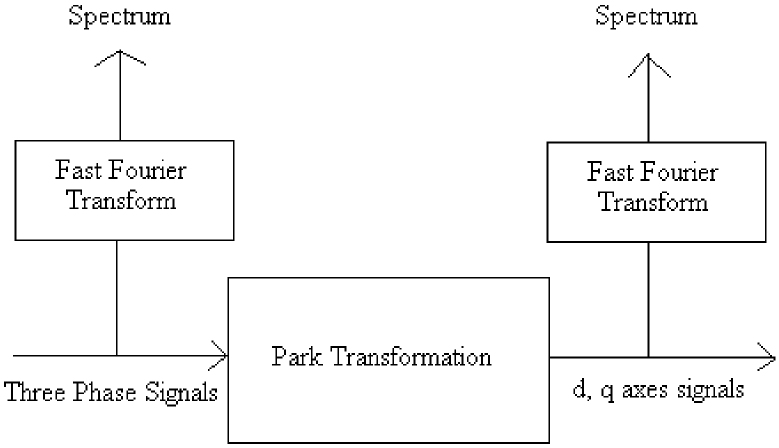


Figure 2: Simulated network

4. Simulated Results

Amplitude of the phase signals have been increased and transformed in Park Domain. Then spectrums of d-axis and q axis signals are obtained. Some such spectrums are presented in tabular form in Table 1. Three parameters FPP, SPP and NP have been introduced to express the peaks of each spectrum, FPP signifies 'first positive peak', SPP signifies 'second positive peak' and NP signifies 'negative peak'. Values of FPP, SPP and NPP of the phase a, phase b, phase c, d-axis and q-axis signals are presented in Table 2, 3, 4, 5 and 6 respectively.

Table 1: FFT based Spectrum in Park plane

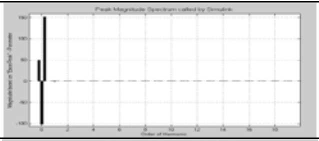
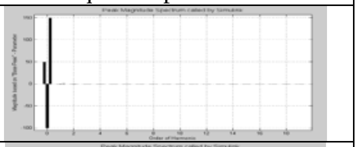
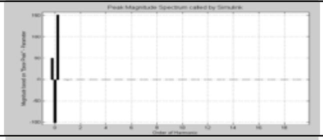
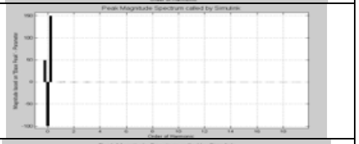
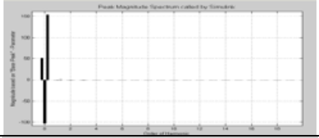
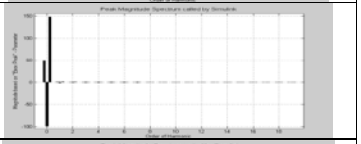
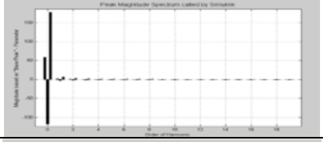
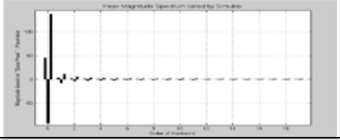
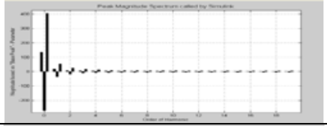
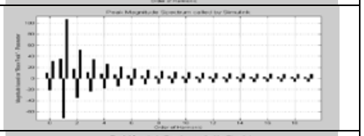
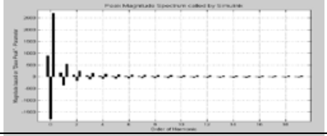
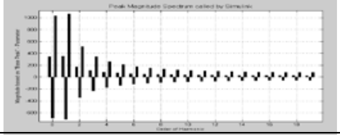
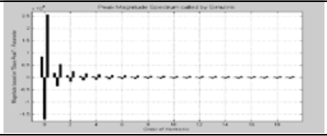
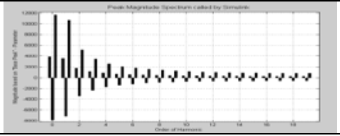
Amplitude	d axis Spectrum	q axis Spectrum
0.01		
0.1		
1.0		
10		
100		
1000		
10000		

Table 2: Data of FFT base spectrum of voltage of phase a

Amplitude	FPP (0)	SP (0)	SPP (0)	FPP (1)	SP (1)	SPP (1)	FPP (2)	SP (2)	SPP (2)	FPP (3)	SP (3)	SPP (3)	FPP (4)	SP (4)	SPP (4)	FPP (5)	SP (5)	SPP (5)
0.01	50.00	100.01	150.01	0.50	1.01	1.57	0.01	0.02	0.02	0.01	0.01	0.02	0.01	0.02	0.02	0.01	0.01	0.02
0.1	50.00	100.01	150.01	0.59	1.19	1.78	0.01	0.02	0.02	0.01	0.01	0.02	0.01	0.02	0.02	0.01	0.02	0.02
1	50.00	100.01	150.01	1.49	2.99	4.48	0.01	0.02	0.02	0.01	0.01	0.02	0.01	0.02	0.02	0.01	0.02	0.02
10	50.00	100.01	150.01	10.49	20.99	31.48	0.01	0.01	0.02	0.01	0.02	0.02	0.01	0.01	0.02	0.01	0.02	0.02
100	50.00	100.01	150.01	100.49	200.99	301.48	0.01	0.01	0.02	0.01	0.02	0.02	0.01	0.01	0.02	0.01	0.02	0.02
1000	50.00	100.01	150.01	1000.49	2000.99	3001.48	0.01	0.01	0.02	0.01	0.02	0.02	0.01	0.01	0.02	0.01	0.02	0.02
10000	50.00	100.01	150.01	10000.49	20000.99	30001.48	0.01	0.01	0.02	0.01	0.02	0.02	0.01	0.01	0.02	0.01	0.02	0.02

Table 3: Data of FFT base spectrum of voltage of phase

Amplitude	FPP (0)	SP (0)	SPP (0)	FPP (1)	SP (1)	SPP (1)	FPP (2)	SP (2)	SPP (2)	FPP (3)	SP (3)	SPP (3)	FPP (4)	SP (4)	SPP (4)	FPP (5)	SP (5)	SPP (5)
0.01	50.00	100.00	150.00	0.51	1.02	1.53	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	50.00	100.00	150.00	0.60	1.20	1.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	50.00	100.00	150.00	1.50	3.00	4.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	50.00	100.00	150.00	10.50	21.00	31.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	50.00	100.00	150.00	100.50	201.00	301.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1000	50.00	100.00	150.00	1000.50	2001.00	3001.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10000	50.00	100.00	150.00	10000.50	20001.00	30001.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

b

Table 4: Data of FFT base spectrum of voltage of phase c

Amplitude	FPP (0)	SP (0)	SPP (0)	FPP (1)	SP (1)	SPP (1)	FPP (2)	SP (2)	SPP (2)	FPP (3)	SP (3)	SPP (3)	FPP (4)	SP (4)	SPP (4)	FPP (5)	SP (5)	SPP (5)
0.01	50.00	100.00	150.00	0.51	1.02	1.53	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	50.00	100.00	150.00	0.60	1.20	1.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	50.00	100.00	150.00	1.50	3.00	4.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	50.00	100.00	150.00	10.50	21.00	31.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	50.00	100.00	150.00	100.50	201.00	301.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1000	50.00	100.00	150.00	1000.50	2001.00	3001.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10000	50.00	100.00	150.00	10000.50	20001.00	30001.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 5: Data for spectrum in d axis

A	FFP (0)	NP (0)	SPP (0)	FFP (1)	NP (1)	SPP (1)	FFP (2)	NP (2)	SPP (2)	FFP (3)	NP (3)	SPP (3)	FFP (4)	NP (4)	SPP (4)	FFP (5)	NP (5)	SPP (5)
0.01	50.44	100.88	151.32	0.09	0.17	0.26	0.04	0.08	0.12	0.03	0.05	0.08	0.02	0.04	0.06	0.02	0.03	0.05
0.1	50.52	101.03	151.55	0.10	0.20	0.31	0.05	0.09	0.14	0.03	0.06	0.09	0.02	0.04	0.07	0.02	0.04	0.05
1	51.53	102.58	153.84	0.26	0.52	0.79	0.12	0.24	0.36	0.08	0.15	0.23	0.06	0.11	0.17	0.04	0.09	0.13
10	58.92	117.83	176.75	1.87	3.74	5.61	0.85	1.71	2.56	0.56	1.12	1.68	0.42	0.84	1.26	0.34	0.67	1.01
100	135.29	270.57	405.86	17.97	35.93	53.90	8.21	16.41	24.62	5.40	10.80	16.20	4.05	8.10	12.15	3.26	6.52	9.78
1000	899.00	1797.99	2696.99	178.90	357.81	536.71	81.74	163.47	245.21	53.79	107.58	161.37	40.37	80.75	121.12	32.50	65.01	97.51
10000	8536.08	17072.16	25608.24	1788.29	3576.58	5364.88	817.03	1634.05	2451.08	537.69	1075.38	1613.06	403.60	807.21	1210.81	324.94	649.88	974.82

Table 6: Data for spectrum in q axis

A	FFP (0)	NP (0)	SPP (0)	FFP (1)	NP (1)	SPP (1)	FFP (2)	NP (2)	SPP (2)	FFP (3)	NP (3)	SPP (3)	FFP (4)	NP (4)	SPP (4)	FFP (5)	NP (5)	SPP (5)
0.01	49.80	99.61	149.41	0.18	0.36	0.54	0.09	0.17	0.26	0.06	0.11	0.17	0.04	0.09	0.13	0.03	0.07	0.10
0.1	49.77	99.53	149.30	0.21	0.43	0.64	0.10	0.21	0.31	0.07	0.14	0.20	0.05	0.10	0.15	0.04	0.08	0.12
1	49.41	98.83	148.24	0.53	1.07	1.60	0.26	0.52	0.77	0.17	0.34	0.51	0.13	0.26	0.38	0.10	0.20	0.31
10	45.87	91.74	137.60	3.73	7.46	11.20	1.81	3.62	5.43	1.20	2.41	3.61	0.91	1.81	2.72	0.73	1.46	2.19
100	10.42	20.84	31.25	35.72	71.44	107.16	17.32	34.63	51.95	11.53	23.06	34.59	8.69	17.38	26.07	7.01	14.02	21.03
1000	344.08	688.17	1032.25	355.60	711.19	1066.79	172.39	344.78	517.16	114.80	229.61	344.41	86.54	173.08	259.62	69.81	139.62	209.43
10000	3889.10	7778.20	11667.30	3554.37	7108.74	10663.11	1723.11	3446.22	5169.33	1147.55	2295.09	3442.64	865.04	1730.07	2595.11	697.82	1395.64	2093.46

5. Observation

Tables 2-6 show that the spectrums are changing with respect to the change of amplitude of the signals. Change of FPP, SPP and NP of the first spectrums with respect to the variation of amplitude are plotted in Figures 2 and 3. The observation are summarized as

- Change of peaks are linear
- The distance between the peaks are increasing with the increase of amplitude.
- Peaks of d axis signals are always increasing in nature.
- Fig. 2 shows that peaks of q axis signals are decreasing up to certain amplitude of the phase signals. The Fig. 3 shows that peaks of q-axis signals are increasing again with the increase of amplitude of phase-signals.
- The most important observation is obtained from the spectrum comparison among phase signals from Table 2, 3 and 4 with the spectrum of d-q signals. It is seen that the higher order spectrums in phase signals are zero where as Park domain signals (d and q axes signals) contain considerable amount of higher order spectrums.

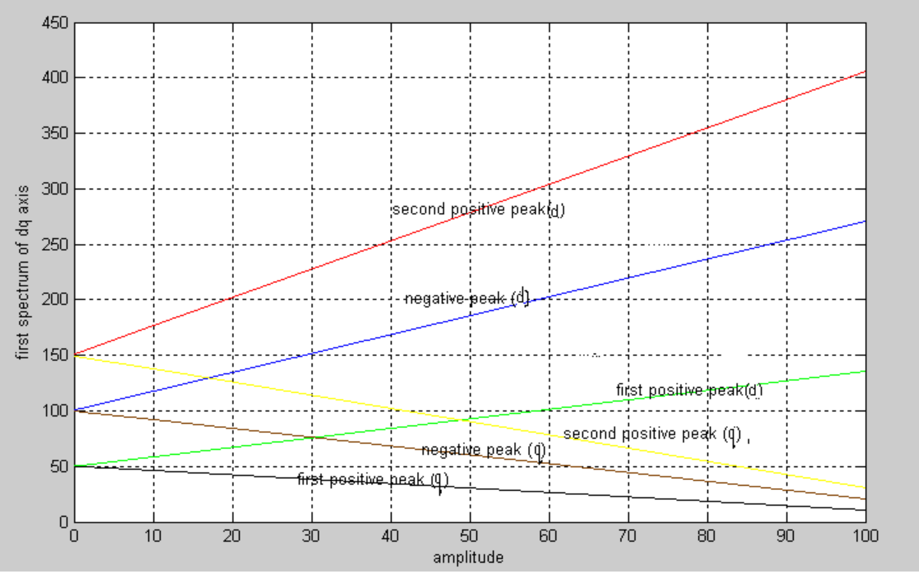


Figure 3: First spectrums versus amplitude of range 0.01-100 in Park Domain

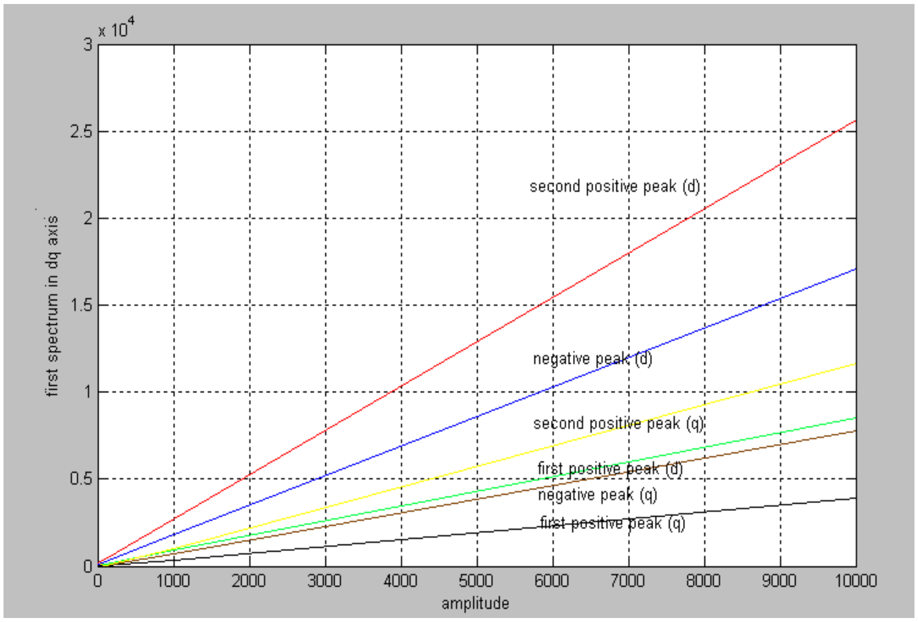


Figure 4: First spectrums versus amplitude of range 0.01-10,000 in Park Domain

6. Conclusion

The observation shows that the nature of spectrums of d axis signals and q axis signals are different. Secondly some considerable amount of higher-order-spectrums is observed to be introduced after Park transformation, which should be taken into consideration in performance analysis and power quality assessment in Park domain. The main cause of high order spectrum introduction after Park transformation may or may not be due the transform method but more research should be done to find out the answer.

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