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Method and Algorithm for solving the Bicriterion Network Problem

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Abstract

In this paper, a bicriterion shortest route problem is formulated as a two criterion linear programming problem. A method and algorithm are presented for solving bicriterion network problem, which are based on introducing the fuzzy sets of the value "near to the optimal values" for each criterion, and transforming the initial problem into a mixed integer linear programming problem. The applicability of the algorithm is demonstrated by considering an example.

Keywords
Bicriterion network problem, compromised shortest route, shortest route

1. Introduction
There are many design and managerial problems, for example, design of telecommunications networks, design of transportation networks, design of large-scale irrigation systems, etc., which can be solved with the aid of network models and/or algorithms. A diverse set of network models and algorithms developed to accommodate the various real applications. Sometimes single objective function may not be sufficient to characterize many practical problems completely. In a real transportation network several objectives, for example, time, cost, distance, etc. can be assigned to each arc. If only one objective is given on each arc the solution of the problem can be obtained by classical shortest route algorithm, given in [1]. When more than one objective is given on each arc the solution of the problem can not be obtained by classical shortest route algorithm. The shortest route may be not wise to use because it could be expensive. To deal with a real problem with more than one objective, variants of classical shortest route algorithm have been developed, which are called the *bicriterion* or *multi-criteria* shortest route algorithms [2, 3, 4, 6]. To the best of our knowledge, one of the first algorithm for solving the bicriterion shortest route problem were proposed in [2].

A label setting algorithm is proposed in [4], which is a multicriteria version of Hansen's bicriteria algorithm, given in [2]. It can be seen as a generalization of classical shortest route algorithm [1] to multiple criteria. The author assumed that all edge-coefficients are non-negative. The algorithm makes a set of label at each node. All labels are put in a set and each iteration one label is removed as a permanently labeled. From all the permanent labels at the destination node, the user will select the label with the cost/time combination that he/she prefers. Then this particular route is obtained tracing backward from destination node through the nodes using label's information. In [6], the authors proposed a method to solve the fuzzy shortest route problem. The weighted additive method is introduced to solve a multiple objective integer programming problem, which met the requirements of the network linear programming constraints. Weights in the weighted additive model show the relative importance of the goals. For simplicity, the authors assumed that the importance of the four objectives is the same.

In a real transportation network, it is very difficult to know exact travel time or cost of an arc (i, j), because it depends on some real factors, for example, traffic jam, cost of fuel, accident, etc. To deal with this imprecise information, the probability concepts or interval analysis could be employed. Another way, these types of uncertainty can be represented by membership functions under the fuzzy set theory [7]. The outline of the paper is as follows. In section 2 method and algorithm for solving the bicriterion network problem are given. An example is given in Section 3. Conclusions are in Section 4.

2. Method and Algorithm for solving the Bicriterion Network Problem

In this section, a mixed integer linear programming (ILP) method is proposed to solve the bicriterion network problem. The new method is based on the approach, proposed in [7], for solving multicriterion continuous problems, which introduces fuzzy sets of the values "near to the optimal values" for each criterion.

Consider a connected directed network G, $G = (N, A)$, where $N = \{1, \ldots, n\}$ is the set of the nodes and $A = \{(i, j), (k, l), \ldots, (y, z)\}$ is finite set of directed arcs joining nodes in N. The cardinality of N and A are denoted by $|N|$ and $|A|$ respectively, and $|N| = n$, $|A| = m$. Each arc $(i, j) \in A$ has two attributes, i.e., $d_{ij} = (d'_{ij}, d''_{ij})$. d'_{ij} is the distance between node i and node j, and d''_{ij} is the travel time from node i to node j.

The bicriterion network problem can be formulated as follows:

$$\min\ J_1 = \sum_i \sum_j d'_{ij} x_{ij}$$

$$\min\ J_2 = \sum_i \sum_j d''_{ij} x_{ij}$$

subject to

$$\begin{aligned} &\sum_j x_{ij} = 1 \\ &\sum_j x_{jn} = 1 \\ &\sum_i x_{ik} = \sum_j x_{kj}\ ,\ \forall\ k \neq 1, k \neq n \\ &x_{ij} \geq 0\ ,\ \forall\ i, j \end{aligned} \tag{1}$$

where x_{ij} is the decision variables, and x_{ij} = 0 or 1. For example, x_{ij} = 0 means the corresponding arc ($i \to j$) is not used (is not included), and x_{ij} = 1 means the corresponding arc ($i \to j$) is used (is included).

The generalized steps of the algorithm of the bicriterion network problem:

Step 1. Obtain the optimal solution x_1^* which satisfies the constraints (1) such that $\min J_1 = f_1^T x_1^* = J_1^*$.

Step 2. Obtain the optimal solution x_2^* which satisfies the constraints (1) such that $\min J_2 = f_2^T x_2^* = J_2^*$.

Step 3. Obtain the solution x'_1 which satisfies the constraints (1) such that $\max J_1 = -f_1^T x'_1 = J'_1$.

Step 4. Obtain the solution x''_2 which satisfies the constraints (1) such that $\max J_2 = -f_2^T x''_2 = J''_2$.

Step 5. Obtain the membership functions of the fuzzy sets of "near to optimal values" of the first objective function

$$\mu_{J_1} = \begin{cases} 1, & for \quad J_1 \leq J_1^* \\ \dfrac{J'_1 - J_1(x)}{J'_1 - J_1^*}, & for \quad J_1^* \leq J_1 \leq J'_1 \\ 0, & for \quad J_1 \geq J'_1 \end{cases}$$

Step 6. Obtain the membership functions of the fuzzy sets of "near to optimal values" of the second objective function

$$\mu_{J_2} = \begin{cases} 1, & for \quad J_2 \leq J_2^* \\ \dfrac{J''_2 - J_2(x)}{J''_2 - J_2^*}, & for \quad J_2^* \leq J_2 \leq J''_2 \\ 0, & for \quad J_2 \geq J''_2 \end{cases}$$

***Step* 7**. Construct the mixed ILP problem and solve the problem

max λ

Subject to

$$\lambda \le \frac{J_1' - J_1(x)}{J_1' - J_1^*}$$

$$\lambda \le \frac{J_2'' - J_2(x)}{J_2'' - J_2^*}$$

$$\sum_i x_{0i} = 1$$

$$\sum_j x_{jN} = 1$$

$$\sum_j x_{ij} = \sum_k x_{ki}$$

$$\forall\ i, j \ne 0, \forall\ j \ne N$$

$$x_{ij} = 0 \text{ or } 1,\ \forall\ (i, j) \in A$$

$$0 \le \lambda$$

3. Numerical Example

Consider the network in Figure 1. Each arc (i, j) has two attributes: distance and travel time.

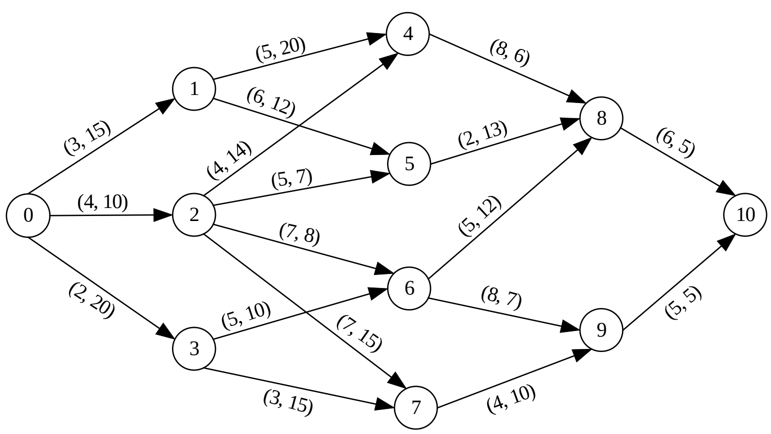

Figure 1 – Network in which each arc has two attributes distance and travel time

We define the bicriterion network problem:

min $J_1 = 3x_{01} + 4x_{02} + 2x_{03} + 5x_{14} + 6x_{15} + 4x_{24} + 5x_{25} + 7x_{26} + 7x_{27} + 5x_{36} + 3x_{37} + 8x_{48} + 2x_{58} + 5x_{68} + 8x_{69} + 4x_{79} + 6x_{810} + 5x_{910}$

min $J_2 = 15x_{01} + 10x_{02} + 20x_{03} + 20x_{14} + 12x_{15} + 14x_{24} + 7x_{25} + 8x_{26} + 15x_{27} + 10x_{36} + 15x_{37} + 6x_{48} + 13x_{58} + 12x_{68} + 7x_{69} + 10x_{79} + 5x_{810} + 5x_{910}$

subject to

$$x_{01} + x_{02} + x_{03} = 1$$

$$x_{810} + x_{910} = 1$$

$$x_{01} - x_{14} - x_{15} = 0$$
$$x_{02} - x_{24} - x_{25} - x_{26} - x_{27} = 0$$
$$x_{03} - x_{36} - x_{37} = 0$$
$$x_{14} + x_{24} - x_{48} = 0$$
$$x_{15} + x_{25} - x_{58} = 0$$
$$x_{26} + x_{36} - x_{68} - x_{69} = 0$$
$$x_{27} + x_{37} - x_{79} = 0$$
$$x_{48} + x_{58} + x_{68} - x_{810} = 0$$
$$x_{69} + x_{79} - x_{910} = 0$$
$$0 \le x_{ij} \le 1$$

The given network meets the requirement of the network linear programming constraints and the feasible solutions x_{ij} are integers (0 or 1).

After step 1 we obtain the following results (MATLAB optimization tools box were used to get the results):

min $J_1 = J_1^* = 14$,

x_1^* = (x_{01} = 0, x_{02} = 0, $x_{03} = 1$, x_{14} = 0, x_{15} = 0, x_{24} = 0, x_{25} = 0, x_{26} = 0, x_{27} = 0, x_{36} = 0, $x_{37} = 1$, x_{48} = 0, x_{58} = 0, x_{68} = 0, x_{69} = 0, $x_{79} = 1$, x_{810} = 0, $x_{910} = 1$).

This solution gives the shortest route for the first objective from source node to destination node as ($0 \to 3 \to 7 \to 9 \to 10$), and the associated distance is $J_1 = 14$.

After step 2 we obtain the following results (MATLAB optimization tools box were used to get the results):

min $J_2 = J_2^* = 30$,

x_2^* = (x_{01} = 0, x_{02} = 1, $x_{03} = 0$, x_{14} = 0, x_{15} = 0, x_{24} = 0, x_{25} = 0, x_{26} = 1, x_{27} = 0, x_{36} = 0, $x_{37} = 0$, x_{48} = 0, x_{58} = 0, x_{68} = 0, x_{69} = 1, $x_{79} = 0$, x_{810} = 0, $x_{910} = 1$).

This solution gives the shortest route for the second objective from source node to destination node as ($0 \to 2 \to 6 \to 9 \to 10$), and the associated travel time is $J_2 = 30$.

After step 3 we obtain the following results (MATLAB optimization tools box were used to get the results):

$J_1' = 24$.

x_1' = (x_{01} = 0, x_{02} = 1, $x_{03} = 0$, x_{14} = 0, x_{15} = 0, x_{24} = 0, x_{25} = 0, x_{26} = 1, x_{27} = 0, x_{36} = 0, $x_{37} = 0$, x_{48} = 0, x_{58} = 0, x_{68} = 0, x_{69} = 1, $x_{79} = 0$, x_{810} = 0, $x_{910} = 1$).

This solution gives the longest route for the first objective from source node to destination node as ($0 \to 2 \to 6 \to 9 \to 10$), and the associated distance is $J_1' = 24$.

After step 4 we obtain the following results (MATLAB optimization tools box were used to get the results):

$J_2'' = 50$.

x_2'' = (x_{01} = 0, x_{02} = 0, $x_{03} = 1$, x_{14} = 0, x_{15} = 0, x_{24} = 0, x_{25} = 0, x_{26} = 0, x_{27} = 0, x_{36} = 0, $x_{37} = 1$, x_{48} = 0, x_{58} = 0, x_{68} = 0, x_{69} = 0, $x_{79} = 1$, x_{810} = 0, $x_{910} = 1$).

This solution gives the longest route for the second objective from source node to destination node as ($0 \to 3 \to 7 \to 9 \to 10$), and the associated travel time is $J_2'' = 50$.

The membership functions of the fuzzy sets of "near to optimal values" of the first objective function are as follows (step 5):

$\mu_{J_1} = 0$, $J_1 \ge 24$

$$\mu_{J_1} = 1,\ J_1 \le 14$$

$$\mu_{J_1} = \frac{24 - J_1(x)}{24 - 14}$$

$J_1(x)$ = $3x_{01}$ + $4x_{02}$ + $2x_{03}$ + $5x_{14}$ + $6x_{15}$ + $4x_{24}$ + $5x_{25}$ + $7x_{26}$ + $7x_{27}$ + $5x_{36}$ + $3x_{37}$ + $8x_{48}$ + $2x_{58}$ + $5x_{68}$ + $8x_{69}$ + $4x_{79}$ + $6x_{810}$ + $5x_{910}$

The membership functions of the fuzzy sets of "near to optimal values" of the second objective function are as follows (step 6):

$$\mu_{J_2} = 0,\ J_2 \ge 50$$

$$\mu_{J_2} = 1,\ J_1 \le 30$$

$$\mu_{J_2} = \frac{50 - J_2(x)}{50 - 30}$$

$J_2(x)$ = $15x_{01}$ + $10x_{02}$ + $20x_{03}$ + $20x_{14}$ + $12x_{15}$ + $14x_{24}$ + $7x_{25}$ + $8x_{26}$ + $15x_{27}$ + $10x_{36}$ + $15x_{37}$ + $6x_{48}$ + $13x_{58}$ + $12x_{68}$ + $7x_{69}$ + $10x_{79}$ + $5x_{810}$ + $5x_{910}$

The mixed ILP problem is as follows (step 7):

max λ

subject to

$\lambda \le$ 2.4 − $0.3x_{01}$ − $0.4x_{02}$ − $0.2x_{03}$ − $0.5x_{14}$ − $0.6x_{15}$ − $0.4x_{24}$ − $0.5x_{25}$ − $0.7x_{26}$ − $0.7x_{27}$ − $0.5x_{36}$ − $0.3x_{37}$ − $0.8x_{48}$ − $0.2x_{58}$ − $0.5x_{68}$ − $0.8x_{69}$ − $0.4x_{79}$ − $0.6x_{810}$ − $0.5x_{910}$

$\lambda \le$ 2.5 − $0.75x_{01}$ − $0.5x_{02}$ − $1x_{03}$ − $1x_{14}$ − $0.6x_{15}$ − $0.7x_{24}$ − $0.35x_{25}$ − $0.4x_{26}$ − $0.75x_{27}$ − $0.5x_{36}$ − $0.75x_{37}$ − $0.3x_{48}$ − $0.65x_{58}$ − $0.6x_{68}$ − $0.35x_{69}$ − $0.5x_{79}$ − $0.25x_{810}$ − $0.25x_{910}$

and the constraint of the network

$$x_{01} + x_{02} + x_{03} = 1$$
$$x_{810} + x_{910} = 1$$
$$x_{01} - x_{14} - x_{15} = 0$$
$$x_{02} - x_{24} - x_{25} - x_{26} - x_{27} = 0$$
$$x_{03} - x_{36} - x_{37} = 0$$
$$x_{14} + x_{24} - x_{48} = 0$$
$$x_{15} + x_{25} - x_{58} = 0$$
$$x_{26} + x_{36} - x_{68} - x_{69} = 0$$
$$x_{27} + x_{37} - x_{79} = 0$$
$$x_{48} + x_{58} + x_{68} - x_{810} = 0$$
$$x_{69} + x_{79} - x_{910} = 0$$
$$x_{ij} = 0 \text{ or } 1,\ \forall\ (i, j) \in A$$
$$0 \le \lambda$$

The solution is obtained by using the software package *LIOP*–1 [5]. The optimal values are as follows:

λ = 0.7

(x_{01} = 0, x_{02} = 1, x_{03} = 0, x_{14} = 0, x_{15} = 0, x_{24} = 0, x_{25} = 1, x_{26} = 0, x_{27} = 0, x_{36} = 0, x_{37} = 0, x_{48} = 0, x_{58} = 1, x_{68} = 0, x_{69} = 0, x_{79} = 0, x_{810} = 1, x_{910} = 0)

Hence, the compromised shortest route is (0 $\rightarrow$ 2 $\rightarrow$ 5 $\rightarrow$ 8 $\rightarrow$ 10), and this route yields the following values of the objective functions: J_1 = 17 and J_2 = 35.

Comments

Using the new algorithm, we obtained the shortest route and the longest route for first objective (distance) from source node to destination node (0 $\rightarrow$ 3 $\rightarrow$ 7 $\rightarrow$ 9 $\rightarrow$ 10) (the associated distance is J_1 = 14) and (0 $\rightarrow$ 2 $\rightarrow$ 6 $\rightarrow$ 9 $\rightarrow$ 10) (the associated distance is $J_1' = 24$), respectively. As well as, we obtained the shortest route and the longest route for second objective (travel time) from source node to destination node (0 $\rightarrow$ 2 $\rightarrow$ 6 $\rightarrow$ 9 $\rightarrow$ 10) (the associated travel time is J_2 = 30) and (0 $\rightarrow$ 3 $\rightarrow$ 7 $\rightarrow$ 9 $\rightarrow$ 10) (the associated travel time is $J_2'' = 50$), respectively. From the obtained results we may conclude that the shortest route for first objective is not the shortest route for second objective and vice versa.

Finally, we obtained a route that is not the shortest route for both objectives, but it is good for both objectives, and we denoted this route as "compromised shortest route".

4. Conclusions

A method and algorithm are proposed for solving bicriterion network problem for acyclic network, which are based on introducing the fuzzy sets of the values "near to the optimal values" for each criterion, and transforming the initial problem in to a mixed ILP problem. Using the new proposed approach, we obtained the shortest routes for both objectives, and finally, we obtained the compromised shortest route that is good for both objectives. Another approach to solve the bicriterion network problem is to transform the problem to a single criterion objective function. Then, this problem can be solved by using traditional linear programming problem. This method requires more operations in order to obtain the compromised shortest route for both objectives. This fact confirms the advantage of the proposed bicriterion network algorithm. Numerical example is given to illustrate the efficient assessment of the solution and the workability of the developed method and algorithm. In principle, this new approach can be generalized to *n*-criterion network problem.

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