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Reliability Modeling of a Manufacturing Cell Operated under Degraded Mode

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Abstract

In this study, a stochastic model is developed to analyze performance measures of a manufacturing cell, which is allowed to operate under degraded mode. The model is used to determine reliability and productivity of the cell, as well as the utilization of its components, under various operational conditions, including equipment failures and fault-tolerant states. The model and the results can be useful for design engineers and operational managers to analyze performance of a system at the design or operational stage.

Keywords
Degraded Machine Operation, Multi-State Reliability Modeling, Manufacturing Cells, Failure Analysis.

1. Introduction

A machining cell consists of one or more machines, served by a loading and unloading system, which could be a robot or an operator, and a pallet handling system to transfer a batch of parts in and out to be machined by the system. Manufacturing cells are designed around flexible machines to produce a high variety of products. Flexibility in manufacturing results in higher utilization of equipment than it would be in traditional manufacturing systems. Consequently, flexible systems have higher failure rates and require well planned maintenance activities. Unexpected changes in machine states can be classified as faults and failures. A fault is a tolerable malfunction rather than a total breakdown or a failure. With a tolerable malfunction, a machine can operate in a degraded performance level as opposed to its normal performance level. Thus, a machine can be in one of three states: *Up-Normal*, *Up-Degraded*, and *Down* states.

Manufacturing cells are widely used in industry to process a variety of parts to achieve high productivity in production environments with rapidly changing product structures and customer demand. They offer flexibility to be adapted to the changes in operational requirements. There are various types of Flexible Manufacturing Cell (FMC) systems with flexible machines for discrete part machining. A FMC consists of a robot, one or more flexible machines including inspection, and an external material handling system such as an automated pallet for moving blanks and finished parts into and out of the cell. The robot is utilized for internal material handling which includes machine loading and unloading. The FMC is capable of doing different operations on a variety of parts, which usually form a part family with selection by a group technology approach.

FMC performance depends on several operational and system characteristics including part scheduling, robot speeds, machine and pallet characteristics. Most of the related researches are directed to scheduling, control, part grouping, and layout aspects of FMCs. Scheduling algorithms are used to determine the sequence of parts, which are continuously introduced to the cell. However, other system characteristics such as machining rate, pallet capacity, robot speed and pallet speed significantly affect FMC performance. Several models have been developed [1-9] for analyzing FMC systems with different configurations. In all of the systems studied, machines are assumed to have only one type of failed state which may not represent all systems in real life. This study differs from previous studies in that fault-tolerant states are incorporated into the model and the machine is allowed to operate in a degraded state in addition to its normal operational state. A Markovian model is developed for the FMC with fault tolerant states to determine reliability and productivity of the system under various operational conditions. The model and the results can be useful for design engineers as well as operational managers in production and maintenance planning.

2. Operation of the Cell

Operation of the FMC system is illustrated in Figure 1. An automated pallet handling system delivers n blanks consisting of different parts into the cell. Initially the robot reaches to the pallet, grips a blank, moves to the machine and loads the blank. After the operation is completed, the robot reaches the machine, unloads the completed part and places it into the pallet, picks another blank and loads it onto the machine. This sequence of operations continues until all parts on the pallet are completed, at which time the pallet with n finished parts moves out and a new pallet with n blanks is delivered into the cell automatically. Since a variety of parts, which require different operations, are introduced into the system, part processing times as well as loading/unloading times are assumed stochastic. Machines are assumed to be unreliable and fail during the operations. Time to failure and time to repair are assumed to follow exponential distribution. Processing times as well as loading/unloading times are random due to the introduction of different parts into the FMC, failures of machines, and random characteristics of system operation. These random operational characteristics present a complication in studying and modeling cell performance.

3. Stochastic Modeling of Cell Operations

In order to analyze an FMC with stochastic operations, the following system states and parameters are defined:

$S_{ijk}(t)$ = state of the FMC at time t

$P_{ijk}(t)$ = probability that the system will be in state $S_{ijk}(t)$

i = number of blanks in FMC (on the pallet, the machine, or the robot gripper)

j = state of the production machine (j=0 if the machine is idle in normal mode; j=1 if the machine is operating in the normal mode; j=2 if the machine is operating in the fault-tolerant (degraded) mode; j=3 if the machine has failed and is under repair; j=4 if the machine is idle in the fault-tolerant state).

k= state of the robot (l=0 if the robot is idle; l=1 if the robot is loading or unloading)

l = loading rate of the robot (parts/unit time); u = unloading rate of the robot (parts/unit time)

z = combined loading/unloading rate of the robot; w = pallet transfer rate (pallets/unit time)

λ_1= failure rate of the machine when in normal mode ($1/\lambda_1$ = mean time between failures)

λ_2= rate at which machine transfers to the degraded (fault-tolerant) mode from normal operational mode ($1/\lambda_2$ = mean time to transfer to fault-tolerant mode from normal mode)

λ_3= failure rate of machine when in degraded mode ($1/\lambda_3$ =mean time between failures in this mode)

μ_1= repair rate of machine from failed to normal operational mode.

μ_2= repair rate of machine from degraded mode to normal mode.

μ_3= repair rate of machine from failed to degraded mode.

v_i= machining or production rate of the machine (parts/unit time) (i=1 normal mode; i=2 degraded mode)

n = pallet capacity (number of parts/pallet)

Q_c= production output rate of the cell in terms of parts/unit time.

We assume that the machine fails at different rates when in normal up state and when in degraded up state. Also it is assumed that a failed machine can be repaired to bring it to normal up state, or partially repaired for temporary reasons, to bring it to the degraded up state. Using the fact that the *net flow* rate at each state is equal to the difference between the rates of *flow in* and *flow out*, a set of differential equations are obtained for the stochastic FMC. For example, for the state (n,001), rate of change with respect to time t is given by:

$dP_{n01}(t)/dt = wP_{000}(t) - lP_{n01}(t)$

The set of differential equations for all states are given by equations 1-19 below. Note that equations are given in three different sets since each set has a unique form. The first set represents the initial system when a new pallet arrives with n blanks; the last set represents the system when the last parts on the pallet are being processed; and the second set represents the intermediate operations. In each term, t has been omitted for simplification. At steady state, $t\rightarrow\infty$; $dP_{n01}(t)/dt\rightarrow 0$ and all of the differential equations change into a difference equations as: $wP_{000} - lP_{n01}=0$. These equations must be solved to obtain the steady state probabilities and system performance measures.

$$\frac{dp_{n,0,1}}{dt} = wp_{0,0,0} - lp_{n,0,1} \quad (1)$$

$$\frac{dp_{n,4,1}}{dt} = wp_{0,4,0} - lp_{n,4,1} \quad (2)$$

$$\frac{dp_{n-1,3,0}}{dt} = \lambda_1 p_{n-1,1,0} + \lambda_3 p_{n-1,2,0} - (\mu_1 + \mu_3) p_{n-1,3,0} \tag{3}$$

$$\frac{dp_{n-1,1,0}}{dt} = lp_{n,0,1} + \mu_1 p_{n-1,3,0} + \mu_2 p_{n-1,2,0} - (\lambda_1 + \lambda_2 + v_1) p_{n-1,3,0} \tag{4}$$

$$\frac{dp_{n-1,2,0}}{dt} = \lambda_2 p_{n-1,1,0} + \mu_3 p_{n-1,3,0} + lp_{n,4,1} - (\lambda_3 + \mu_2 + v_2) p_{n-1,2,0} \tag{5}$$

$$\frac{dp_{n-1,0,1}}{dt} = v_1 p_{n-1,1,0} + \mu_2 p_{n-1,4,1} - zp_{n-1,0,1} \tag{6}$$

$$\frac{dp_{n-1,4,1}}{dt} = v_2 p_{n-1,2,0} - (\mu_2 + z) p_{n-1,4,1} \tag{7}$$

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$$\frac{dp_{n-x,3,0}}{dt} = \lambda_1 p_{n-x,1,0} + \lambda_3 p_{n-x,2,0} - (\mu_1 + \mu_3) p_{n-x,3,0} \tag{8}$$

$$\frac{dp_{n-x,1,0}}{dt} = zp_{n-x+1,0,1} + \mu_1 p_{n-x,3,0} + \mu_2 p_{n-x,2,0} - (\lambda_1 + \lambda_2 + v_1) p_{n-x,1,0} \tag{9}$$

$$\frac{dp_{n-x,2,0}}{dt} = \lambda_2 p_{n-x,1,0} + \mu_3 p_{n-x,3,0} + zp_{n-x+1,4,1} - (\lambda_3 + \mu_2 + v_2) p_{n-x,2,0} \tag{10}$$

$$\frac{dp_{n-x,0,1}}{dt} = v_1 p_{n-x,1,0} + \mu_2 p_{n-x,4,1} - zp_{n-x,0,1} \tag{11}$$

$$\frac{dp_{n-x,4,1}}{dt} = v_2 p_{n-x,2,0} - (\mu_2 + z) p_{n-x,4,1} \tag{12}$$

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$$\frac{dp_{0,3,0}}{dt} = \lambda_1 p_{0,1,0} + \lambda_3 p_{0,2,0} - (\mu_1 + \mu_3) p_{0,3,0} \tag{13}$$

$$\frac{dp_{0,1,0}}{dt} = zp_{1,0,1} + \mu_1 p_{0,3,0} + \mu_2 p_{0,2,0} - (\lambda_1 + \lambda_2 + v_1) p_{0,1,0} \tag{14}$$

$$\frac{dp_{0,2,0}}{dt} = \lambda_2 p_{0,1,0} + \mu_3 p_{0,3,0} + zp_{1,4,1} - (\lambda_3 + \mu_2 + v_2) p_{0,2,0} \tag{15}$$

$$\frac{dp_{0,0,1}}{dt} = v_1 p_{0,1,0} + \mu_2 p_{0,4,1} - up_{0,0,1} \tag{16}$$

$$\frac{dp_{0,4,1}}{dt} = v_2 p_{0,2,0} - (\mu_2 + u) p_{0,4,1} \tag{17}$$

$$\frac{dp_{0,4,0}}{dt} = up_{0,4,1} - (\mu_2 + w) p_{0,4,0} \tag{18}$$

$$\frac{dp_{0,0,0}}{dt} = up_{0,0,1} + \mu_2 p_{0,4,0} - wp_{0,0,0} \tag{19}$$

The system consists of 14+5(n-2) equations and equal number of unknowns. For example, for n=4, number of system states, as well as number of equations, is 14+5(4-2) =24 and for n=10, it is 14+5(10-2) =54. It is possible to obtain an exact solution for this system of equations given by PT=0, where P is the steady state probabilities vector to be determined and T is the probability transition rate matrix. It is known that all of the equations in PT=0 are not linearly independent and thus the matrix T is singular, which does not have an inverse. We must add the normalizing condition given by equation (20) below to the sets of equations above by eliminating one of them to assure that sum of all state probabilities is 1.

$$\sum_{i=0}^{n}\sum_{j=0}^{2}\sum_{k=0}^{2}\sum_{l=0}^{2} P_{ijkl} = 1 \tag{20}$$

Theoretically it may be possible to manipulate the equations (1-19) in order to determine closed form solutions for state probabilities. However, because of large number of equations involved, it is difficult to obtain closed form solutions. Exact numerical solutions can be obtained for all state probabilities by solving the set of linear equations by any method. We have solved the equations by MAPLE software for the state probabilities. Once the state probabilities, $P_{i,j,k}$, are determined, it is then possible to determine various system and subsystem performance measures. The following notations and performance measures in equation set (21) are calculated in the next section.

M_u = Percent of time machine is in operating in normal state (*Up-Normal State*).
M_g = Percent of time machine is in operating in degraded state (*Up-Degraded State*).
M_d = Percent of time machine is not operating (*Down State)*.
M_i = Percent of time machine is idle waiting for loading, unloading and pallet transfers.
R_u = Percent of time robot is being utilized in loading and unloading state.
P_u = Percent of time pallet is being utilized in transferring pallets in and out of the system.
Q = System production rate based on machine rates of production in different up states.

$$M_u = \sum_{k=0}^{k=n-1} P_{k,2,1} \ ; \quad M_g = \sum_{k=0}^{k=n-1} P_{k,3,1} \ ; \quad M_d = \sum_{k=0}^{k=n-1} P_{k,4,1} \ ; \quad M_i = \sum_{k=0}^{k=n} (P_{k,1,2} + P_{k,5,2}) + P_{0,1,1} + P_{0,5,1}$$

$$R_u = \sum_{k=0}^{k=n} (P_{k,1,2} + P_{k,5,2}) \ ; \quad P_u = P_{0,1,1} + P_{0,5,1} \ ; \ Q_c = v_1 M_u + v_2 M_g \tag{21}$$

4. Numerical Results

In this section, we present some numerical results for a case problem with different parameters for an FMC system allowed to operate under degraded mode. The parameter values for the unreliable FMC system were as follows: Operation time per part at Normal Up state ($1/v_1$)=4 time units; Operation time per part at Degraded Up state ($1/v_2$)= 8 time units; Robot loading time for the first part ($1/\ell$)=1/6 time units; Robot load/unload time for subsequent parts (1/z)=1/3 time units; Robot unloading time for the last part (1/u)=1/6 time units; Mean time to failure of the machine at Normal Up state ($1/\lambda_1$)=100 time units; Mean time machine transfers from Normal Up to Degraded state ($1/\lambda_2$)= 50 time units; Mean time to failure of the machine at Degraded state ($1/\lambda_3$)=80 time units; Mean time to repair (MTTR) the machine when in failed state ($1/\mu_1$)=10 time units; MTTR the machine to move it from Degraded to Normal Up state ($1/\mu_2$)=5 time units; MTTR the machine to move it from failed to Degraded state ($1/\mu_3$)=8 time units; Pallet transfer time (1/w) =4, 8, 12 time units/pallet; Pallet capacity (n) = 2,…..,20 units. It should be noted that the mean is the inverse of the rate in each case.

Figure 2 shows the production output rate as a function of pallet capacity (n) at different pallet transfer rates of w=0.25, w=0.125, and w=0.0833. As it is seen from the figure, production rate increases with increasing pallet capacity and pallet transfer rates. While the rate of increase is higher initially, it levels off at higher values of n. Figures 3 and 4 show machine utilizations with respect to pallet capacity and pallet transfer rate at normal up state and degraded state respectively. An increasing trend is observed in machine utilizations as the pallet capacity and pallet transfer rates are increased. The increase levels off at higher pallet capacities exceeding 10 units. Figure 5 shows the percent of time machine is down under repair, while figure 6 shows the percent of time machine is idle waiting for the robot loading/unloading or for the pallet transferring parts. While down time percentages increase with increasing pallet capacity due to increased utilization and failures, percent idle time decreases with increasing pallet capacities. Figures 7 and 8 show robot and pallet utilizations as functions of pallet capacity and pallet transfer rates respectively. Robot utilization increases with increasing pallet capacity due to increased loading/unloading frequency per pallet. However, pallet utilization decreases as the pallet capacity is increased or pallet transfer rate is decreased. This is because more parts are transferred at each transfer time and frequency of transfer reduces. While some of these relations may be more or less obvious, accurate relations need to be established through these results.

5. Conclusions

In today's dynamic manufacturing environment, manufacturing firms produce a variety of products which need flexible manufacturing equipment with automated material handling equipment. Because of high utilization of these equipments, different levels of operations, including fault-tolerant operation modes, may be needed. It is important

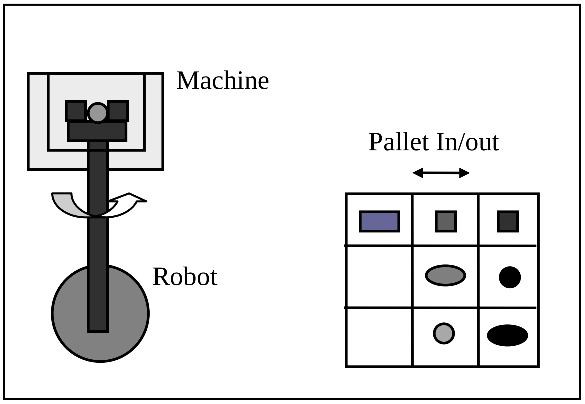

Figure 1. A Flexible Manufacturing Cell

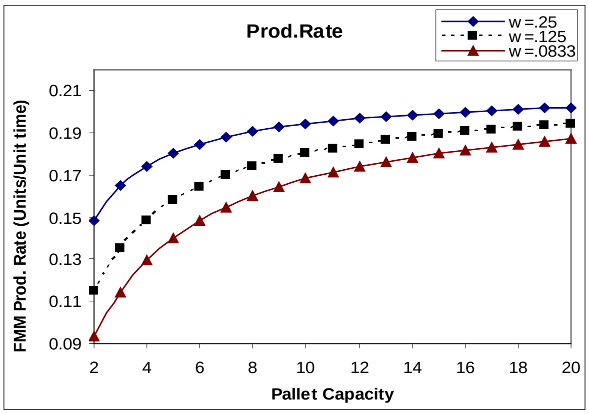

Figure 2. Production output rate vs. pallet capacity.

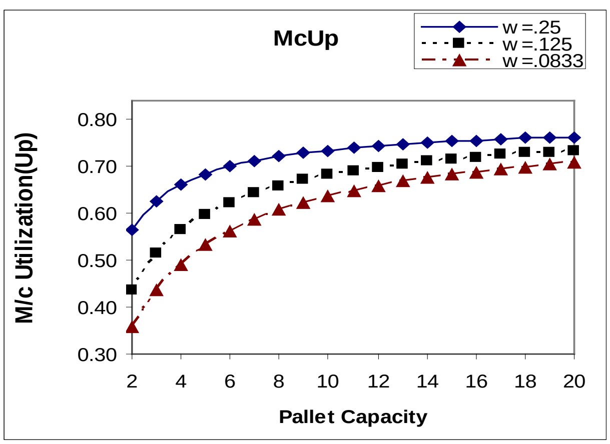

Figure 3. Machine utilization at normal up state.

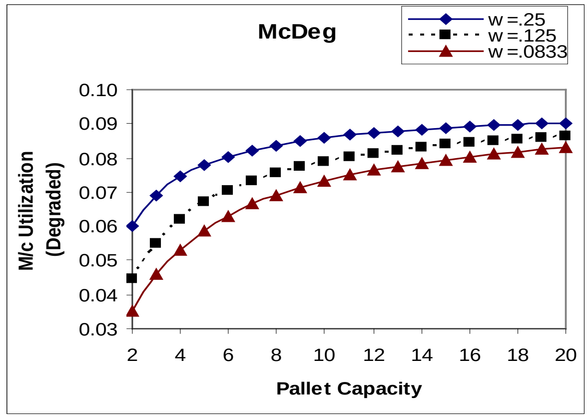

Figure 4. Machine utilization at degraded state.

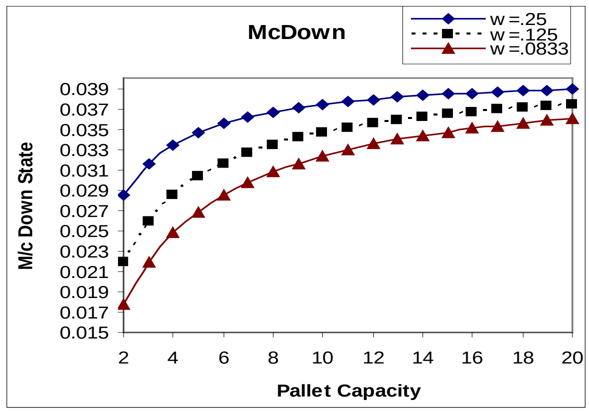

Figure 5. Machine down state vs. pallet capacity

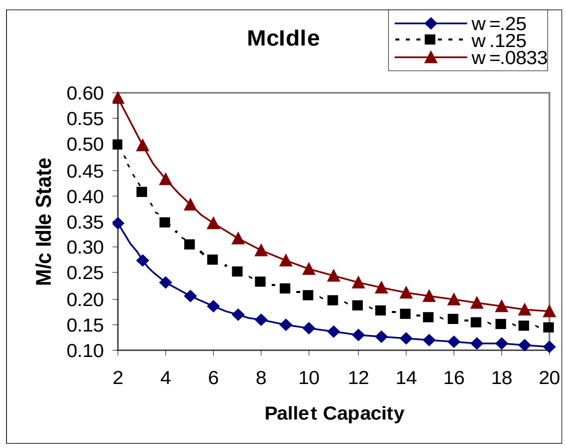

Figure 6. Machine idle state vs. pallet capacity.

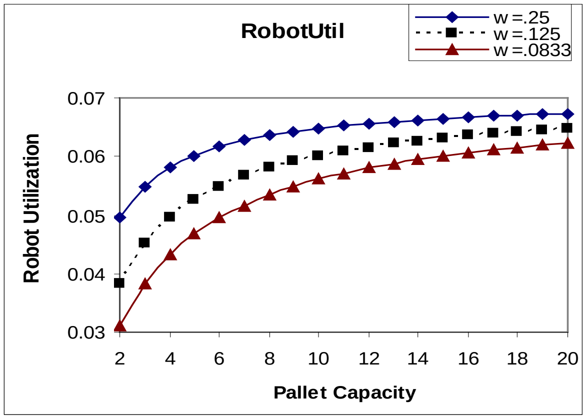

Figure 7. Robot utilization vs. pallet capacity.

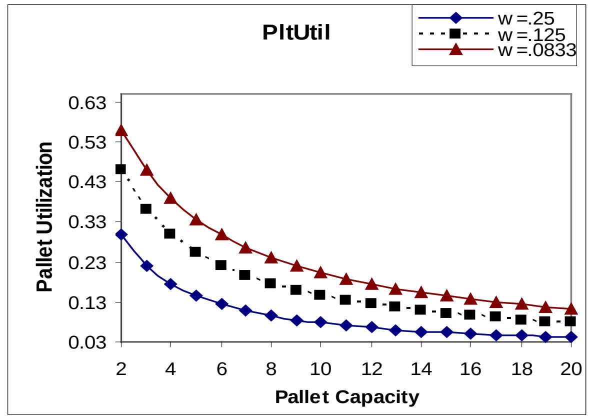

Figure 8. Pallet utilization vs. pallet capacity

to be able to analyze such systems in order to gain full benefits in their implementation and subsequent operations. Stochastic models and formulations obtained in this paper are used to analyze and optimize the productivity and performance measures of FMC systems under different machine, robot, and pallet operational characteristics. Best parameter combinations can be determined for a given system. In particular, best machining rates, machine repair rates, robot loading and unloading rates, pallet capacity, and pallet transfer rates can be determined for specific FMC characteristics. It is possible to optimize machine repair and maintenance rates to achieve maximum production output rates and other performance measures.

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