Performance of Phase II Approaches for Monitoring Multivariate Multiple Linear Profiles based on Estimated Parameters

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Abstract
Profile monitoring is usually performed by establishing control charts. In most of the cases, the in-control values of the profile parameters are assumed to be known in Phase II, whereas it is not valid in many practical situations. In this article, we investigate the effect of parameters estimation from in-control Phase I samples on the in-control and out-of-control performance of two Phase II control charts for monitoring multivariate multiple linear profiles designated as MEWMA and MEWMA/χ^2. The out-of-control performance of the methods is evaluated by using corrected limits to consider the variability due to parameters estimation. The performance of the monitoring approaches is compared in terms of statistical properties of *ARL* distribution including *AARL*, *SDARL* and *CVARL* in order to consider practitioner-to-practitioner variability trough a Monte Carlo simulation algorithm named as ARLS. The results showed that parameters estimation severely effects on the performance of the monitoring schemes.

Keywords
Estimation effect; multivariate multiple linear profile; Phase II; profile monitoring; Statistical process control

1. Introduction
In some applications, the quality of a product or process needs to be explained through a relationship between a response variable and one or several explanatory variables, which is named as "*profile*". Profiles are categorized based on the structure of this relationship, i.e. simple linear, multivariate linear, multiple linear and non-linear profiles which are more complicated.

Profile monitoring is commonly performed in two phases: Phase I and Phase II. The methods of profile monitoring in Phase I and Phase II are usually different. In Phase I, the main goals are monitoring the process stability over time and estimating the process parameters through in-control data set; whereas in Phase II, we are intended to detect the shifts in parameters, quickly. Furthermore, different metrics are implemented for evaluating the in-control and out-of-control performance of the profile monitoring approaches in Phase I and Phase II including probability of an out-of control signal and Average run length (ARL). Many researchers have studied on the simple linear profile monitoring in Phase II, i.e. Kang and Albin (2000); Kim et al. (2003); Mahmoud et al. (2010); Saghaei et al. (2009); Zhang et al. (2009). Some simple linear profile monitoring approaches in Phase I were proposed by researchers such as Kang (2000); Mahmoud, Parker, Woodall, and Hawkins (2007); Mahmoud and Woodall (2004). Moreover, there are several approaches for monitoring non-linear profiles developed by Ding et al. (2006); Jensen and Birch (2009); Jin and Shi (1999); Steiner et al. (2016); Vaghefi et al. (2009); Walker (2002).

In multivariate multiple linear (MML) profiles, which is the main focus of the current study, several correlated response variables follow a linear regression relationship with several explanatory variables. Noorossana et al. (2010) proposed four different methods including Likelihood Ratio Test (LRT), T^2, Wilk's lambda and principal components analysis (PCA) for monitoring this type of profiles in Phase I. Four methods were also developed by Eyvazian et al. (2011) for monitoring MML profiles in Phase II. The results of their research will be used in the current study.

Based on the literature, it is usually assumed that the process parameters are known in Phase II. However, in many practical applications, the process parameters are rarely known and should be estimated based on an in-control data set collected in Phase I. A variety of studies has been performed to survey the parameters estimation effect on the performance of monitoring schemes for non-profile characteristics, i.e. Chakraborti (2000), Jones et al. (2004), Jones et al. (2001), Zwetsloot and Woodall (2017), Castagliola et al. (2016), Khoo (2005), Saleh et al. (2015).

When it is supposed that the process parameters are known, the *ARL* metric would be a parameter. However, violating this assumption leads to changing *ARL* from a parameter to a random variable that follows a right-skewed distribution (Jensen et al. 2006). In the related studies in the context of parameters estimation effect, the *ARL* metric and its statistical distribution properties including average of *ARL* (*AARL*), standard deviation of *ARL* (*SDARL*) and coefficient of variation of *ARL* (*CVARL*) are usually used for investigating the effect of parameter estimation.

There are little work on the context of parameters estimation effect for the profile characteristics. Woodall and Montgomery (2014) stated that "*There is also work needed on the effect of parameter estimation error on the Phase II performance of profile monitoring methods.*"

For the first time, Mahmoud (2012) studied the parameters estimation effect on three well-known methods of simple linear profile monitoring under in-control and out-of-control conditions in terms of *ARL* and standard deviation of run length (*SDRL*) metrics. The results of his study showed that the parameters estimation significantly affects both in-control and out-of-control performance of the monitoring approaches. He also applied corrected limits to evaluate the out-of-control performance of the control charts. Mahmoud et al. (2010) used corrected limits for evaluating the out-of-control performance of MEWMA control charts when parameters are estimated. For more information about corrected limits and its applications, see Champ et al. (2005), Jones (2002) and Quesenberry (1993).

Sampling by different practitioners in Phase I may cause different estimations of the process parameters that will be used in Phase II. Consequently, a new source of variation is added to the process, called *practitioner-to-practitioner* variability that can be measured by the statistical properties of *ARL* distribution such as: average of *ARL* (*AARL*) or standard deviation of *ARL* (*SDARL*). Aya et al. (2015) surveyed the parameters estimation effect on the in-control performance of three simple linear profile monitoring schemes proposed by Kang and Albin (2000), Kim et al. (2003) and Mahmoud et al. (2010) in terms of *AARL* and *SDARL* metrics. Aya et al. (2016) used *CVARL* metric to compare the performance of adaptive MEWMA control chart under in-control and out-of-control conditions. This metric can be used to compare the performance of monitoring approaches based on *AARL* and *SDARL* metric, simultaneously.

To the best of our knowledge, there is no study in the literature, which surveyed the parameters estimation effect on the performance of the MML profile monitoring approaches. The current study investigates the effect of parameters estimation on the in-control and out-of-control performance of the two Phase II monitoring approaches of MML profiles proposed by Eyvazian et al. (2011) using a new Monte Carlo simulation algorithm. In addition, the out-of-control performance of mentioned methods are evaluated using simulated corrected limits. Three metrics including *AARL*, *SDARL*, and *CVARL* are also applied to measure the estimation effect in order to consider practitioner-to-practitioner variability and compare the performance of competing approaches.

The structure of this paper is organized as follows. In Section 2, the model of MML profile and its assumptions are discussed. Two control chart schemes for monitoring MML profiles based on known parameters developed by Eyvazian et al. (2011) are presented briefly in Section 3. We proposed a Monte Carlo simulation algorithm for measuring the effect of parameters estimation on the performance of the MML profile monitoring approaches in Section 4. Also, the in-control and out-of-control performance of the monitoring approaches are investigated in terms of three metrics including *AARL*, *SDARL* and *CVARL* in Section 5 and Section 6, respectively. The conclusions and future study suggestions are given in the last section.

2. Multivariate multiple linear profile regression model

Supposed that n observations $(x_{1i}, x_{2i}, \ldots, x_{qi}, y_{1ik}, y_{2ik}, \ldots, y_{pik})$, $i = 1, 2, \ldots, n$ are available at kth sample over time where p and q denote the number of response and explanatory variables, respectively. Under in-control conditions, the relationship between response and explanatory variables can be illustrated by following multivariate multiple linear regression model:

$$\mathbf{Y}_k = \mathbf{XB} + \mathbf{E}_k, \tag{1}$$

or

$$\begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1p} \\ y_{11} & y_{22} & \cdots & y_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{np} \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & \cdots & x_{1q} \\ 1 & x_{21} & \cdots & x_{2q} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \cdots & x_{nq} \end{bmatrix} \begin{bmatrix} \beta_{01} & \beta_{02} & \cdots & \beta_{0p} \\ \beta_{11} & \beta_{12} & \cdots & \beta_{1p} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{q1} & \beta_{q2} & \cdots & \beta_{qp} \end{bmatrix} + \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \cdots & \varepsilon_{1p} \\ \varepsilon_{21} & \varepsilon_{22} & \cdots & \varepsilon_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{n1} & \varepsilon_{n2} & \cdots & \varepsilon_{np} \end{bmatrix}, \tag{2}$$

where $\mathbf{Y}_k$ is an $n \times p$ matrix of response variables at sample k, $\mathbf{X}$ is an $n \times (q+1)$ matrix of explanatory variables, $\mathbf{B}$ is an $(q+1) \times p$ matrix of the profile parameters including all MML profile intercepts and slopes and $\mathbf{E}_k$ is an $n \times p$ matrix of error terms. It is assumed that the error vector follows a p-variate normal distribution with mean vector zero and covariance matrix $\mathbf{\Sigma}$ Eyvazian et al. (2011). The matrix $\mathbf{B}$ for kth sample can be estimated by Ordinary Least Squares (OLS) approach as follows (see Rencher (2003) for more information):

$$\hat{\mathbf{B}}_k = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{Y}_k. \tag{3}$$

3. Multivariate multiple linear profile monitoring approaches in Phase II

Eyvazian et al. (2011) have developed four Phase II control chart schemes for monitoring MML profiles based on known parameters. In the current study, we investigate the parameters estimation effect on the in-control and out-of-control performance of the proposed monitoring methods by Eyvazian et al. (2011).

3.1. MEWMA control chart

In this method matrix $\hat{\mathbf{B}}_k$ is rewritten as a $((q+1)p) \times 1$ multivariate random normal vector denoted by $\hat{\boldsymbol{\beta}}_k$ as follows (Eyvazian et al., 2011):

$$\hat{\boldsymbol{\beta}}_k = (\hat{\beta}_{01k}, \hat{\beta}_{11k}, \ldots, \hat{\beta}_{q1k}, \hat{\beta}_{02k}, \hat{\beta}_{12k}, \ldots, \hat{\beta}_{q2k}, \ldots, \hat{\beta}_{0pk}, \hat{\beta}_{1pk}, \hat{\beta}_{01k}, \ldots, \hat{\beta}_{qpk})^{\mathrm{T}}. \tag{4}$$

When the process is in statistical control, the expected value and the covariance matrix of $\hat{\boldsymbol{\beta}}_k$ are:

$$\mathrm{E}(\hat{\boldsymbol{\beta}}_k) = (\beta_{01}, \beta_{11}, \ldots, \beta_{q1}, \beta_{02}, \beta_{12}, \ldots, \beta_{q2}, \ldots, \beta_{0p}, \beta_{1p}, \ldots, \beta_{qp})^{\mathrm{T}}, \tag{5}$$

$$\mathbf{\Sigma}_{\hat{\boldsymbol{\beta}}_k} = \begin{bmatrix} \mathbf{\Sigma}_{11} & \mathbf{\Sigma}_{12} & \cdots & \mathbf{\Sigma}_{1p} \\ \mathbf{\Sigma}_{21} & \mathbf{\Sigma}_{22} & \cdots & \mathbf{\Sigma}_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Sigma}_{p1} & \mathbf{\Sigma}_{p2} & \cdots & \mathbf{\Sigma}_{pp} \end{bmatrix}. \tag{6}$$

The elements of matrix $\mathbf{\Sigma}_{\hat{\boldsymbol{\beta}}_k}$ has been given by Eyvazian et al. (2011). The MEWMA control chart proposed by Lowry et al. (1992) is used for monitoring MML profile parameters. The statistic of this chart is as follows:

$$T^2_{\mathbf{z}_k} = \mathbf{z}_k^{\mathrm{T}}\mathbf{\Sigma}_{\mathbf{z}}^{-1}\mathbf{z}_k, \tag{7}$$

where

$$\mathbf{z}_k = \lambda(\hat{\boldsymbol{\beta}}_k - \boldsymbol{\beta}) + (1-\lambda)\mathbf{z}_{(k-1)}, \tag{8}$$

$$\boldsymbol{\Sigma}_{\mathbf{z}} = \frac{\lambda}{2-\lambda}\boldsymbol{\Sigma}_{\hat{\boldsymbol{\beta}}_k}. \tag{9}$$

The smoothing parameter (λ) can take values between 0 and 1, and $\mathbf{z}_0$ is a $((q+1)p)\times 1$ zero vector. The upper control limit of this control chart can be obtained in a way to achieve a specific in-control *ARL* by simulation runs.

3.2. MEWMA/χ^2 control chart

This method is an extension of the MEWMA/χ^2 method proposed by Noorossana et al. (2010) for monitoring MSL profiles. Supposed that a data set is available at the *k*th random sample. Let define $\bar{\mathbf{e}}_k = (\bar{e}_{1k}, \bar{e}_{2k}, \dots, \bar{e}_{pk})$ as the $1\times p$ vector of average errors, where $\bar{e}_{jk} = n^{-1}\sum_{i=1}^{n} e_{ijk}$ for $j = 1, 2, \dots, p$. It should be noted that $\bar{\mathbf{e}}_k$ follows a *p*-variate normal distribution with mean vector zero and known variance-covariance matrix $\boldsymbol{\Sigma}_{\bar{\mathbf{e}}} = n^{-1}\boldsymbol{\Sigma}$. For the *k*th sample, the MEWMA statistic of this method is given by Eyvazian et al. (2011):

$$\mathbf{z}_k = \lambda\bar{\mathbf{e}}_k^{\mathrm{T}} + (1-\lambda)\mathbf{z}_{k-1}, \tag{15}$$

$$\boldsymbol{\Sigma}_{\mathbf{z}} = \frac{\lambda}{n(2-\lambda)}\boldsymbol{\Sigma}, \tag{16}$$

where $\mathbf{z}_0$ is a $1\times p$ vector of zeros. The upper control limit of MEWMA control chart is obtained such that a specific value of *ARL* is achieved. Eyvazian et al. (2011) have extended the χ^2 statistic proposed by Noorossana et al. (2010) for monitoring the process variability using following statistic:

$$\chi_k^2 = \sum_{i=1}^{n} \mathbf{e}_{ik}\boldsymbol{\Sigma}^{-1}\mathbf{e}_{ik}^{\mathrm{T}}, \tag{17}$$

where $\mathbf{e}_{ik}$ is a random vector which follows a *p*-variate normal distribution with mean vector zero and covariance matrix $\boldsymbol{\Sigma}$. Under in-control conditions, χ_k^2 is a chi-square random variable with *np* degrees of freedom. Therefore, the upper control limit of this statistic can be obtained by $\chi^2_{np,\alpha}$.

4. Parameters estimation effect on the in-control performance of Phase II MML profile monitoring approaches

As mentioned before, the profile parameters matrix (**B**) is usually assumed to be known in Phase II. However, we tend to evaluate the effect of violating this assumption on the performance of the monitoring approaches. Hence, in this study, it is assumed that matrix **B** is unknown and is estimated using an in-control Phase I data set. In this section, the in-control performance of four methods for monitoring MML profiles are evaluated when profile parameters are assumed to be unknown using a Monte Carlo simulation algorithm. It should be noted that we apply the multivariate profile model applied by Eyvazian et al. (2011) in order to perform a better comparison of the monitoring approaches performance, which is given by:

$$\begin{cases} Y_1 = 3 + 2X_1 + X_2 + \varepsilon_1 \\ Y_2 = 2 + X_1 + X_2 + \varepsilon_2 \end{cases}. \tag{18}$$

Four paired observations are considered for the explanatory variables (X_1, X_2) as $(2,1), (4,2), (6,3), (8,2)$, which are fixed in all sampling points. In addition, vector $(\varepsilon_1, \varepsilon_2)$ follows a bivariate normal distribution with mean vector $\underline{0}$ and known covariance matrix of $\boldsymbol{\Sigma} = \begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}$. The steps of the proposed simulation algorithm named as *ARL* simulation (ARLS) are given as follows:

1- Specify the upper control limits for each control chart to achieve $ARL_0 = 200$ by 10000 simulation runs based on known $\mathbf{B}$ and $\mathbf{\Sigma}$. Table 1 illustrates the upper control limits of the two mentioned MML profile monitoring methods based on $\lambda = 0.2, 0.1, 0.05$. Point to notice is that the upper control limits of the mentioned methods are calculated by Eyvazian et al. (2011) only for $\lambda = 0.2$. Using different values of λ helps practitioners to investigate the effect of λ on the control limits values. Based on Table 1, increasing λ leads to achieve wider control limits. Not that, as the χ^2 statistic does not depend on λ value, the upper control limit of this control chart is unchanged based on different values of λ.

Table 1. The in-control upper control limits of the monitoring approaches based on the known parameters

λ	Method		
	MEWMA	MEWMA/χ^2	
0.05	14.6	MEWMA	9.04
		χ^2	23.77
0.1	16.3	MEWMA	10.22
		χ^2	23.77
0.2	17.55	MEWMA	11.1
		χ^2	23.77

2- Create *m* in-control MML profiles applying a multivariate normal distribution with mean vector zero and known covariance matrix $\mathbf{\Sigma}$.
3- Quantify $\hat{\mathbf{B}}_j$ for each generated in-control profile ($j = 1, 2, ..., m$) using Eq. 3, then calculate $\bar{\hat{\mathbf{B}}}$ by $\bar{\hat{\mathbf{B}}} = \frac{\sum_{j=1}^{m} \hat{\mathbf{B}}_j}{m}$.
4- Generate an MML in-control profile using known parameters and estimate the matrix ($\mathbf{B}$) using Eq. 3. Then, calculate the chart statistic substituting $\bar{\hat{\mathbf{B}}}$ for the known matrix ($\mathbf{B}$) and put *RL*=1.
5- If the value of calculated statistic is more than UCL, go to step 6; otherwise, put RL=RL+1 and go to step 4.
6- Record RL values and go back to step 4.
7- Repeat steps 4-6, 5,000 times and calculate the ARL by averaging available *RL* values. Then go back to step 2.
8- Repeat steps 2-6, 5,000 times to achieve 5,000 different *ARL* values and calculate the *AARL*, *SDARL* and *CVARL*.

Table 2 shows simulated in-control *AARL*, *SDARL* and *CVARL* values for each monitoring approaches by ARLS simulation algorithm based on different values of λ and *m*. It is observed that the in-control *AARL* increases by increasing *m* and approaches to the desired value of *ARL*=200, because increasing *m* leads to more accurate estimation of the matrix $\mathbf{B}$. In addition, by increasing λ and *m*, the *AARL* value generally increases in all the methods and consequently, the number of false alarms decreases.

Table 2. The in-control *AARL*, *SDARL* and *CVARL* comparisons of the monitoring approaches when *m* Phase I samples are used for parameters estimation

λ =0.2						
m	*AARL*		*SDARL*		*CVARL* (%)	
	MEWMA	MEWMA/χ^2	MEWMA	MEWMA/χ^2	MEWMA	MEWMA/χ^2

30	80.30	125.30	33.81	42.02	42.11	33.54
100	133.27	170.12	27.06	23.59	20.30	13.87
200	163.25	180.75	19.00	15.31	11.63	8.47
500	184.96	193.74	10.11	8.64	5.40	4.46
1000	193.37	197.87	7.25	6.32	3.70	3.20
2000	198.48	199.43	4.91	4.72	2.47	2.37
3000	199.98	200.96	3.64	4.34	1.82	2.16
λ =0.1						
m	*AARL*		*SDARL*		*CVARL* (%)	
	MEWMA	MEWMA/χ^2	MEWMA	MEWMA/χ^2	MEWMA	MEWMA/χ^2
30	58.50	105.75	35.37	44.76	60.46	42.33
100	117.37	152.80	30.62	34.11	26.09	22.32
200	145.23	176.45	21.99	23.69	15.14	13.42
500	173.81	190.28	13.32	13.00	7.66	6.83
1000	186.80	196.34	9.06	7.45	4.85	3.79
2000	194.23	201.00	5.92	5.48	3.05	2.73
3000	196.99	200.97	5.37	5.66	2.73	2.82
λ =0.05						
m	*AARL*		*SDARL*		*CVARL* (%)	
	MEWMA	MEWMA/χ^2	MEWMA	MEWMA/χ^2	MEWMA	MEWMA/χ^2
30	50.47	96.28	31.58	42.16	62.57	43.79
100	98.05	143.55	27.91	35.37	28.47	24.64
200	129.97	167.90	25.44	25.63	19.57	15.26
500	162.90	186.01	16.10	17.60	9.88	9.46
1000	180.05	195.40	11.46	10.64	6.37	5.44
2000	189.85	201.34	6.80	6.07	3.58	3.01
3000	193.35	202.55	5.21	5.60	2.69	2.77

According to Table 2 illustrates the outperformance of MEWMA/χ^2method in terms of *AARL* metric for all the values of λ rather than MEWMA method. It shows that implementing of combined control chart schemes can improve the performance of monitoring methods when parameters are estimated.

The results shows the trend of in-control *SDARL* for each monitoring approach based on different values of *m*. It is usually suggested by researchers to consider in-control *SDARL* within 5%-10% of desired in-control *ARL* value. Based on the obtained results, in terms of SDARL, MEWMA method performs better than MEWMA/χ^2 method in small shifts. By increasing *m*, the performance of MEWMA/χ^2 method improves both methods perform a similar performance.

Choosing the superior method based on both *AARL* and *SDARL* is more complicated because the method that has a better performance in terms of *AARL* may not perform the same way in terms of *SDARL*. Hence, it is suggested to apply the *CVARL* metric to facilitate the evaluation of the methods performance and choosing the superior one. It is obvious that smaller values of *CVARL* shows the better performance of a monitoring scheme. Based on the obtained results, the outperformance of MEWMA/χ^2 method for all the values of *m* and λ is inevitable.

5. Parameter estimation effect on the out-of-control performance of the MML profile monitoring approaches

To investigate the out-of-control performance of the monitoring schemes based on estimated parameters, determination of corrected control limits is initially required for each approach. As mentioned before, to ensure that occurring an out-of-control signal is due to the effect of parameter shift, the corrected limits are distinguished wider than original limits which are based on known parameters. To evaluate the out-of-control performance of simple linear profile monitoring methods, Mahmoud (2012) calculated the corrected limits to achieve *ARL*=200 based on estimated parameters. In the current study, the corrected control limits are calculated for all the mentioned MML profile monitoring methods based on different values of m and $\lambda = 0.2$ to achieve *AARL*=200 using 10,000 simulation runs. The numerical results are summarized in Table 3. Table 3 shows that decreasing m and consequently increasing the variance of parameters estimations leads to achieve wider corrected limits.

Table 3. The simulated corrected limits when m Phase I samples are used for parameters estimation

$\lambda = 0.2$			
m	MEWMA	MEWMA/χ^2	
30	21	MEWMA	12.55
		χ^2	24.34
100	18.65	MEWMA	11.8
		χ^2	23.97
500	17.75	MEWMA	11.4
		χ^2	23.87
1000	17.65	MEWMA	11.3
		χ^2	23.8

In the current study we consider two types of shifts in the profile parameters using the same MML profile model mentioned in Section 4: I) shifts in the intercept of the first profile from β_{01} to $\beta_{01} + \delta_1\sigma_1$ in units of σ_1 , II) shifts in the slope of the first profile from β_{11} to $\beta_{11} + \delta_2\sigma_1$ in units of σ_1 . Note that all the performance metrics including *AARL*, *SDARL* and *CVARL* are calculated by ARLS algorithm with the difference that in the step 4 of the mentioned algorithm, an out-of-control profile is generated instead of an in-control profile applying considered shifts. For a better comparison, the applied shifts are considered in accordance with applied shifts in Eyvazian et al. (2011) study.

The simulated out-of-control performance metrics based on shift I and different values of m have been summarized in Table 4. The last row of the Tables (4-6) shows the out-of-control *ARL*s obtained by Eyvazian et al. (2011) based on known parameters.

It is observed that MEWMA/χ^2 method performs better than MEWMA method in terms of all performance metrics. But in some cases it is possible that the superior method in terms of *AARL* and *SDARL* metric is not the same. In this situations, we can use *CVARL* metric as the basis of comparison in which consider both *AARL* and *SDARL*, simultaneously.

Table 4. The out-of-control performance comparison of the monitoring approaches when m Phase I samples are used for parameters estimation under the shifts from β_{01} to $\beta_{01} + \delta_1\sigma_1$

m	Method	Metric	δ_1									
			0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
30	MEWMA	*AARL*	25.92	6.46	3.70	2.72	2.10	1.98	1.80	1.49	1.17	1.03
		SDARL	14.55	1.05	0.30	0.14	0.07	0.06	0.07	0.07	0.05	0.01
		CVARL	56.14	16.19	7.98	5.14	3.33	2.98	3.83	4.70	3.89	1.42
	MEWMA/χ^2	*AARL*	18.28	4.94	2.86	1.88	1.37	1.11	1.02	1.00	1.00	1.00

		SDARL	8.89	0.75	0.24	0.14	0.07	0.03	0.01	0.00	0.00	0.00
		CVARL	48.65	15.22	8.53	7.41	5.36	2.86	0.77	0.12	0.02	0.00
100	MEWMA	*AARL*	19.54	5.80	3.46	2.54	2.09	1.91	1.65	1.31	1.08	1.01
		SDARL	4.80	0.56	0.15	0.07	0.03	0.02	0.03	0.04	0.02	0.00
		CVARL	24.54	9.64	4.22	2.82	1.40	1.12	2.11	2.83	1.56	0.41
	MEWMA/χ^2	*AARL*	15.55	4.69	2.74	1.84	1.35	1.10	1.02	1.00	1.00	1.00
		SDARL	3.38	0.39	0.14	0.07	0.04	0.02	0.00	0.00	0.00	0.00
		CVARL	21.71	8.28	4.98	3.67	2.95	1.72	0.48	0.10	0.02	0.00
500	MEWMA	*AARL*	17.55	5.52	3.35	2.47	2.06	1.87	1.59	1.25	1.07	1.01
		SDARL	1.63	0.22	0.06	0.03	0.01	0.01	0.02	0.02	0.01	0.00
		CVARL	9.33	3.89	1.71	1.28	0.65	0.72	1.25	1.27	0.94	0.25
	MEWMA/χ^2	*AARL*	14.32	4.58	2.69	1.83	1.34	1.10	1.02	1.00	1.00	1.00
		SDARL	1.30	0.16	0.06	0.03	0.02	0.01	0.00	0.00	0.00	0.00
		CVARL	9.06	3.53	2.14	1.84	1.57	0.82	0.32	0.07	0.01	0.00
1000	MEWMA	*AARL*	17.48	5.51	3.34	2.46	2.06	1.87	1.59	1.25	1.07	1.00
		SDARL	1.24	0.13	0.04	0.02	0.01	0.01	0.01	0.01	0.01	0.00
		CVARL	7.09	2.43	1.42	1.00	0.65	0.71	0.75	0.96	0.93	0.22
	MEWMA/χ^2	*AARL*	14.11	4.54	2.68	1.82	1.34	1.10	1.02	1.00	1.00	1.00
		SDARL	0.97	0.11	0.04	0.03	0.02	0.01	0.00	0.00	0.00	0.00
		CVARL	6.91	2.42	1.62	1.47	1.30	0.77	0.29	0.08	0.02	0.00
∞	MEWMA	*ARL*	17.29	5.43	3.34	2.45	2.06	1.87	1.59	1.24	1.06	1.01
	MEWMA/χ^2	*ARL*	13.66	4.53	2.65	1.82	1.34	1.09	1.02	1.00	1.00	1.00

Table 5 shows the out-of-control performance of monitoring methods based on shift II. It is observed that MEWMA/χ^2 and Modified MEWMA methods have uniformly better performance than other methods in terms of *AARL*. However, in small shifts, Modified MEWMA method performs better than MEWMA/χ^2 method. In addition, MEWMA performs generally better than LRT method.

Table 5. Out-of-control performance comparison of the monitoring approaches when *m* Phase I samples are used for parameter estimation under the shifts from β_{11} to $\beta_{11}+\delta_2\sigma_1$

m	Method	Metric	δ_2									
			0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
30	MEWMA	*AARL*	56.32	12.84	6.23	4.19	3.24	2.63	2.27	2.05	1.95	1.84
		SDARL	39.92	4.10	0.94	0.41	0.23	0.13	0.10	0.04	0.03	0.04
		CVARL	70.89	31.94	15.11	9.82	7.05	5.10	4.18	2.17	1.36	2.32
	MEWMA/χ^2	*AARL*	54.91	11.52	5.42	3.53	2.56	1.91	1.48	1.22	1.09	1.03
		SDARL	41.29	4.45	0.96	0.44	0.22	0.15	0.10	0.06	0.03	0.01
		CVARL	75.20	38.63	17.68	12.41	8.64	7.84	6.79	4.94	2.74	1.11
100	MEWMA	*AARL*	42.44	10.59	5.60	3.84	2.99	2.47	2.16	1.99	1.88	1.70
		SDARL	13.81	1.51	0.41	0.19	0.11	0.06	0.04	0.02	0.02	0.03
		CVARL	32.54	14.29	7.40	4.85	3.65	2.62	1.75	0.96	1.18	1.96
	MEWMA/χ^2	*AARL*	39.19	9.87	5.08	3.40	2.48	1.88	1.46	1.21	1.08	1.02

		SDARL	13.03	1.52	0.45	0.19	0.11	0.09	0.05	0.03	0.02	0.01
		CVARL	33.25	15.42	8.84	5.46	4.53	4.64	3.69	2.60	1.43	0.60
500	MEWMA	*AARL*	37.62	9.99	5.34	3.71	2.91	2.41	2.11	1.97	1.84	1.64
		SDARL	5.79	0.61	0.18	0.08	0.05	0.03	0.02	0.01	0.02	0.02
		CVARL	15.39	6.10	3.47	2.07	1.70	1.22	0.85	0.51	0.83	1.25
	MEWMA/χ^2	*AARL*	37.21	9.49	4.97	3.33	2.44	1.85	1.45	1.21	1.08	1.02
		SDARL	6.54	0.66	0.19	0.09	0.06	0.04	0.03	0.02	0.01	0.00
		CVARL	17.59	6.97	3.82	2.65	2.38	2.19	1.89	1.32	0.81	0.40
1000	MEWMA	*AARL*	36.70	9.85	5.31	3.71	2.89	2.40	2.11	1.96	1.83	1.64
		SDARL	3.30	0.45	0.12	0.06	0.03	0.02	0.01	0.01	0.01	0.01
		CVARL	9.00	4.57	2.31	1.62	1.18	0.93	0.68	0.42	0.64	0.84
	MEWMA/χ^2	*AARL*	35.97	9.41	4.92	3.30	2.42	1.84	1.44	1.20	1.07	1.02
		SDARL	4.14	0.47	0.14	0.07	0.05	0.03	0.02	0.01	0.01	0.00
		CVARL	11.50	4.98	2.94	2.00	1.91	1.56	1.52	1.13	0.66	0.32
∞	MEWMA	*ARL*	35.67	9.66	5.26	3.66	2.89	2.39	2.10	1.96	1.81	1.63
	MEWMA/χ^2	*ARL*	34.40	9.06	4.91	3.26	2.41	1.84	1.44	1.19	1.07	1.02

As we can see, MEWMA/χ^2 and Modified MEWMA methods have smaller values of *SDARL* compared to other competing methods. However, in small shifts, the performance of Modified MEWMA method is better than MEWMA/χ^2 method. Furthermore, LRT method performs uniformly better than MEWMA/χ^2 method. The noteworthy point is that despite of the worst performance of LRT method in terms of *AARL* and *SDARL*, this method has the best *CVARL* performance among all competing methods. This point can potentially affects choosing the superior method when the process monitoring is performed based on the estimated parameters.

6. Conclusion remarks and future study suggestions

In this study, we evaluated the performance of two Phase II control charts for monitoring multivariate multiple linear profiles: MEWMA and MEWMA/χ^2 control charts proposed by Eyvazian et al. (2011) when parameters are estimated. Three different criteria: *AARL*, *SDARL* and *CVARL* are applied to compare the in-control and out-of-control performance of monitoring approaches by a new Monte Carlo simulation algorithm named as ARLS. The results showed the significant impact of parameters estimation on the both in-control and out-of-control performance of the monitoring approaches. The simulation study illustrated that the number of Phase I samples should be as large as possible in order to achieve an accurate estimation.

According to the results, MEWMA/χ^2 method performs better than other MEWMA method in terms of in-control *AARL* and *CVARL*. This method also performs better than MEWMA method in terms of *SDARL* metric except in small shifts.

The out-of-control performance of each method was evaluated using ARLS algorithm in terms of all three metrics. The results showed that MEWMA/χ^2 method is the best method in detecting shifts in regression coefficients in terms of out-of-control *AARL*, *SDARL* and also *CVARL* metric.

It can be concluded that the MEWMA/χ^2 method that is the extension of MEWMA method is better than its classical version in about all the cases based on estimated parameters

As future researches, we recommend evaluating the effect of parameters estimation on the performance of the control charts for monitoring other types of profile such as: polynomial and nonlinear profile. Furthermore, proposing new metric for measuring estimation effect can be remarkable.

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