Optimal Vendor-Buyer Cooperative Inventory Policy for Order Size Dependent Transportation Cost

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Abstract

In a vendor-buyer cooperative inventory system several factors including planned shortage, controlled sales price, shipment size and frequency affect the total profit. Though a larger order size reduces the number of shipments, it conversely increases the lead time and transportation cost. On the other hand, planned shortage of items reduces the buyer's storage cost; at the same time it increases the backorder cost for the buyer. By controlling sales price, the buyer can affect the rate of market demand which eventually affects the total profit. The problem is to find an appropriate sales price, backorder quantity, ordering size and frequency that lead to maximum joint total profit. Considering all of these aspects for an imperfect production system, this paper presents two joint total profit models for two different cases depending on the location of the quality inspection. A search approach, based on *random walk with random restart* method, is used to solve both models for obtaining suboptimal solutions. A numerical instance is presented to illustrate the solution approach. The numerical instance presents that the solution noticeably differs when quality inspection is done by the buyer instead of vendor.

Keywords

Cooperative inventory, planned shortage, imperfect production, controlled sales price.

1. Introduction

Recently, joint economic lot size (JELS) problems have been in the spotlight of researches on supply chain. Many firms have adopted this type of lot-size models which coordinate inventory replenishment decisions between the buyer and the vendor. It has been proved that in most case if vendor and buyer make decisions jointly to determine the best policy, it is more rewarding for them than if they make decisions independently. Since JELS models have received more attention by firms, it has encouraged lot of researches to determine the best policy which can get the minimum total cost or the higher supply chain profit under an integrated vendor-buyer cooperation.

In many researches on JELS models the imperfect production, lead time, pricing decisions or planned shortages have been studied separately. Indeed, since Goyal (1977) introduced the joint total cost for a vendor and a buyer and Benerjee (1986) extended Goyal's work to determine the joint optimization of the cost functions of the vendor and the buyer, many researchers have studied JELS model in a lot of streams.

The *imperfect production* has been introduced in an economic order quantity model by Porteus (1986). Salameh and Jaber (2000) extended Porteus's work by finding the expected annual profit considering a 100% screening process where the rate of defective items is a random proportion. They also considered that imperfect items will be sold at the end of the screening process, in a single batch. Salameh and Jaber's work has inspire many others researches. Huang (2002) considered a two-stage supply chain where a penalty cost is applied to the vendor if a shipment contains defective items. As well as Huang, researchers as Eroglu and Ozdemir (2007) added shortages to Salameh and Jaber's model. Wee *et al.* (2007) also studied an optimal inventory model for items with imperfect quality and shortage backordering. Since this study, Chang and Ho (2010) extended Wee *et al.*'s work by using the renewal-reward theorem to modify the expected profit. Ha and Kim (2012) also reported a cooperative inventory model for imperfect quality.

On the other hand, *pricing decisions* have impact on the profit of a model and on the demand. Researches that consider pricing decisions in JELS model have been conducted in many streams. Viswanathan and Wjang (2003) studied discount pricing decisions in distribution channels with price-sensitive demand. Ordering and pricing policies with price-sensitive demand have been studied in the research of Sajadieh and Jokar (2009a). In that research the maximize profit have been found by assuming a linear demand.

Besides, imperfect production and pricing decisions the *lead time* has been often studied in the case of a JELS model, especially deterministic lead time. Sajadieh and Jokar (2009b) studied expected total cost of JELS models where the lead time is a stochastic variable. Since Kim and Benton (1995) discussed about considering the impact on lead time of the shipment size, researchers started to consider lead time as a function or decision variable instead of a given parameter. Ben-Daya and Hariga (2004) developed a single vendor single buyer model in which lead time is considered as a linear function. They assumed that lead time is proportional to the quantity ordered.

In previous researches, several models have been studied on JELS considering lead time, imperfect production, pricing decisions and planned shortages separately. However, the combination of all these three factors is lacking in the above discussed research publications. It is reported that Rad *et al.* (2014) have developed JELS models considering imperfect production, shortages and pricing decisions. The aim of this current research is to extend the works of Rad *et al.* (2014) by introducing a more practical aspect when the lead time is significantly correlated with the order quantity. Lead time is assumed to be a function of the order quantity and lead time affects the fixed transportation cost as well. Hence, this paper presents an inventory model which considers the impact of order quantity on lead time and on the transportation cost.

The objective of this research is to derive an expected joint total profit function of the system, and then it is used to find the optimal order quantity, selling price, number of shipments and backorder quantity that maximize the profit of the joint system considering. The paper is structured in a sequential order. Section 2 introduces the problem and defines the assumptions and notations used in formulating the problem. Section 3 develops the mathematical models. Section 4 presents the solution methodology and Section 5 presents a numerical example. Section 6 concludes on the overall outcomes and results.

2. Problem Description

This research considers a single vendor and a single buyer for a single product case. In practice, production can be imperfect and the vendor can decide to apply quantity discounts to influence the buyer and encourage him to buy in large quantity. The buyer orders a certain quantity of an item for each shipments. When buyer has the responsibility to do 100% screening as a quality inspection, each one of the shipments received by the buyer may contain a certain proportion of defective items. The inspection can occur at the vendor's facility as well. In that case buyer will receive a 100% good quality product. These two options are presented as two different cases in this paper. The buyer can decide to make planned shortage to reduce his holding cost, so the backordering cost and the buyer's backorder quantity are considered.

As we consider the vendor making price decisions to influence the buyer, the demand is known as a function of the price selling and the transportation cost is known as a function of lead time, while lead time is a function of the order quantity. Thus, if quantity discounts are applied, buyer will want to buy in large quantity to have a lower price. Though a lower price will increase the demand and sales, a large quantity will increase the transportation cost, buyer's holding cost and the lead time. Hence, an appropriate policy has to be determined for the order quantity, selling price, as well as number of shipments and backorder quantity in order to maximize the joint profit. The following assumptions will be helpful to understand the problem clearly.

(i) A single-vendor single-buyer supply chain for a single product is considered.
(ii) The demand rate is known as a function of the selling price.
(iii) All products are subjected to an error free screening process, to identify the defective items.
(iv) The unit purchase price from the vendor, is reduced at a proportion of defect rate.
(v) Both the screening and production rates are higher than the demand rate.
(vi) The defective items are scrapped without incurring any cost or generating any salvage value.
(vii) The expected number of good items is equal to the demand rate during the cycle time.
(viii) The lead time is an exponential function of the order quantity.
(ix) The fixed transportation cost is a function of the lead time.
(x) Time of the system is infinite and inventory is continuously reviewed all along.
(xi) All the items produced are delivered in full shipments during the cycle time.

To formulate this problem some notations are required to be stated first. The notations used for this paper are divided into three categories, namely parameters, variables and measures. Here the parameters are, D = demand rate (units/year), δ = coefficient in the demand rate function, θ = exponent of the selling price, S = vendor's set up cost (\$/setup), A = buyer's ordering cost (\$/order), h_v = vendor's holding cost (\$/unit/year), h_b = buyer's holding cost (\$/unit/year), π = buyer's backordering cost (\$/unit/year), c_p = unit production cost (\$/unit), $c_{b,s}$ = buyer's screening cost (\$/unit), $c_{v,s}$ = vendor's screening cost (\$/unit), $c_{v,w}$ = vendor's warranty cost for defective items

(\$/unit), c_v = unit cost for handling or receiving an item (\$/unit), γ = defect rate, $f(\gamma)$ = unit purchase price from the vendor (\$/unit), f_{max} = maximum unit purchase price (\$/unit), x = reducing coefficient of the purchase price (\$/unit/ γ), s_b = buyer's screening rate (unit/year), P = vendor's production rate (units/year), $r = D/P$, $r_b = D/s_b$, τ = lead time (year), τ_0 = minimum lead time (year), β = exponent of the order quantity in the lead time function, $g(\tau)$ = fixed transportation cost per shipment (\$/order), F_0 = minimum fixed transportation cost per shipment (\$/order/year), T = buyer's replenishment cycle time (year/order), I_v = vendor's inventory level (units), I_b = buyers inventory level (units). On the other hand, the decision variables are denoted as p = buyer's unit selling price (\$/unit), Q = buyer's order quantity (units/order), b = buyer's backorder quantity (units/order), n = number of shipments per production cycle of the vendor. The objective function are defined as JTP_b = joint total profit when buyer inspects the items (\$/year), JTP_v = joint total profit when vendor inspects the items (\$/year). Using these notations, the mathematical models for both Cases are derived and solved for obtaining a suitable strategy for the collaborative system.

3. Mathematical Modeling

This section develops two JELS models when planned shortages occur and when the rate of defective is a given value. In Case 1 the buyer inspects the items and Case 2 the screening is done by the vendor. In each model the total profit of the vendor and the buyer are calculated separately and then the combined is found to express the joint total profit (JTP) of the integrated vendor-buyer system. Here, we consider four functions for the demand rate, unit price, lead time and transportation cost which are listed in the assumptions, respectively. Thus, the demand rate $D = \delta p^{-\theta}$ where $\delta > 0$ and $\theta > 1$, the unit purchase price $f(\gamma) = f_{\max} - x\gamma$ where $f_{\max} > x > 0$, the lead time $\tau = \tau_0 Q^{\beta}$, where $\beta = [0,1]$, and the fixed transportation cost $g(\tau) = F_0\tau = F_0\tau_0 Q^{\beta}$.

3.1 Case 1: Inspection done by the buyer

For the first case, we consider a situation where the inspection of the items is done by the buyer. In this situation, the shipment contains defective items when it arrives at the buyer's facility. The buyer's screening cost is added to the total cost for the buyer. In this model the inventory levels for both vendor and buyer are shown graphically in Figure 1 on a continuous time scale. The average inventory level for the vendor is obtained by the method developed by Sarker and Parija (1994). In order to obtain the joint total profit, the total profit of the buyer and of the vendor are calculated separately. Buyer's total profit includes inventory holding, ordering, backordering, transportation, quality inspection, and items handling or receiving costs, as well as purchasing and selling prices. The vendor's total profit includes inventory holding, setup, production and warranty costs, as well as his selling price to the buyer.

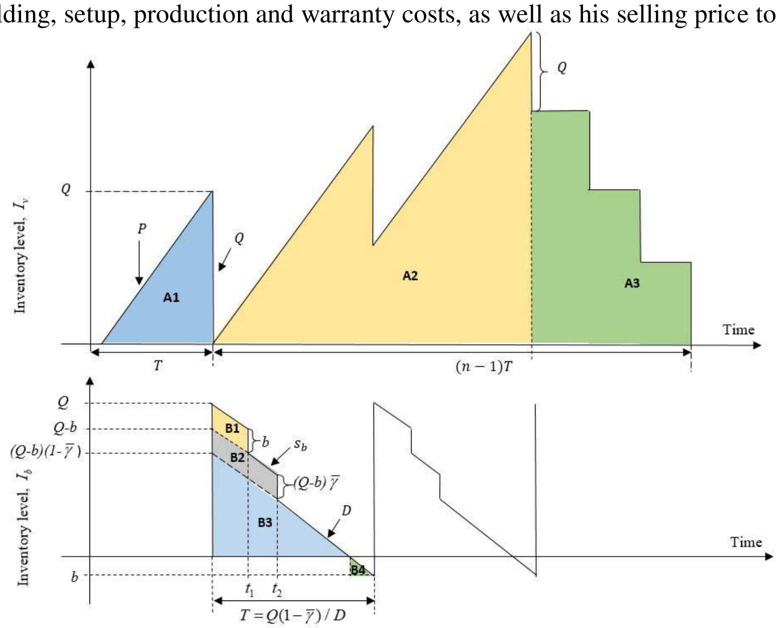

Figure 1. Vendor's and buyer's inventory level when *buyer* inspects the items (Case 1)

Finally, the joint total profit, $JTP_b(p,Q,b,n)$ for Case 1 is obtained by adding the total profit of the buyer in and the total profit of the vendor. After simplification, the joint total profit becomes

$$JTP_b(p,Q,b,n) = \left(p - \frac{c_{b,s}+c_v+c_p+\gamma c_{v,w}}{1-\gamma} \right)\delta p^{-\theta} - \frac{A+g(\tau)+S/n}{Q(1-\gamma)}\delta p^{-\theta} - \frac{h_b(1-\gamma)}{2}\left[\frac{b^2}{Q}+Q\right] - \frac{\pi b^2}{2Q(1-\gamma)} + b(1-\gamma)h_b$$
$$- \frac{h_b}{(1-\gamma)}\left[\frac{b^2}{\left((1-\gamma)/r_b-1\right)Q} + (Q-b)\gamma r_b\right] - \frac{h_v Q}{2}\left[2n(1-\gamma) - \frac{n(1-\gamma)D}{P} - (n-1)\right]. \qquad (1)$$

3.2 Case 2: Inspection done by the vendor

For Case 2, the inspection takes place at the vendor's facility. This means that the shipments contain only perfect items at their arrival to the buyer. Hence, in this case, the vendor's total profit includes screening cost i.e., quality inspection cost, in addition to the holding [refer to Figure 2], setup and production. It is to be noted that, the warranty cost is not present in the vendor's total profit; similarly, the unit price reduction and screening cost is absent in the buyer's total profit function.

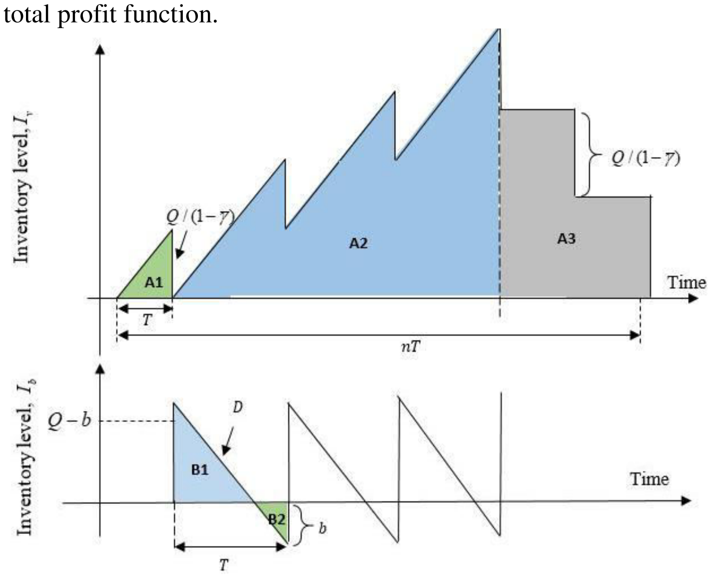

Figure 2. Vendor's and buyer's inventory level when *vendor* does the quality inspection (Case 2).

Thus, the joint total profit, $JTP_v(p,Q,b,n)$ in Case 2 is obtained by adding the total profit of the buyer, and the total profit of the vendor. The simplified final version of the joint total profit function is presented in Eq. (2).

$$JTP_v(p,Q,b,n) = \left(p - c_v - \frac{c_p+c_{v,s}}{1-\gamma} \right)\delta p^{-\theta} - \frac{A+g(\tau)+S/n}{Q}\delta p^{-\theta} - \frac{h_b(Q-b)^2}{2Q} - \frac{h_v Q}{2}\left[\frac{2n}{1-\gamma} - \frac{nD}{(1-\bar{\gamma})^2 P} - (n-1)\right] - \frac{\pi b^2}{2Q}. \qquad (2)$$

Based on the situations (Case 1 or 2) the problem is solved for obtaining appropriate values of p, Q, b and n that maximize JTP_b or JTP_v.

4. Solution Methodology

This section presents the methodology to find the best values of the decision variables, *p, Q, n* and *b* that maximize the joint total profit in any case. It can be proved that the function $JTP_b(p,Q,b,n)$ is concave in *b* as well as concave in n for a given value of p and *Q*. The first partial derivatives of JTP_b with respect to b and n lead to the optimal values of the backorder quantity, b^* and number of shipments, n^*. This derivatives yield $b^* = Qh_bR_P$ and $n^* = V_p/Q$, respectively, where $R_p = \dfrac{\gamma r_b + (1-\gamma)^2}{2h_b/\{(1-\gamma)/r_b - 1\} + h_b(1-\gamma)^2 + \pi}$ and $V_p = \sqrt{2SD/\left[h_v(1-\gamma)\{(1-\gamma)(2-r)-1\}\right]}$. Substituting b^* and n^* in Eq. (1), JTP_b can be obtained as

$$JTP_b(p,Q)=\left(p-\frac{c_{b,s}+c_v+c_p+\gamma c_{v,w}}{1-\gamma}\right)\delta p^{-\theta}-\frac{A+g(\tau)}{Q(1-\gamma)}\delta p^{-\theta}-\frac{SD}{(1-\gamma)V_p}-\frac{h_b(Q-Qh_bR_p)\gamma r_b}{(1-\gamma)}-\frac{\pi Q h_b^{2}R_p^{2}}{2(1-\gamma)}-\frac{Qh_b^{3}R_p^{2}}{(1-\gamma)\left((1-\gamma)/r_b-1\right)}$$
$$-\frac{h_b(1-\gamma)}{2}\left[Qh_b^{2}R_p^{2}+Q\right]-\frac{h_v}{2}\left[2V_p(1-\gamma)-V_p(1-\gamma)r-V_p\right]-\frac{h_vQ}{2}. \quad (3)$$

Similarly for Case 2, the function $JTP_v(p,Q,b,n)$ is concave in b, as well as concave in *n* for given values of *p* and *Q*. Hence, from the first partial derivatives of $JTP_v(p,Q,b,n)$ with respect to b and *n* we have $b^*=h_bQ/(h_b+\pi)$ and $n^*=W_p/Q$, respectively, where $W_p=\sqrt{\dfrac{2SD(1-\gamma)}{h_v\left[2-r/(1-\gamma)-(1-\gamma)\right]}}$. These results yield

$$JTP_v(p,Q)=\left(p-c_v-\frac{c_p+c_{v,s}}{1-\gamma}\right)\delta p^{-\theta}-\frac{A+g(\tau)}{Q}\delta p^{-\theta}-\frac{S\delta p^{-\theta}}{W_p}-\frac{h_bQ}{2}\left[1-\frac{h_b}{h_b+\pi}\left(2-\frac{h_b}{h_b+\pi}\right)\right]-\frac{\pi h_b^{2}Q}{2(h_b+\pi)^2}-\frac{h_v}{2}\left[\frac{2W_p}{(1-\gamma)}-\frac{W_pr}{(1-\gamma)^2}-W_p\right]-\frac{h_vQ}{2}. \quad (4)$$

Hence, if the optimum value of (p,Q) is known the corresponding optimum values of b and n can be obtained from the corresponding expressions of b^* and n^* noted here. In order to maximize the objective functions $JTP_b(p,Q)$ in Eq. (3) or $JTP_v(p,Q)$ in Eq. (4) for appropriate values of p and Q, a *random walk with random restart* search approach is followed in this paper. This search approach starts with a random point (p,Q) and updates the solution point by searching multiple random directions (random walk) for increasing the objective function value. The random walk continues until the objective function reaches to a local optimal. The procedure restarts with a new random point and it is repeated for a preset number of iterations.

5. Numerical Example

In this section the two Cases are illustrated with the same numerical example. Here a supply chain of a cell phone screen is considered. A similar numerical example is presented in Rad *et al.*'s (2014) paper. The screen has a decreasing demand rate $D_p=300000p^{-1.25}$ units/year. The manufacturer has a holding cost h_v =\$0.25/unit/year, a production cost c_p =\$2.5/unit, a screening cost $c_{v,s}$ =\$0.1/unit, a warranty cost $c_{v,w}$ =\$11/unit, a set up cost S =\$1200/setup. The buyer has a holding cost h_b =\$0.86/unit/year, an ordering cost A=\$100/order, a screening cost $c_{b,s}$ =\$0.1/unit and a handling cost C_v =\$1/unit. Furthermore, the transportation cost is given by $g(\tau)=F_0\tau_0Q^{0.2}$ where F_0 =\$100/order, τ_0 =1week=1/52 year. The unit purchase price is given by $f(\gamma)=9-20\gamma$. The shortage cost is π=\$1.5/unit/year, and the rate of defect is given by $\gamma=0.02$. The ratios are r =0.8, r_b =0.3 and r_v =0.4.

5.1 Solution to the numerical instance

At first let us consider the case when the vendor inspects the shipment. Thus, the joint total profit function in Eq. (4) is considered to be maximized. The joint total profit $JTP_v(p,Q)$ for this example is represented in Figure 3. A sub-optimal solution $(p,Q)^*$ is obtained by *random walk with random restart* search approach. Then the corresponding b^* and n^* are yielded from the expressions listed in Section 4. Both the lower and upper integers of n^* are checked and the best solution is evaluated for the appropriate integer value of n [see Figure 4].

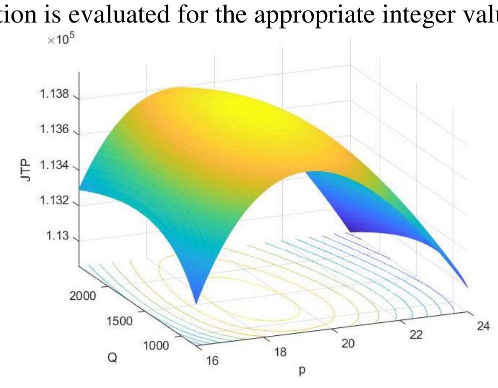

Figure 3. Representation of $JTP_v(p,Q)$ for the numerical instance.

In Figure 4, it is shown that $JTP^*_{v,1} < JTP^*_{v,2}$. As we are maximizing JTP^*_v , the best known solution is $Q^* = 1395,\ p^* = 17.73$, $b^* = 508$ and $n^* = 14$, which maximize the joint total profit JTP^*_v = \$113,865.94. Similarly, in Case 1, when quality inspection is done by the buyer, the best known solution is obtained as, $Q^* = 1259,\ p^* = 20.24$, $b^* = 339$ and $n^* = 16$, which yields JTP^*_b =\$112,169.28.

p^*	Q^*	b^*	n^*	$JTP_v(p,Q,n,b)^*$
17.71	1487	542	13.14	113866.02

$\lfloor n_{opt} \rfloor$

Q^*	p^*	b^*	$JTP^*_{v,1}(n=13)$
1503	17.70	548	113,865.45

$\lceil n_{opt} \rceil$

Q^*	p^*	b^*	$JTP^*_{v,2}(n=14)$
1395	17.73	508	113,865.94

Figure 4. Sub-optimal solutions for $JTP_v(p,Q,n,b)$.

5.2 Paired t-test

Since the resolution of the models has shown that the concavity cannot be easily proved in closed form, two search procedures have been employed within a certain feasible range. The first one is *random walk with random restart* and it is presented in the previous section. The second one is *goal seek*. This search procedure gives the best known input value that we need to have a specific result of a formula. This search procedure starts with $Q = 1$ and p is found by searching the best value to have the first partial derivative with respect to p of the expected total profit equal to zero. Then Q is found by searching the best value to have the first partial derivative with respect to Q of the expected total profit equal to zero. The two steps are repeated until the differences between the two last values of Q is less than ε . These two search procedures do not guarantee that a global optimal solution can be found. However, the comparison of the two methods will help to ensure the existence of a solution and help to find the best known optimal solution.

Table 1. Parameters for 20 examples for $JTP_v(p,Q,n,b)$.

N°	y	A	S	h_b	h_v	r	$f_{\max}$	x	δ	F_0	τ_0	β
1	0.02	100	1200	0.86	0.25	0.8	9	20	300000	100	1/52	0.1
2	0.02	100	1200	0.25	0.25	0.8	9	20	200000	100	1/52	0.2
3	0.02	100	1200	0.86	0.86	0.8	9	40	100000	100	2/52	0.5
4	0.02	200	1400	0.86	0.25	0.5	20	22	30000	100	2/52	0.7
5	0.02	200	1200	0.86	0.25	0.5	25	20	3000	100	2/52	0.95
6	0.2	100	1200	0.86	0.25	0.5	9	20	300000	100	2/52	0.1
7	0.2	100	1400	0.25	0.25	0.5	9	20	200000	100	2/52	0.2
8	0.2	100	1200	0.86	0.86	0.5	9	40	100000	100	2/52	0.5
9	0.2	200	1400	0.86	0.25	0.5	20	22	30000	100	2/52	0.7
10	0.2	200	1200	0.86	0.25	0.5	25	20	3000	100	2/52	0.95
11	0.3	100	1200	0.86	0.25	0.5	9	20	300000	100	2/52	0.1
12	0.2	100	1800	0.4	0.1	0.5	10	20	200000	200	2/52	0.2
13	0.3	100	1200	0.86	0.86	0.5	9	40	100000	100	2/52	0.5
14	0.3	200	1400	0.86	0.25	0.5	20	22	30000	100	2/52	0.7
15	0.3	200	1200	0.86	0.25	0.5	25	20	3000	100	2/52	0.95
16	0.35	200	1500	0.6	0.2	0.1	9	20	300000	300	1/52	0.1
17	0.45	100	1200	0.4	0.1	0.1	9	40	200000	100	1/52	0.2
18	0.45	100	1400	0.25	0.25	0.1	5	20	100000	200	2/52	0.5
19	0.35	200	1800	0.86	0.25	0.1	20	22	30000	100	2/52	0.7
20	0.45	300	1400	0.86	0.25	0.1	10	20	3000	100	1/52	0.95

Hence in this part 20 examples have been resolved with the two search procedures for the two cases. The parameters for these 20 example are listed in Table 1. The two set of solutions (*random walk with random restart* and *goal seek*) in each cases are seen as paired observations (total profit). A paired t-test have been realized and it proves that the two methods give equal solutions. This result convinces us that a local optimal solution exists and that the search procedures developed give a correct approximate solution.

These examples consider the case when the inspection occurs at the vendor. The parameters of the 20 examples and the result of the paired t-test are shown in Table 2. The parameters c_p , c_v , $c_{v,s}$, $c_{v,w}$ and π are not changed.

Table 2. Sub-optimal solutions for $JTP_v(p,Q,n,b)$ of the twenty examples.

	VENDOR with *random walk*					VENDOR with *goal seek*					Differences
N°	p^*	Q^*	b^*	n^*	JTP_v^*	p^*	Q^*	b^*	n^*	JTP_v^*	d
1	18,95	1404	1027	14	114290,38	17,73	1395	508	14	113866,94	423,44
2	19,11	1228	579	12	75943,91	17,51	1462	209	11	75785,93	157,98
3	20,42	830	387	7	36587,75	16,72	788	287	8	36187,06	400,69
4	23,64	1121	1064	3	10531,33	22,72	607	221	6	10281,18	250,15
5	46,51	221	221	3	848,22	41,64	121	44	6	802,39	45,83
6	2,29	1551	998	6	109398,07	21,6	1325	483	7	109058,14	339,93
7	22,39	2063	978	4	72488,75	21,6	1364	195	6	72365,44	123,31
8	24,42	939	783	3	34657,84	22,76	562	205	5	34427,27	230,57
9	27,41	912	865	3	10129,69	25,64	573	209	5	9903,33	226,36
10	52,37	184	184	3	819,54	43,9	119	43	5	773,23	46,31
11	24,72	1528	977	5	106569,18	23,88	1155	421	7	106239,14	330,04
12	25	1290	268	5	70540,68	23,76	1428	204	5	70477,26	63,42
13	27,17	859	513	3	33694,26	24,62	617	225	4	33510,51	183,75
14	30,13	821	786	3	9876,96	27,91	506	184	5	9659,82	217,14
15	56,56	166	166	3	802,39	45,5	109	40	5	755,31	47,08
16	30,69	1006	870	4	100696,76	30,25	1395	508	3	100399,46	297,3
17	31,15	1121	788	3	66558,67	30,75	1220	174	3	66455,67	103
18	35,63	573	395	2	31356,75	35,18	399	145	3	31221,17	135,58
19	38,79	608	545	2	9183,92	39,22	402	147	3	9028,26	155,66
20	65,5	136	136	2	747,99	63,56	127	46	2	712,47	35,52

Here, mean value of the differences $\bar{d} = 190.653$ and standard deviation of the differences $s_{\bar{d}} = 121.13$ results $t = 0.35$. The corresponding critical value is $t_{19,0.95} = 1.729$. Since $t < t_{19,0.95}$, we can not reject the null hypothesis which states that both of the two solution methods give the solution. Similarly, in Case 1, when quality inspection is done by the buyer, we obtained $\bar{d} = 402.683$, $s_{\bar{d}} = 1353.42$ and $t = 0.07$. The null hypothesis can not be rejected neither.

6. Conclusion

A joint economic lot size (JELS) problem with imperfect production, pricing decisions and planned shortages is presented in this paper, when the lead time and transportation cost are significantly correlated with the order size. The objective of the problem was to maximize the joint total profit, by optimally determining the order size, unit selling price, backorder quantity and number of shipments. The problem is mathematically formulated for two cases and solved heuristically. A closed form solution for order quantity and unit selling price is not on hand due to lack of global concavity. The solution method follows derivative approach for finding optimum number of shipments and backorder quantity. Whereas, the order quantity and unit selling price were determined by a search heuristic namely, *random walk with random restart.* An efficient heuristic development is the potential extension of this research. Depending on the instance, a case based concavity in a certain solution range can lead to the finding of a local optimum solution.

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