A Robust Optimization Approach for Supply Network Design under Uncertainties

Carles Sitompul, Yogi Yusuf Wibisono, and Reynold Andika
Department of Industrial Engineering
Universitas Katolik Parahyangan
Jl. Ciumbuleuit 94 Bandung 4014 Indonesia
carles@unpar.ac.id, yogi@unpar.ac.id, chevalier.treizieme.ciel@gmail.com

Abstract

In supply chain practices, designing supply network is a strategic decision that affects the survival of an organization. Although demand and supply uncertainties are well observed in supply chains, it has not been addressed pro-actively in the design model. Vast literatures on network design model had been dealing the problem using either deterministic (using mean value) or stochastic programming. We propose a mathematical model using robust approach where the variability of system performance is taken into account early in the modeling process. Using this robust optimization approach, the knowledge of demand and supply distributions is not critical. Furthermore, only a small number of scenarios are needed to solve the problem and yet it is able to maintain the adequacy of the model. The result from an illustrative example shows that total costs vary following a normal distribution where its variance depends on the risk parameters.

Keywords
Robust optimization, Network design, Uncertainties, Scenarios

1. Introduction
Nowadays, most companies believe that decision making based on the supply chain concept is necessary to ensure the survival of their organizations. In an emerging environment of demanding customers, companies must correctly respond to their needs in timely and swiftly manners. Consistencies of meeting demands at the right amount, at the right time and at the right qualities are becoming more important to the customer.

It is well observed in supply chain practices that uncertainties are present in many ways. Demand forecasts are subject to errors which may results in real demands higher or lower than their forecast average. In some countries, transportation lead times are very difficult to forecast due to heavy congestion, weather and accidents. If companies do not deal with these uncertainties appropriately, they are prone to poorer performance in particular the total cost of meeting demands. It is therefore necessary for companies to take these uncertainties into account when developing their strategic plans. According to Geary et al. (2002), uncertainties in supply chains can be categorized into process, supply, demand, and control uncertainties. Gupta and Maranas (2003) stated that those uncertainties needs to be tackled with supply chain planning because decisions made in supply chains are very critical to the survival of the company.

Mulvey et al. (1995) introduced the robust optimization approach when dealing with uncertainties. Their approach is proactively seeking the "best" solution in term of robustness which is insensitive to uncertainties. This robust optimization concept has been widely applied in many areas of decision making including in supply chains (e.g. Ben Tal, 2005, Bertsimas and Thiele, 2004). To the best of our knowledge, however, these methods have not been implemented for supply network design. The problem has been solved by using stochastic programming (Nagar and Jain, 2008, Santoso et al, 2005, Kallrath, 2005). We believe that solving the supply network design problem with robust optimization approach will bring about some advantages, for example there is no need for the assumption of parameter distribution.

In this paper, we propose a robust optimization approach for supply network design under demand uncertainty. Although uncertainties from customer demands have been widely investigated (e.g. Ben Tal, 2005), it is still relevant in today's changing environment. It will be shown that even with a small number scenarios, the model gives a rather good solution in term or robustness.

2. Problem Description

A network can be described as a graph G (V,A), where V is a set of vertices and A is a set of arcs connecting vertices in the graph. Set V consists of three sets, i.e. I (set of suppliers), J (set of facilites) and K (set of customers) as shown in Figure 1. Variable y_j equals to 1 if facility j is opened and 0 otherwise. The cost associated with building facility j is f_j. A quantity of x_{jk} shows the amount transported from node j to node k with costsq_{jk}.

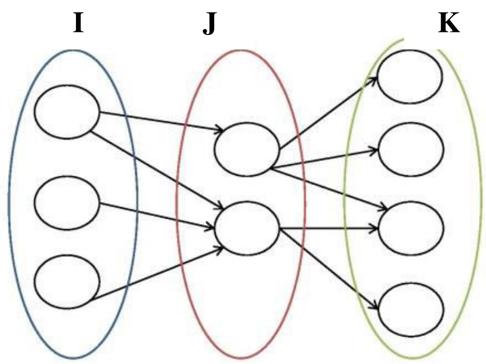

Figure1. A Network Graph

The deterministic network design problem as modified from Santoso et al. (2005) can be formulated as follows:
Minimize

$$\sum_{j\in J} f_j y_j + \sum_{(j,k)\in A} q_{jk}x_{jk} \tag{1}$$

Subject to:

$$\sum_{j\in J} x_{jk} \geq d_k, \forall k \in K \tag{2}$$

$$\sum_{j\in J} x_{ij} \leq s_i, \forall\ i \in I \tag{3}$$

$$\sum_{i\in I} x_{ij} - \sum_{k\in J} x_{jk} = 0, \forall j \in J \tag{4}$$

$$\sum_{k\in K} x_{jk}\ \leq My_j, \forall j \in J \tag{5}$$

$$y_j \in \{0,1\}, \forall j \in J$$

$$x_{jk} \geq 0, \forall j \in J, k \in K$$

Equation (1) shows the objection function that is minimizing the sum of fixed costs of building facilities and transportation costs. Constraint (2) makes sure that the total inflow to customer j be greater or equal than its demand d_j. In constraint (3), the total outflow from supplier i must not be greater than its supply s_i. Constraint (4) requires that the total inflow equals the total outflow for each facilities in J. Constraint (5) makes sure that fixed building costs of a facility are realized whenever there is an outflow of material from the facility.

First, we model supply uncertainties at a facility $j \in J$ as given in Figure 2. Let us assume that supplies from $i \in I$ during a period of time are random variables having normal distributions with parameters $(\mu S_i, \sigma S_i^2)$, then the resulting supplies at facility j during a period of time are also following normal distributions with parameters $(\mu S_j, \sigma S_j^2)$, where: $\mu S_j = \sum_{i \in I} \mu S_i$ and $\sigma S_j^2 = \sum_{i \in I} \sigma S_i^2$.

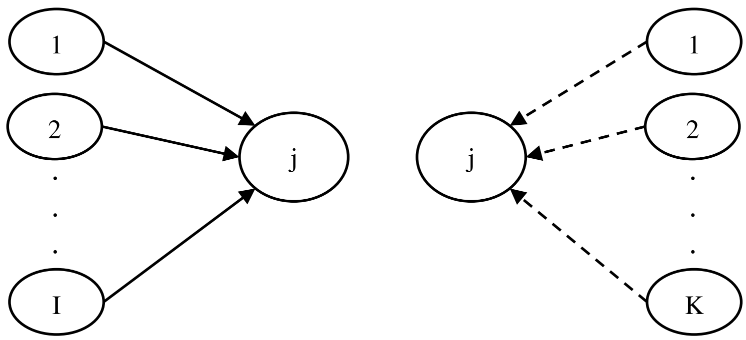

Figure 2. Supply and Demand Models

Consequently, we can also model the demand uncertainties at facility j as given in Figure 2. Assuming that demands at customer $k \in K$ are following normal distributions with parameters $(\mu D_k, \sigma D_k^2)$, then the resulting demands experienced by facility $j \in F$ also follow normal distributions with parameters $(\mu D_j, \sigma D_j^2)$, where: $\mu_j = \sum_{k \in K} \mu D_k$ and $\sigma D_j^2 = \sum_{k \in K} \sigma D_k^2$. Now, if there are products transported from facility j to customer k, then demands accounted at facility j are only those coming from k where $x_{jk} > 0$. The same reasoning applied to products that are transported to facility j are coming only from s where $x_{ij} > 0$. For example, the average demand at facility j can then be formulated follows:

$$\mu_j = \sum_{\forall k \backslash x_{jk} > 0} \mu D_k$$

The above equation is however still difficult to reformulate into a linear programming. To solve such problem, we use a robust optimization approach using set of scenarios denoted by $s \in S$. The set of scenarios contains both supply scenarios and demand scenarios. Suppose that demands in node $k \in K$ is realized as a scenario d_{ks} then we can formulate a balance constraint as follows:

$$\sum_{j \in F} x_{jk} + i_{ks}^- - i_{ks}^+ = d_{ks}, \forall k \in K, s \in \mathrm{S} \tag{6}$$

where i_{ks}^- and i_{ks}^+ can be seen as unmet demands and excess inventory respectively which in turn can be penalized. Consequently, supposed that supply to facility $j \in J$ is also realized as a scenario p_{js} then we can formulate a balance constraint at facility j as follows:

$$p_{js} + u_{js}^- - u_{js}^+ = \sum_{k \in C} x_{jk}, \forall j \in F, s \in \mathrm{S} \tag{7}$$

where u_{js}^- and u_{js}^+ can be seen as lack of supplies and excess inventory at facility j which can be penalized too.

Bear in mind that, controlled variables in network designs in this case are y_j, $\forall i \in I, j \in J$ and x_{jk} $\forall j \in J, k \in K$. It is also practical to use holding costs to penalize excess inventory both at facility j and customers (retailers) k denoted as h_j and h_k. Costs of lack of supplies at facility j and unmet demands at customer k, however, are very difficult to quantify. Therefore, we introduce a parameter ω as a penalty to the objective functions. The robust optimization model is basically a multi objective programming, where we attempt to minimize the average costs, standard deviation costs, and penalties.

Under scenario *s*, the total costs are formulated as:

$$\xi_s = \sum_{j \in F} f_j y_j + \sum_j \sum_k q_{jk} x_{jk} + \sum_j h_j u_{js}^+ + \sum_k h_k i_{ks}^+$$

The resulting robust optimization model can be formulated as follows:

Minimize

$$z = \bar{\xi} + \lambda \left(\sum_{s' \in \Omega} \pi_{s'} (\xi_{s'} - \bar{\xi})^2 \right) + \omega \left(\sum_s \left(\sum_j u_{js}^- + \sum_k i^\wedge ks^- \right) \right) \tag{8}$$

Subject to

$$\xi_s = \sum_j f_j y_j + \sum_j \sum_k q_{jk} x_{jk} + \sum_j h_j u_{js}^+ + \sum_k h_k i_{ks}^+ \tag{9}$$

$$\bar{\xi} = \sum_s \pi_s \xi_s \tag{10}$$

$$\sum_{j \in F} x_{jk} + i_{ks}^- - i_{ks}^+ = d_{ks}, \forall k \in K, \forall s \in S \tag{11}$$

$$p_{js} + u_{js}^- - u_{js}^+ = \sum_{k \in K} x_{jk}, \forall j \in J, \forall s \in S \tag{12}$$

$$\sum_{k \in K} x_{jk} \leq M y_j, \forall j \in F \tag{13}$$

$$y_j \in \{0,1\}, \forall j \in J$$

$$x_{jk} \geq 0, \forall j, k$$

where:
Parameters and indices:

i	: Suppliers, $i \in I$
j	: Facilities, $j \in J$
k	: Customers, $k \in K$
s	: Scenarios, $s \in$ S
f_j	: Fixed costs of opening facility j
q_{jk}	: Variable costs of transportation from supplier i to facility j or from facility j to customer k
h_j	: Holding costs at facility or customer j
d_{ks}	: Demands of customer k under scenario s realizations
p_{js}	: Supplies to facility j under scenario s realizations

Decision Variables:

y_j	: Binary variables which equal to 1 if facility j is opened
x_{jk}	: Quantity transported from facility j to customer k

i_{ks}^{-} : Unmet demands at customer k under scenario s realizations
i_{ks}^{+} : Excess inventory at customer k under scenario s realizations
u_{js}^{-} : Lack of supplies at facility j under scenario s realizations
u_{js}^{+} : Excess inventory at facility j under scenario s realizations

Equation (8) is the objective function minimizing total costs which consist of average costs, mean absolute deviation of costs, and mean unmet demands or lack of supplies. Constraints (9) compute the scenario costs which include the fixed and variable costs and holding costs while average cost is computed by Equation (10). In Equation (11), inventory balance constraint is shown at the customer, while balance constraint in the facility is shown in Equation (12). Equation (13) makes sure that fixed costs are realized whenever there are products transported from facility j.

Bear in mind that for simplicity, this formulation consists of only decision variables from facilities to customers, where supplies are considered as given even though they are limited and uncertain. The model can naturally be extended to include decision variables from supplier to facilities.

The realization of scenarios can be modeled as a triangular-like distribution where normal distribution is discretized using three numbers as shown in Figure 3. The scenarios are represented by $\mu - \sigma, \mu, \mu + \sigma$ each with probabilities 0.16, 0.68, and 0.16 respectively.

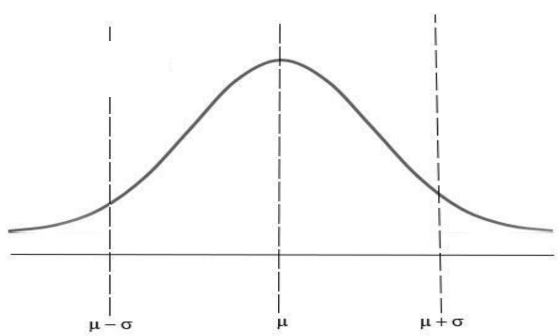

Figure 3. Scenario realizations

To linearize the problem, we use mean absolute deviation instead of standard deviation of costs. We introduce variables $\theta_s, s \in S$ to calculate the absolute deviation, i.e.

$$AD_s = \xi_s - \bar{\xi} + 2\theta$$

given that

$$\xi_s - \bar{\xi} + \theta \geq 0 \qquad (14)$$

Constraint 14 is added to the above formulation to make sure that absolute deviations are correctly computed.

3. An illustrative example

As an illustrative example, we use the following problem where a network consists of 2 potential facilities and 2 customers where the parameters are given in Table 1.

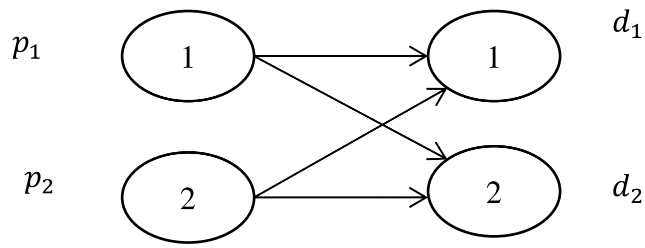

Figure 4. An illustrative example

If each facility's supply and customer's demand is realized using 3 number of scenarios then the full combination of scenarios will be $3^4 = 81$ scenarios.

Table 1 Illustrative parameters

	Facility 1	Facility 2	Customer 1	Customer 2
Average Demand			50	50
Demand Standard Deviation			10	10
Average Supply	100	100		
Supply Standar Deviation	20	40		
Holding costs	1	1	1	1
Fixed costs	100	100		

As an illustrative, we use $\lambda = 1$ and $\omega = 10$ to show the cost scenario realizations as depicted in Figure 5. It is shown that the costs are normally distributed. The problem is modeled in AMPL (A mathematical Programming Language) and solved using CPLEX available at Neos Server (https://neos-server.org/).

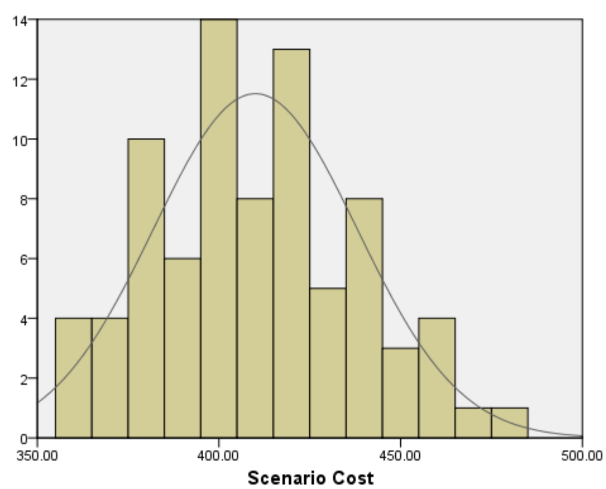

Figure 5. Scenario Cost Histogram

The results are naturally depending on parameters λ and ω which show the decision maker's attitude toward risks. The effect of parameter ω which is used to penalize unmet demands or supplies can be seen in Table 2.

Table 2. The Effect of Unmet Demands and Supplies Penalties

$\lambda = 1$	$\omega = 5$	$\omega = 10$	$\omega = 50$	$\omega = 100$
Total cost	463	511	591	591
Average Cost	399	399	570	570

$\lambda = 1$	$\omega = 5$	$\omega = 10$	$\omega = 50$	$\omega = 100$
Mean Cost Deviation	16.3	16	21.25	21.25
Mean Unmet Demand/Supplies (unit)	9.6	9.6	0	0

The effect of parameter λ which is used to penalize mean deviation of costs can be seen in Table 3.

Table 3. The Effect of Mean Deviation Penalties

$\omega = 10$	$\lambda = 1$	$\lambda = 10$	$\lambda = 100$	$\lambda = 200$
Total cost	511	659	1020	1064
Average Cost	399	399	410	390
Mean Cost Deviation	16	16	1.1	0.3
Mean Unmet Demand/Supplies (unit)	9.6	9.6	50	60

4. Conclusion and further research

In this paper, we develop a robust optimization model to network designs taking into account both supply and demand uncertainties. As expected, the resulting solutions are depending on how decision makers view risks and their implications both to costs realization and unmet demands or supplies. The total costs vary following a normal distribution where its variance depends on the risk parameters. Using the same λ parameter, the mean cost deviation is larger with larger ω while the unmet demands and supplies is getting smaller. Conversely, the results also show that when using the same ω, the mean cost deviation is smaller with larger λ. The mean unmet demands/suppliers, however, is larger with larger λ.

In the future, we still have to investigate the model and its solution approach, in particular the effect of discretization of demand and supply distributions. Furthermore, computational complexities of the problems still need to be investigated.

Acknowledgement

Part of this research is funded by research grant from the Directorate of Research and Community Services of the Ministry of Research, Technology and Higher Education Republic of Indonesia.

References

Ben Tal, A., Golany, B., Nemirovski, A. Vial,J.,-P., Retailer-Supplier Flexible Commitments Contracts: A Robust Optimization Approach, *Manufacturing & Service Operation Management* 7, 248—271, 2005.

Bertsimas, D. & Thiele, A., A Robust Optimization Approach to Supply Chain Management, *Integer Programming and Combinatorial Optimization*, 145 – 156, 2004.

Chopra, S. & Meindl, P., *Supply Chain Management: Strategy, Planning, and Operation*, 3rd Edition, Pearson Prentice Hall, New Jersey, 2007.

Geary, S., Childerhouse, P. & Towill, D., Uncertainty and The Seamless Supply Chain. *Supply Chain Management Review* 6, 52-60, 2002.

Gupta, A. & Maranas, C.D., Managing Demand Uncertainty In Supply Chain Planning. *Computer & Chemical Engineering*, 27, 1219 – 1227, 2003.

Kallrath, J., Solving Planning and Design Problems in the Process Industry Using Mixed Integer and Global Optimization, *Annals of Operation Research* 140, 339 – 373, 2005.

Mulvey,J.M., Vanderbei,R.J. & Zenios, S.A., Robust Optimization of Large Scale Systems. *Operation Research* 43, 264-281, 1995.

Nagar, L., & Jain, K. Supply Chain Planning Using Multi-Stage Stochastic Programming. *Supply Chain Management: An International Journal* 13 (3), 251 – 256, 2008.

Santoso, S.A.T, Goetschalckx, M., & Shapiro, A., A Stochastic Programming Approach for Supply Chain Network Design Under Uncertainty. *European Journal of Operational Research* 167, 96-115, 2005.

Biographies

Carles Sitompul is a lecturer, and head of the Department of Industrial Engineering Parahyangan Catholic University Bandung, Indonesia. He earned his Bachelor degree from Bandung Institute of Technology, Master of Engineering from the same institute and the Katholike Universiteit Leuven Belgium. He later finished his doctoral study at the Ghent University Belgium. His research interest includes supply chain, manufacturing, optimization, and production system.

Yogi Yusuf Wibisono earned his Bachelor and Master degree from Bandung Insitute of Technology. He currently is doing his Ph.D. at the same insititute.

Reynold Andika earned his Bachelor degree from Parahyangan Catholic University, Bandung.