A Study on the Effects of the Future Value of Fares for Airline Revenue Management

H.C.C. HO
Department of Industrial and Systems Engineering
The Hong Kong Polytechnic University
Hong Kong, China
chichiu.ho@connect.polyu.hk

F.C.K. YEUNG
Department of Industrial and Systems Engineering
The Hong Kong Polytechnic University
Hong Kong, China
chun-kit.yeung@connect.polyu.hk

Abstract

A novel EMSR (Expected Marginal Seat Revenue) concept is proposed by considering the future value of airfares and seat inventory allocation. The seat allocation resulting from the extensions of EMSRa & EMSRb will be examined in terms of the protection limit, booking limit, and the possible maximum revenues. In our findings, we suggest that the decision makers should recognize the actual value of the revenues from the lower fare classes in the early stage of a flight booking period and adjust the seat inventory allocation in real practice.

Keywords
Seat Inventory Control, EMSRa, EMSRb, Future Value, Revenue Management

1. Introduction
Over the decades, Revenue Management (RM) has played a crucial role in the success of airlines, which can be classified into 1) pricing, 2) demand forecasting, 3) seat inventory control and 4) overbooking (McGill & Van Ryzin, 1999). Seat inventory control has long been a research area for airline industry researchers and it was the main subject of this research as well. Littlewood (1972) was introduced the Littlewood's Rule, which is designed for more complicated models, from the expected marginal seat revenue (EMSR) control for multiple classes (Belobaba, 1987), to optimal booking limits for each class and bid-price control in the RM of airline alliance (Wright et al., 2010). In most of the seat inventory control models, multiple classes models assume that the demand for the lower fare class will come first when the reservations for the lower class have all been received or equal to the booking limits, after which the customers booking a higher fare class start their booking. In the process of calculating the protection levels and setting the booking limits for each fare class, however, the future values of the lower fare are not considered, especially the value of EMSR. In this research, the effects of the future value of fares on the revenues were analyzed, especially in the seat inventory control area.

Many airlines apply a wide range of fares structures for the travel market, which is price discrimination, including discounted fares directed at the price-sensitive travelers as well as the higher-priced business and first-class fares (Botimer & Belobaba, 1999). It is assumed that the lower fare or discounted fare demand comes before the higher fare demand. On one hand, this pricing strategy can help the airline to capture more revenues from the travelers who are price-sensitive. On the other hand, the scheme ensures the cash flow of the airline. As airlines can gain the revenues from the lower fare demand first, which can be a type of liquidity resources, they can use it to invest in internal operational improvement or other financial investments that enable airlines to gain the returns on investments from improved services as well as combat inflation.

Inflation is one of the main causes of the net loss of airlines (Wensveen, 2015), and the revenues from the early stage of booking periods help airlines to combat inflation, especially major international airlines are involved in the global market and which face different inflation factors in different countries. Also, it is proved that the survival probabilities of airlines are positively related to airline growth and profitability (Chin & Tay, 2001). By improving their forecasting techniques, the capacity flexibility and responsiveness to the changing environment, airlines can increase their survival probabilities. Therefore, the future value of the revenues from lower fare classes is larger than its face value if the revenues are used for the investment activities properly. And, the future value of any lower fare class should be a consideration in the seat inventory control system.

Seat inventory control is the practice of maximizing total passenger revenues and/or load factors by balancing the number of reservations accepted in discount and full-fare classes for a flight (Belobaba, 1987). Given a price structure, an effective seat inventory control is important to maintain the revenues on a flight which includes seat allocation to each fare class and setting up appropriate booking limits and even the overbooking practices. In the calculation process, the expected marginal seat revenue (EMSR) of the remaining capacity is the key value to determine the optimal protection levels and booking limits for each class. EMSRa (Belobaba, 1987) and EMSRb (Belobaba, 1989) were described by Belobaba, which are used to calculate the booking limits in multiple fare classes.

In this research paper, seat inventory control is the focus by assuming a given multiple-fares classes and forecast demand. The EMSRa and EMSRb models are extended to consider the future value of each fare by which we can analyze the difference of booking limits results in terms of protection limit, booking limit and the possible maximum revenues.

2. Problem formulation

The two heuristics in Revenue Management introduced by Belobaba – EMSRa and EMSRb – are used in the analysis. We introduce extensions to these two models by applying the concept of the future value of fares. The followings are the formulations of extensions of EMSRa and EMSRb. Both models are designed to address multiple-classes, static, fixed source problems. Here are the assumptions:

Let C be the fixed capacity of a flight, and we have n classes with n fares for each one. Class one has the highest price and the n^{th} class has the lowest. $p_1 >. p_2 > \cdots > p_n$. p_i represents the fare price in class i. We assume the bookings arrive in increasing fare order, so class n customers book first. Within the total booking period, excluding the highest fare, other fare classes have an equal booking time open for customers. Demand arrives in stages that are denoted by d_j, representing the demand in each fare class j and distributed with CDF $F_j(x)$. The demand arrival stage orders are $j, j-1, \ldots, 1$. We assumed that each demand in class j is an independent random variable.

Given a total booking period k, let the returns of investment or inflation rate be $r\%$. If the booking period is 1 year, and the investment or inflation rate is 5%, at the beginning stage j, the future value of that p_i is $p_i(1+5\%)$. The expression of the future value of any p_i ,is as follows.

$$\tilde{p}_i = p_i(1 + r\%)^{\frac{i-1}{n-1}}. \quad (1)$$

Noted that the future value of p_1 ($\tilde{p}_1$) equals to p_1.

In both models, we assume that there are no cancelations or no-shows and we must find out the best multiple fare class capacity allocation in which the maximum expected revenue can be achieved. Suppose that we are starting in stage $j+1$ and we must to calculate the protection level for stages $j, j-1, \ldots, 1$.

2.1. The extension of EMSRa

Let y_j be the protection limits for class j, and we must determine the protection levels for the current class j against each higher fare class $j-1, j-2, \ldots, 1$, where $j \geq 2$ indexed with w. For every combination of w and j, we can get the protection levels for that class (w) by applying Littlewood's rule:

$$P\left(d_w > y_w^j\right) = \frac{\tilde{p}_j}{\tilde{p}_w} \text{ or } y_w^j = F_w^{-1}\left(\frac{\tilde{p}_w - \tilde{p}_j}{\tilde{p}_w}\right) \quad (2)$$

Then the protection limits for y_j is found by adding all these protection levels together.

$$y_j = \sum_{w=1}^{j-1} y_w^j \quad (3)$$

2.2. The extension of EMSRb

In this model, the statistical averaging effect is considered based on the approximation of comparing the higher fare classes that is the future demand and the fare of each fare class will be aggregated and weighted. Again, suppose we are starting in stage $j+1$. The future demand for classes $j, j-1, \ldots, 1$, indexed with w, is aggregated:

$$\hat{d}_j = \sum_{w=1}^{j} d_w \quad (4)$$

and the weighted revenues are

$$\bar{p}_j = \frac{\sum_{w=1}^{j} \tilde{p}_w * d_w}{\sum_{w=1}^{j} d_w} \quad (5)$$

Then, we can get the protection limit for classes j with Littlewood's rule $P\left(\hat{d}_{j-1} > y_j\right) = \frac{\tilde{p}_j}{\bar{p}_{j-1}}$, where $j \geq 2$. Rearranging gives:

$$y_j = F_j^{-1}\left(\frac{\bar{p}_{j-1} - \tilde{p}_j}{\bar{p}_{j-1}}\right) \quad (6)$$

After we have determined the protection limits for each fare j through the extensions of EMSRa and EMSRb, we can obtain the booking limits of class j denoted by b_j. And we will have n booking limits. Assuming we are at the beginning stage of j, we can know the number of bookings we already accepted:

$$\sum_{i=j+1}^{n} x_i. \quad (7)$$

Since we do not allow no-shows and cancelations, we can obtain the remaining capacity by subtracting the booked numbers from our total capacity; we denote this remaining capacity by C_j.

$$C_j = \left[C - \sum_{i=j+1}^{n} x_i\right] \quad (8)$$

Then, we can get the booking limit of class j by deducting the protection limit of j from the remaining capacity.

$$b_j = C_j - y_j \quad (9)$$

Noted that we have no no-shows or cancelations, so we allow all the unbooked or available capacity to be booked. In the end, we do not want to spoil our resources. So, the first class booking limit is $b_1 = C_1$.

2.3. Comparison of the Possible Maximum Revenues

To have a comprehensive understanding of the difference of EMSRa&b and their extensions, it is not enough to simply compare the results of protection levels and limits, but we need to put focus on the possible maximum revenues of difference seat inventory allocations from different methods, especially the extension of EMSRa and EMSRb in different values of return rate on investment or inflation.

As we assumed that the fares would be sold in an increasing order, and we have already set the protection limit in any fare class for higher fare classes by using the methods mentioned above. Then we can determine the possible maximum number of seats which can be sold of that fare class by deducting the protection limits from the current remaining capacity. For the lowest fare class, the possible maximum number of seats can be sold equal to the booking limit. $b_j = C_j - y_j$.

We assume that all the available seat bookings will be sold out, so the remaining capacity of C_{j-1} equals the protection limits of class j (y_j) and the possible maximum number of seats which can be sold of class $j-1$ equals the difference between protection limit j and protection limit $j-1$. It will be the same pattern for the higher fare classes until the highest fare class. As the highest fare class does not have a protection limit, and the possible maximum number of seats which can be sold will equal to the protection limit of class 2, in other words, this is the number of seats reserved for class 1.

After we determine the possible maximum number of seats which can be sold at different fare classes, we can obtain the maximum revenues by multiplying the fares of different classes. In the end, we can obtain the total possible maximum revenues of a seat inventory allocation by aggregating the maximum revenues of each fare class.

3. Analysis

To assess the difference of seat inventory allocation results between the EMSRa & EMSRb and their extensions, an identical test sample of four fare classes was used, the details are shown in Table 1.

Table 1
Test sample - four fare classes

Class (j)	Fare (p_j)	Mu (d_i)	Sigma (σ)
1	\$500	16.5	5.6
2	\$425	44.2	15
3	\$320	35.1	11.2
4	\$165		

In this test sample, we assumed that a flight has 150 fixed capacity and four fare classes. The demand for each fare class is independent and distributed with a given mean and variance. As we only want to know the protection levels for the first three highest fares and the booking limits for each class, there is no need to have the demand forecast for the lowest fare. The following analysis processes are divided into three parts, (1) the analysis of seat inventory allocation by using EMSRa and its extension; (2) the analysis of seat inventory allocation by using EMSRb and its extension and (3) the possible maximum revenues under different seat inventory allocation methods.

3.1. EMSRa and its extension

With the demand forecasting information, we can obtain the protection levels for the higher fares, that is, how many seats we need to protect against the lower fares by using the EMSRa method, as shown in
Table 2. The third column shows the protection levels for fare classes 1-3 against class 4 and we can obtain the protection limits for class 4, which means how many seats we need to reserve for the higher fare classes, by aggregating all the protection levels for the higher fares, shown on the bottom line. The fourth and fifth columns, they show the protection limit results for classes 3 and 2 respectively.

Table 2
Protection limits - EMSRa

Class (j)	Fare (p_j)	Protection Levels (w, y_4)	Protection Levels (w, y_3)	Protection Levels (w, y_2)
1	$500	18.96	14.49	10.70
2	$425	48.46	33.94	
3	$320	34.66		
4	$165			
		102.08	48.44	10.70

Table 3 shows the protection limit results by using the extension of EMSRa. Notice that the first step is to calculate the future values of each fare, and here we show a sample assuming the return on investment or inflation rate equals to 5%. Then, we use the future values to calculate the protection levels for each higher fare class and obtain the protection limits for the relative lower fare class.

Table 3
Protection limits- EMSRa's extension

Class (j)	*Future Value ($\tilde{p}_i$)	Protection Levels (w, y_4)	Protection Levels (w, y_3)	Protection Levels (w, y_2)
1	$500	18.71	14.17	10.35
2	$432	47.96	33.35	
3	$331	34.42		
4	$173			
		101.09	47.52	10.35

Table 4 is a summary of the results of seat inventory control (protection limit and booking limit) by using EMSRa and its extension methods. We notice that the main difference is that the protection level for class 1 against lower fares is decreased by 1 unit and the same amount is given to class 4. This means that the lowest fare class deserves to be given one more booking limit under a 5% return on investment or inflation rate assumption.

Table 4
Summary of the PL&BL results (*PL=Protection Limit; *BL=Booking Limit)

EMSRa				EMSRa's Extension		
Class j	Fare (p_j)	*PL (y_j)	*BL (b_j)	Future Value ($\tilde{p}_i$)	*PL (y_j)	*BL (b_j)
1	$500		150	$500		150
2	$425	11	139	$432	10	140
3	$320	48	102	$331	48	102
4	$165	102	48	$173	101	49

3.2. EMSRb and its extension

In this part, the results of seat inventory allocation using EMSRb and its extension are shown in the following tables. The structure of this part is like the previous section. We first to describe the protection limit results of EMSRb and its extension, then their booking limits summary is shown. Table 5 shows the protection limit results by using the methods of EMSRb. The last column shows how many seats we need to reserve for the higher classes in each fare class. As class 1 is the highest fare class, there is no protection limit for the higher fare class.

Table 5
Protection limits - EMSRb

Class (j)	Fare (p_j)	Weighted Avg Fare ($\bar{p}_j$)	Agg mu ($\hat{d}_j$)	Agg Sigma ($\bar{\sigma}$)	Protection Limit (y_j)
1	$500	$500.00	16.5	5.6	
2	$425	$445.39	60.7	16.01	10.70
3	$320	$399.45	95.8	19.54	51.44
4	$165				100.09

Table 6 shows the protection limit results by using the extension of EMSRb. Notice that we first to calculate the future value of each fare and use the future value to calculate the protection limits for each class with the help of Littlewood's rule. Like the example shown in TABLE 3, we also assume the return on investment or inflation rate equals to 5%. In the last column, the protection limits for classes 2-4 are shown.

Table 6
Protection limits - EMSRb's extension

Class (j)	*Future Value ($\tilde{p}_i$)	Weighted Avg Fare ($\bar{p}_j$)	Agg mu ($\hat{d}_j$)	Agg Sigma ($\bar{\sigma}$)	Protection Limit (y_j)
1	$500	$500.00	16.5	5.6	
2	$432	$450.46	60.7	16.01	10.35
3	$331	$406.54	95.8	19.54	50.70
4	$173				99.44

A summary of the results of seat inventory allocation by using EMSRb and its extension is shown in Table 7. We can observe the main difference between these two methods is similar to section 3.1 that is the protection level for class is decreased by 1 unit and it is profitable to allow the lowest fare class to book one more unit.

Table 7
Summary of the PL&BL results (*PL=Protection Limit; *BL=Booking Limit)

	EMSRb			EMSRb - Extension		
Class j	Fare (p_j)	*PL (y_j)	*BL (b_j)	Future Value ($\tilde{p}_i$)	*PL (y_j)	*BL (b_j)
1	$500		150	$500		150
2	$425	11	139	$432	10	140
3	$320	51	99	$331	51	99
4	$165	100	50	$173	99	51

3.3. The possible maximum revenues

In Table 8, we summarize the possible maximum revenues of each method using the analysis examples mentioned above. We list the possible maximum number of seats which can be sold in different fare classes in each method. Then, we can obtain the maximum revenue by multiplying the fares. For the EMSRa and EMSRb methods, we multiply the maximum number of seats to be sold by the original fare prices. As for the extensions of EMSRa & EMSRb, we multiply the maximum number of seats to be sold by the future value of each fare, assuming a 5% return rate on investment of inflation in this case. The bottom line shows the changes of maximum revenue in percentages when we compare the result from extensions to the original method's results. We can notice that the change between EMSRb and its extension is larger than the change between EMSRa and the extension of EMSRa.

Table 8
Summary of maximum revenue of different seats inventory control methods

		EMSRa	EMSRb		EMSRa_ Extension	EMSRb_ Extension
Class j	Fare(j)	Max. Seats Sold Out	Max. Seats Sold Out	Future Value	Max. Seats Sold Out	Max. Seats Sold Out
1	$500	11	11	$500	10	10
2	$425	38	40	$432	37	40
3	$320	54	49	$331	54	49
4	$165	48	51	$173	49	51
Maximum Revenue		$46,461	$46,435		$47,415	$47,477
Changes in Percentage					2.05%	2.24%

4. Findings and implications

In the last part, we demonstrate how we consider the future value of each fare to find out another seat inventory allocation and compare the results of extensions with the original methods in terms of protection limits, booking limits and the possible maximum revenues. However, the examples shown in the last part only consider the future value of fares for a 5% of return rate on investment or inflation. We can have a comprehensive understanding of the effects of time value of fares on seat inventory management by considering different return rates on investment or inflation.

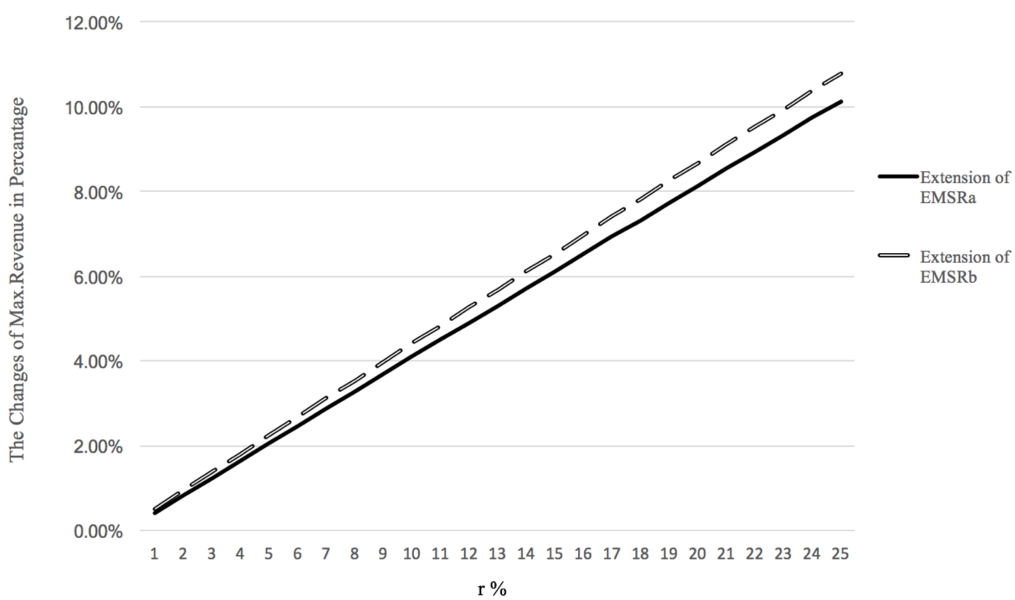

Figure 1. Percentage changes of the maximum revenue of the two extensions compared to the original methods at different return rate of the investment or inflation

Figure 1 shows the percentage changes of maximum revenue when we compare the extensions to original methods at different return rates on investment or inflation. From the figure, we see that there is a positive relationship between the return rate and the changes of maximum revenues. When the return rate increases, the change of maximum revenues in term of percentage will also rise. The extension of EMSRb will cause a slightly greater change in the maximum revenue compared to the extension of EMSRa.

The implication of the finding is that the decision makers of seat inventory control need to recognize the actual values of the relatively low fares. As the revenues from the lowest fare class will be obtained at an early stage that it will help airlines to ensure their internal cash flows. Airlines may make use of such revenues to make internal investments or utilize other investment tools to combat inflation. Under the global airline network, airlines have customers from different countries with different inflation rates. Decision makers need to identify the inflation rate of different currencies and consider the actual value of each fare price and implement appropriate seat inventory allocation over the whole booking period.

5. Conclusion

We have examined the results of protection limits, booking limits and the possible maximum revenues from different EMSR models, especially the extensions of EMSRa & EMSRb. We found that, if we consider the future value of fares, as the return rate on investment or inflation rate rises, the protection levels for the higher fare classes will be decreased and the booking limits for lower fare classes will be increased and the possible maximum revenues will increase. The results reflect that the managers of seat inventory control need to recognize the actual value of the revenues from lower fare classes and should apply the concept to their seat inventory allocation practices. If the return rate of revenues from the early stage of booking period is high enough, it is more appropriate to allow more bookings in the lower fare classes in terms of maximizing the revenues.

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Biographies

H.C.C. HO is currently a fresh graduate and holds a Bachelor of Science degree in Enterprise Engineering with Management from The Hong Kong Polytechnic University. During the last two years, Mr. Ho was one of the founders and committee members of The Hong Kong University Students' Union 1st Chess and Boardgames Club served as promotion secretary. He was mainly responsible for promotional activities include planning and executing related events and assisting in organizing various competitions for members.

F.C.K. YEUNG is also currently a fresh graduate and holds a Bachelor of Science degree in Enterprise Engineering with Management from The Hong Kong Polytechnic University. Mr. Yeung found The Hong Kong Polytechnic University Students' Union 1st Chess and Boardgames Club with Mr. Ho and served as internal vice president. He mainly assisted in internal administration and organized internal activities for committee members to establish cheerful relationships between each member.