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Fuzzy Clustering for Initialization of Simulated Annealing Algorithm to Solve a Capacitated Vehicle Routing Problem

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Abstract

Vehicle Routing Problem (VRP) has been an interesting research area since its introduction. There are various types of VRP models and different solution techniques proposed for this problem. This paper uses several clustering algorithms in initialization of Simulated Annealing to solve VRP. The main contribution of this research is to assess the effect of using some clustering methods in building the initial solution. For this purpose, Hard C-Means (HCM), Fuzzy C-Means (FCM), and Possibilistic C-Means (PCM) are implemented versus random (RANDOM) and heuristically-built (HB) solutions. These algorithms are compared using two well-known standard datasets and results show that using FCM and PCM in initialization of solutions can lead to promising results.

Keywords
Capacitated Vehicle Routing Problem (CVRP), Fuzzy Clustering, Possibilistic clustering, Best Fit Decreasing (BFD), Simulated Annealing.

1. Introduction and Related Works

There are a lot of problems to be administered in a supply chain network such as warehousing, facility location, information flow management and a variety of transportation problems such as Vehicle Routing Problem (VRP). Among various problems considered in a supply chain network design, VRP may be considered as one of the most challenging problems. VRP is a well-known combinatorial optimization problem and was first introduced by Dantzig and Ramser [17]. It has numerous applications such as in Emergency preparedness and disaster relief, solid waste and dairy industries, street cleaning, transportation of handicapped people, school bus routing, etc. The single-depot Capacitated Vehicle routing problem (CVRP) deals with finding routes of minimum cost for a fleet of vehicles, serving a set of customers with known demands. CVRP considers the rules that each vehicle should begin and end at the depot, each customer must be visited exactly once by a single vehicle and the capacities of vehicles must be taken into account. VRP and its extensions (will be called VRP from now on) has been a focus area of many scientists all over the world for nearly 50 years. There have been a great number of publications devoted to this problem. ([5] surveyed more than 1000 journal papers and 30 books/book chapters for VRP). Due to the complexity of the model, numerous heuristic/metaheuristic approaches have been proposed in the literature varying from the well-known Clarke-Wright algorithm [2], Column Generation [9], Genetic Algorithm ([6], [21]), Variable Neighborhood Search [11], Ant Colony Optimization[20], Simulated Annealing [18], Memetic Algorithm ([14], [12]). There are also some hybrid algorithms such as hybrid Genetic-Particle Swarm Optimization ([8], [19]) and hybrid Simulated Annealing-Tabu Search algorithm [13].

Recently, some more complex VRP extensions (often called rich VRPs [16]) have received considerable attention from academicians all over the world. An instance of these problems is Open VRP (OVRP) considered in [7] and many other papers. The uncertain VRP is another research area which has attracted the attention of many scholars. Papers by Liu and Lai [10] and Erbao and Mingyong [1] are instances of these publications considering fuzzy demands. Grouping customers in solving VRP is not new to the literature, Yoshiike and Takefuji [3] proposed a two phase algorithm in which the first phase incorporates using maximum neuron model to group the customers and the second phase deals with solving a TSP for each group of customers using the elastic net model. Knowing that Location-Routing problem (LRP) is an extension to VRP, the paper by Barreto et al. [4] may be considered as another publication devoted to grouping customers in VRP and its extensions. This paper considered a discrete version of LRP and presented some hierarchical and non-hierarchical clustering methods in addition to several proximity measures in solving VRPs.

The objective of the single-depot VRP is to find routes that impose the least cost, considering constraints such as visiting each customer exactly once and respecting vehicle capacities (Figure 1). Moreover, some outstanding types of VRPs are shown in Figure 2. In this paper, a CVRP is considered and a Simulated-Annealing-based procedure is presented in order to solve it. Since local search algorithms are quite sensitive to their initial solutions, the role of clustering algorithms in building initial solutions and reaching better solutions is discussed in this paper. Five algorithms are used to generate initial solutions and their performance is compared. These include using a completely random solution; a feasible initial solution generated using Best Fit Decreasing (BFD) heuristic, and solutions generated using three known clustering methods: Hard C-means, Fuzzy C-means and Possibilistic C-means. The fuzzy clustering procedures are proposed due to their flexibility in assignment of memberships to nodes.

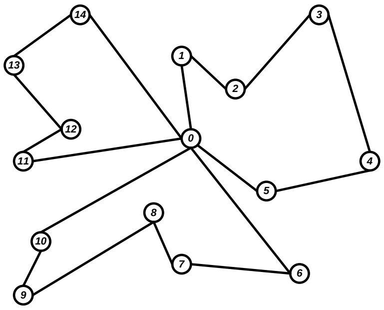

Figure 1: Typical solution of a VRP

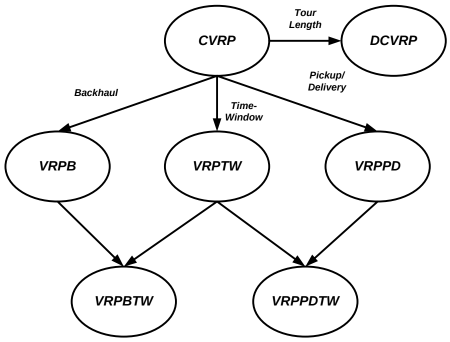

Figure 2: The basic models of VRP and their interconnections [15]

2. Simulated Annealing and the Proposed Approach

2.1 Our Proposed Method

2.1.1 Formulation and General Procedure

In this section, the mathematical model for CVRP is presented. In this model, the binary variable x indicates whether a vehicle traverses an arc in a solution or not. In other words, if a solution contains traversing a specific arc such as (i, j), then x_{ij} takes value 1 and 0 otherwise, K is the number of vehicles to be used and c_{ij} is the cost to traverse the route (i,j). The model is given below:

$$Min \quad \sum_{i\in V}\sum_{j\in V} c_{ij}\, x_{ij} \tag{1}$$

$$\sum_{i\in V} x_{ij} = 1 \quad \forall j \in V \setminus \{\,0\,\} \tag{2}$$

$$\sum_{j\in V} x_{ij} = 1 \quad \forall i \in V \setminus \{\,0\,\} \tag{3}$$

$$\sum_{i\in V} x_{i0} = K \tag{4}$$

$$\sum_{j\in V} x_{0j} = K \tag{5}$$

$$\sum_{i\notin S}\sum_{j\in S} x_{ij} \geq r(S) \quad \forall S \subseteq V \setminus \{\,0\,\}, S \neq \phi \tag{6}$$

$$x_{ij} \in \{0,1\} \quad \forall i, j \in V \tag{7}$$

The Objective function (1) tries to minimize the total cost incurred to the system. Constraints (2) and (3) are called in-degree and out-degree constraints and state that exactly one arc enters and leaves a vertex associated with a customer. Constraints (4) and (5) impose the degree requirements of the depot vertex. The capacity-cut constraints (CCC) of (6) impose connectivity of the solution and the vehicle capacity requirements. They guarantee that each cut (V\S, S) defined by a customer set S is crossed by a number of arcs not smaller than r(s) (minimum number of vehicles needed to serve set S). Finally, (7) guarantees that the values of x_{ij} are binary.

To solve CVRP, numerous methods have been proposed, including heuristics and metaheuristics. One of the main classes of these methods is the local search algorithms such as Simulated Annealing (SA) and Tabu Search (TS). Local search algorithms begin by an initial solution, search through the solution space and try to navigate to better regions of solution space gradually. Therefore, initial solutions play a pivotal role in the performance of these algorithms. In this paper, the effect of using some clustering-based initial solution on the performance of SA is studied. The role of grouping customers in VRP has been considered by many authors such as Kuehn and Hamburger [22], Bodin [23], Min [24], Srivastava [25]. Our proposed algorithm first constructs groups of customers based on a clustering approach and then within each of the clusters, SA is used to improve the sub-problem as much as possible. Hard c-means, Fuzzy c-means and possibilistic C-means are used in this paper and the results are compared from a performance point of view. The PCM used in this paper is the New Possibilistic C-Means (NPCM) presented in Melek et al. [26].

2.1.2 Neighborhood Generation Procedure

A solution is represented using a string of numbers comprising two-sections. The solution representation can be explained as follows.

Assuming m customers and k vehicles, the length of the proposed chromosome is equal to m+k in which m genes belong to the first part and the rest of the genes build the second part. The first part of the chromosome represents the order of the visited customers without consideration of depot. The second part shows the indices of served nodes by each vehicle. To decode any chromosome, both parts are needed.

Take the solution representation in Figure 3 for example. It is comprised of two parts, the first part shows the sequence of serving customers and the second part shows the customers each vehicle must serve. It should be noted that concerning the second part of the chromosome, the last gene ought to equal to the number of demand nodes. In addition, none of the two parts are self-explanatory and one needs both parts to decode the solution. The solution in Figure 4 conveys that customers 5 and 7 are served by vehicle 1, vehicle 2 serves customers 6 and 2 and finally third vehicle serves customers 4, 1, 3 and 8 respectively.

Let the set N(X) to be the set of solutions neighboring a solution X. In each iteration, the next solution Y is generated from N(X) by three kind of moves. There are four local search moves used in this paper called 2-opt, 3-opt, shuffle and perturb as shown in Figure 5. The first three moves modify the first part of the chromosome and perturb is used to change the second part. Generally, in an *r*-opt move, the values of *r* randomly chosen genes are substituted. A solution is r-optimal when it cannot be improved by any r-opt and is called r-opt*.In this paper, 2-opt and 3-opt moves follow the same rule. In a shuffle move, two random indices are selected and the values inside the resulting segment between these two indices are re-ordered randomly. Such a move has a stochastic character and is used in order to diversify the solutions. Finally, in a perturb move, the second part is re-generated, in other words, such a move changes the grouping of customers served by a vehicle.

In the SA phase, the starting temperature is considered to be 450 and the stopping criterion is to reach a temperature less than or equal to 1. These temperatures were used, so 1000 epochs is done in our proposed SA. After some iterations from the last decrease in the temperature, temperature is updated using the formula $T=\alpha T$, where $0<\alpha<1$.

To find these probabilities, various combinations for the ratios of these moves were tested and the fitness of each is shown on a contour chart as shown in Figure 4. Obviously, one of the regions with minimum fitness lies in the region where the probabilities of moves are 0.75, 0.2 and 0.05 for 2-opt, 3-opt and shuffling respectively. It should be stated that the first three moves are mutually exclusive. In other words only one of them is used per iteration. In addition, there is a 40% chance of perturbation in each iteration. Thus, it is possible that in any iteration, a move of the first group is used simultaneously with a perturbation move. The number of iterations in each temperature is a monotonically increasing function. The algorithm begins by 50 iterations per temperature and with each new temperature; the number of iterations is multiplied by 1.005. Then, the upper bound of result is used as the number of iterations. This procedure is devised in order to intensify the search as algorithm goes on. The initial solutions are found using five distinct procedures and the results are compared. These include random initialization of solutions, using the Best Fit Decreasing (BFD) heuristic, hard c-means, fuzzy c-means and new Possibilistic c-means of Melek et al. [26]. None of these methods build a feasible solution, except the BFD heuristics. The BFD heuristic which is normally used in bin-packing problem is a heuristic to build feasible solutions. Therefore, in problems of larger sizes, using BFD can lead to better solutions, since there is no need to search in infeasible regions of space for a long time.

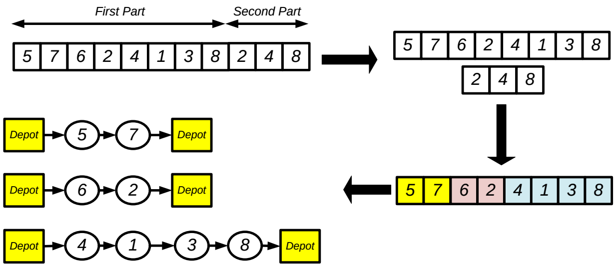

Figure 3: The encoding scheme of a solution

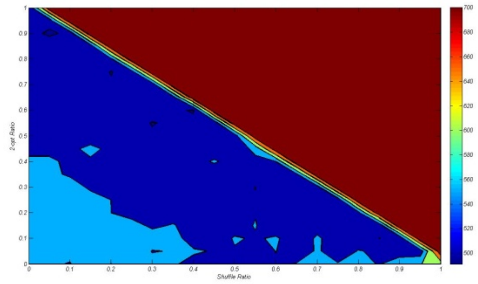

Figure 4: Contour chart of fitness using various move ratios

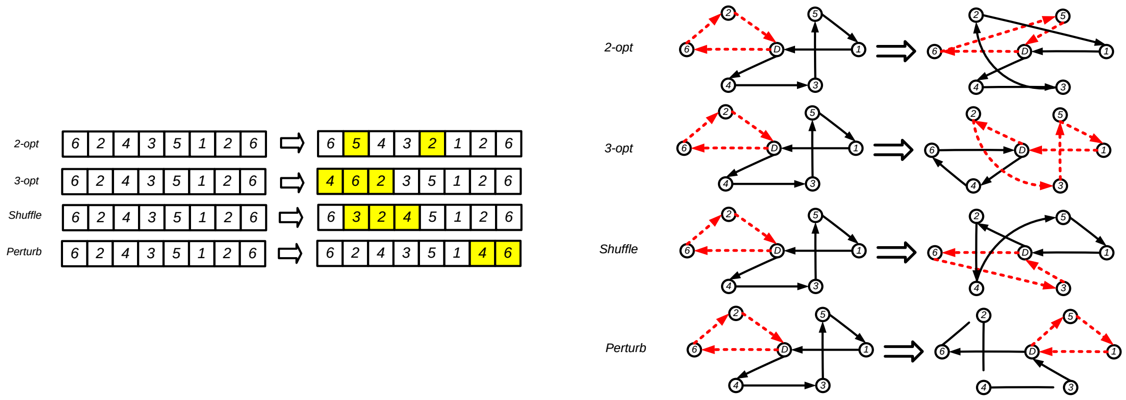

Figure 5: The local search mechanisms of the proposed method

3. Numerical Experiments

Computational experiments were conducted by applying the proposed algorithm to two benchmark datasets of CVRP in order to evaluate the performance of the method. The first used dataset is the one designed by Christofidis and Eilon [27] where the number of customers is less than or equal to 32. The second dataset is the one presented by Augerat et al. [28] which comprises problems with less than 40 customers. The information concerning these datasets is given in table 1. It is seen that problems of various settings are considered in this paper. Each solution method was run 10 times and the worst, average and the best results are given in table 2. The last column of the table shows the gap between the best method and the optimal solution. The gap is calculated using the following formula:

$$Gap = \frac{Z - Z^*}{Z^*} * 100\% \quad (8)$$

where Z is the objective function and Z^* is the optimal solution. It is observed that using PCM or FCM is a good way to initialize solutions that are good enough. Moreover, gaps of the methods are all below 2.8% in all the instances solved.

4. Conclusion and Future Research Areas

Initializing solutions plays a pivotal role in performance of Simulated Annealing. In this paper, we study how using clustering-based initial solutions can affect the performance of SA in solving a Capacitated Vehicle Routing Problem. The computational results show that using a clustering-based initial solution is effective in reaching better solutions compared to previous algorithms in the literature. There are some possible further research areas, such as tuning the clustering algorithm parameters and analyzing how they contribute to the quality of the solution. Moreover, one may apply the proposed algorithm to other variants of the VRP such as VRP with time windows (VRPTW) or VRP with pickup and delivery (VRPPD). Furthermore, the comparison of SA and some other well-known heuristics and metaheuristics such as Genetic Algorithm or Tabu Search is possible.

Table 1: The standard test problems used in this paper

Christofides and Eilon [27]						
Problem Instance	Demand Type	Distance Type	# of Customers	# of Vehicles	Vehicle Capacity	Tightness (Demand/Capacity)
E-n22-k4	General	Euclidean	21	4	6000	0.94
E-n23-k3	General	Euclidean	22	3	4500	0.75
E-n30-k3	General	Euclidean	29	3	4500	0.94
E-n33-k4	General	Euclidean	32	4	8000	0.92
Augerat, et al. Set P [28]						
Problem Instance	Demand Type	Distance Type	# of Customers	# of Vehicles	Vehicle Capacity	Tightness (Demand/Capacity)
P-n16-k8	General	Euclidean	15	8	35	0.88
P-n19-k2	General	Euclidean	18	2	160	0.97
P-n20-k2	General	Euclidean	19	2	160	0.97
P-n40-k5	General	Euclidean	39	5	140	0.88

Table 2: The results of using five algorithms

Problem Name	Random			Feasible			HCM			FCM			PCM			Optimal Solution	Gap
	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max		
E-n22-k4	384.1	411.6	458.8	**375.3***	404.4	496.9	**375.3***	400.9	445.4	**375.3***	396.1	438.9	**375.3***	398.4	425.6	375.3	0%
E-n23-k3	569.7	619.8	678.5	569.7	617.9	659.3	570.0	615.7	675.2	**568.6***	604.0	676.9	569.7	604.4	652.2	568.6	0%
E-n30-k3	550.7	578.9	685.7	551.4	595.1	678.6	550.3	576.1	660.8	*549.9****	569.4	593.5	553.5	568.6	596.8	539.0	2%
E-n33-k4	865.8	896.2	950.2	869.0	900.4	938.6	866.8	888.6	916.5	847.8	880.3	910.7	*846.9****	887.0	914.3	838.7	0.9%
P-n16-k8	451.9	460.7	470.3	**451.3***	453.4	470.3	454.4	460.0	466.3	**451.3***	459.4	470.3	**451.3***	453.0	454.4	451.3	0%
P-n19-k2	**212.7***	222.6	233.2	**212.7***	221.7	226.8	**212.7***	225.4	236.1	**212.7***	219.3	231.2	**212.7***	221.4	225.0	212.7	0%
P-n20-k2	218.3	226.2	243.8	**217.4***	223.7	235.4	218.3	226.6	235.8	**217.4***	223.5	230.7	**217.4***	223.2	229.9	217.4	0%
P-n40-k5	495.7	525.6	565.8	489.0	523.6	569.6	495.5	525.8	560.4	491.5	519.8	555.9	*475.1****	515.7	535.2	461.7	2.8%

*A value equal to the optimal solution is found.
*** The best solution among the 5 methods, although the optimum is not found using none of the 5 methods.

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