Implementation of Accelerated Degradation Testing Methods using R-Excel interface

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Abstract

The requirements for timely information on the reliability of product components and materials facilitate the use of accelerated degradation test (ADT). The ADT approach supports the experimenter in drawing quick inference on the lifetime distribution of testing units at normal use condition. This paper provides an Excel add-in program (called RExADT) for analysis of ADT data. RExADT is designed to support two popular ADT approaches: single-stage and two-stage approach. RExADT is implemented in Visual Basic for Applications (VBA) and R based on the R-EXCEL environment. RExADT is expected to help the users with diverse backgrounds perform the ADT analysis conveniently with a familiar graphical user interface in Excel.

Keywords
Reliability, Accelerated Degradation, Excel, R

1. Introduction

Reliability test has been conducted to obtain product lifetime data in manufacturing industries. For some tests, covariate information, which is related to the wear on the product during the reliability test, is available. The wear measurement (or degradation) can be the physical parameters of the product (e.g., corrosion thickness on a metal plate (Bogdano and Kozin 1985)) or mere indicators of product performance (e.g., the luminosity of an organic light emitting diodes). As sufficient time-to-failure data cannot be secured for highly reliable products, degradation data have been widely used to provide inference on product lifetime distribution from the degradation data. General references for degradation models and their applications can be found in Nelson (1990) and Meeker and Escobar (1998). Recently, Ye and Xie (2015) reviewed two broad categories of degradation models; a stochastic process model and a general path model, and they comprehensively compared advantages and disadvantages of these two modeling approaches.

Degradation test is closely tied with an accelerated life test (ALT) in that both methods have evolved to suit reliability tests for which product lifetimes are expected to last far beyond testing duration. ALT expedites product failures within a relatively short time by stressing testing units beyond their normal use condition. For modern highly reliable products, however, few failures are expected even at elevated testing conditions during the allotted testing time. For timely information on the reliability of product components and materials, the accelerated degradation test (ADT) can be used. The ADT approach combines degradation analysis and ALT by testing products in harsher environments and measuring the evidence of product degradation during the accelerated test. In general, the ADT model is built on the relationship between stresses and parameters of the presumed model for empirical or theoretical degradation paths.

These paths are observed from testing samples under the underlying assumption that the model parameters follow a specified parametric distribution (e.g., Weibull or lognormal distribution). The ADT approach supports the experimenter in drawing quick inference on the lifetime distribution of testing units at normal use condition, provided there is a known functional link relating elevated testing environments to normal use condition.

Microsoft Excel is the most popular commercial tool for storing and working with a dataset. Many users are familiar with performing basic data analysis using Excel, but it does not provide an in-depth statistical analysis of lifetime and degradation data. When reliability engineers need to assess the degradation data, but they are not familiar with statistics, it will be a challenging task to execute accelerated degradation analysis using commercial or free software packages which require a certain level of statistical backgrounds. To the best of our knowledge, there is no Excel-based program for ADT analysis which allows users to perform whole ADT procedures. In this paper, we provide the EXCEL add-in software for ADT analysis (called RExADT hereafter) based on the REXCEL interface as a seamless and direct interface, which is expected for reliability engineers to easily derive inference on reliability information at normal use condition by analyzing ADT data.

The remainder of this paper is organized as follows. Section 2 briefly introduces two different ADT models, single-stage and two-stage approach. Section 3 describes the functions of RExADT and the application of RExADT to an accelerated degradation testing data. Finally, concluding remarks and future possible extensions of the package are given in Section 4

2. ADT model

ADT of an item is conducted at l stress levels, $V_i, i = 1,2, \dots, l$ such that $V_0 \leq V_1 \leq \cdots \leq V_l$, where V_0 denotes the stress at normal use condition. Denote a true degradation path at time t by $D(t)$. The observed sample degradation path at the kth measurement time t_{ijk} on the jth individual testing item under stress level V_i is given by

$$y_{ij}(t_{ijk}) = D(t_{ijk}; V_i, \boldsymbol{\Theta}_{ij}) + \varepsilon_{ij}(t_{ijk}), \;\; i = 1, \dots, l, j = 1, \dots, n_i, k = 1, \dots, m_{ij}, \tag{1}$$

where $\boldsymbol{\Theta}_{ij}$ is the parameter vector. Hereafter, we will omit the subscript of t_{ijk} for notational convenience. $D(\cdots)$ may be a linear or nonlinear function of t and $\boldsymbol{\Theta}_{ij}$. $\varepsilon_{ij}(t)$, a deviation error from the assumed model for item j at ith level of stress, is assumed to be s-independent of $\boldsymbol{\Theta}_{ij}$ for all i and j. The time t, with the total number of inspections on item j under stress level V_i denoted by m_{ij}, could be real operating times, some surrogates such as kilometers or miles, and loading cycles in fatigue tests.

At this point, there are two popular approaches to analyze accelerated degradation data, single-stage and two-stage approach. The most important difference between the two approaches is that how the stress acceleration V_i is to be incorporated in the analysis. The first approach, as the equation (1) implies, incorporates the stress acceleration and measurement time into a unified model to predict degradation path at normal use condition. A failure-time distribution can be directly derived by using a random-coefficients model and pseudo failure-times. The pseudo failure-time is defined as a time when degradation path crosses a pre-determined critical level of failure D_f. This approach is generally called "accelerated degradation analysis" in the literature, but for the purpose of this discussion it will be named as single-stage approach hereafter. The second approach infers lifetime distribution at normal used condition using two-stages. In the first stage, the degradation function $D(t)$ is used to individually extrapolate pseudo failure-time for each of testing items. In the second stage, the pseudo failure-times are treated as complete failure data to estimate the lifetime distribution based on the life-stress relationship. This stage is basically identical to the traditional ALT analysis. In RExADT, we provide both single-stage and two-stage approaches to allow users more flexible choices.

2.1 Single-stage approach

Random Coefficients model

Under a general formulation of the random-coefficients model, the degradation model (1) can be represented as

$$y_{ij}(t) = D(t; V_i, \boldsymbol{\Theta}_{ij}) + g(D(t; V_i, \boldsymbol{\Theta}_{ij}), \alpha, \rho) \cdot \varepsilon_{ij}(t), \;\; i = 1, \dots, l, j = 1, \dots, n_i, k = 1, \dots, m_{ij}, \tag{2}$$

where individual-specific regression parameters $\boldsymbol{\Theta}_{ij} \equiv (\boldsymbol{\beta}_i, \mathbf{b}_{ij})^{\mathrm{T}}$ are represented with a p dimensional vector of fixed-effects $\boldsymbol{\beta}_i$, which is common for all individuals at stress V_i, and a q-dimensional vector of random-effects $\mathbf{b}_{ij}$, which expresses between-individual variation. The random-coefficients are assumed as $\mathbf{b}_{\mathrm{ij}} \sim MVN(\mathbf{0}, \boldsymbol{\Sigma}_i)$ for the jth item at stress V_i. The error terms $\varepsilon_{\mathrm{ij}}(t)|\boldsymbol{\Theta}_{ij} \sim N(0, \sigma_i^2)$. Function $g(\cdot)$ denotes a variance function which expresses the heteroscedasticity with parameter α or correlation among within-individual measurements ruled by correlation parameter ρ. It is reasonable to assume that $\varepsilon_{ij}(t)$ depends on stress level i, but not on item j because the level of

degradation is generally a function of the stress level i only. Note that fixed-effects $\boldsymbol{\beta}_i$ and variance-covariance matrices $\boldsymbol{\Sigma}_{\mathrm{i}}$, and error σ_{i}^2, as aforementioned, can be common across all stress levels.

Stress acceleration

An increase of stress levels accelerates the degradation process related to failure mechanisms for testing items. Interpretation of accelerated test data requires physical or theoretical models relating accelerated stresses to failure-time acceleration. If a model for the relationship is known in priori, the model can be used to estimate degradation rates or lifetime at normal use condition. In RExADT, we considered only a single stress variable model, such as an Arrhenius model, an inverse power law model, and an Eyring model.

The Arrhenius model describes temperature effects on the rate of simple chemical reaction. The reaction rate at temperature V is represented as

$$R_{\mathrm{AR}}(V) = v_0 \exp\left[-\frac{E_a}{k_B \times (V + 273.15)}\right],$$

Where E_a the activation energy in electron volts(eV), $k_B = 8.6171 \times 10^{-5} = 1/11605$ is Boltzmann's constant in electron volts per °C, and $V + 273.15$ is temperature in the absolute Kelving scale. Here, E_a and v_0 are the parameters specifying product or material characteristics. In the Arrhenius relationship, the acceleration factor at stress level $V_i (V_i \geq V_0)$ is

$$AF_{AR}(V_i; V_0) = \frac{R(V_i)}{R(V_0)} = \exp\left[E_a\left(\frac{11605}{V_0} - \frac{11605}{V_i}\right)\right]. \tag{3}$$

Parameter Estimation of ADT model

At a particular stress V_i, maximum likelihood (ML) estimation for the model parameters in (2) is based on the marginal density of $\mathbf{y}_{\mathrm{ij}} = \left(\mathrm{y}_{\mathrm{i}1}(t)^{\mathrm{T}}, \cdots, \mathrm{y}_{\mathrm{i}j}(t)^{\mathrm{T}}\right)^{\mathrm{T}}$

$$p\left(\mathbf{y}_{\mathrm{ij}}|V_i, \boldsymbol{\beta}_i, \sigma_i^2, \boldsymbol{\Sigma}_i\right) = \int p\left(\mathbf{y}_{\mathrm{ij}}|V_i, \boldsymbol{\beta}_i, \mathbf{b}_{ij}, \sigma_i^2\right) p\left(\mathbf{b}_{ij}|\boldsymbol{\Sigma}_i\right) d\mathbf{b}_{ij}, \tag{4}$$

where $p\left(\mathbf{y}_{\mathrm{ij}}|V_i, \boldsymbol{\beta}_i, \mathbf{b}_{ij}, \sigma_i^2\right)$ is the conditional density of $\mathbf{y}_{\mathrm{ij}}$ given the random-effects $\mathbf{b}_{\mathrm{ij}}$ having the marginal distribution $p\left(\mathbf{b}_{ij}|\boldsymbol{\Sigma}_i\right)$. In general, the right hand side of (4) does not have a closed-form expression when the model function is nonlinear in $\mathbf{b}_{\mathrm{ij}}$. Then, approximation methods, such as Lindstrom & Bates' (LB) algorithm (Lindstrom and Bates 1990), and adaptive Gaussian quadrature (Pinheiro and Bates 1995), can be used to estimate the marginal density of $\mathbf{y}_{\mathrm{ij}}$. Bae and Kvam (2004) introduced various approximation methods to numerically optimize the log-likelihood corresponding to (4). Lu and Meeker (1993) proposed a two-stage method for estimating the parameters in the random-coefficients model..

The RExADT estimates the parameters of ADT model using LB algorithm. The LB algorithm approximates the nonlinear (or linear) function D not around the population mean but around the subject specific means which include the estimate of random-effects, $\hat{\mathbf{b}}_{\mathrm{ij}}$. We skip details on the estimation and inference for the parameters in the mixed-effects model here, instead, see Pinheiro and Bates (2000).

Derivation of lifetime distribution

To derive the failure-time distribution and its quantiles from accelerated degradation data, define failure-time T as the first crossing time that the actual degradation path $D\left(t; V_i, \boldsymbol{\Theta}_{ij}\right)$ reaches D_f. Using the form of $D(t; V, \boldsymbol{\Theta})$ for simplicity, the distribution of the failure-time is

$$F(t) = \Pr(T \leq t) = \Pr\left(\inf\left\{u: D(u; V, \boldsymbol{\Theta}) \leq D_f\right\} \leq t\right).$$

At a specific level of the accelerating variable, the failure-time distribution depends on the distribution of the random-coefficients $\mathbf{b}$, which is determined by $\boldsymbol{\Sigma}$. Denote the true value of $\boldsymbol{\Psi} \equiv (\boldsymbol{\beta}, \mathbf{b}, \sigma^2)$ as 0. Within the framework of (2), the failure probability at a given time t can be expressed as

$$F(t; \boldsymbol{\Psi}_0) = \Pr\left(\inf\left\{u: y(u) \leq D_f\right\} \leq t\right) = \int \Pr\left(\inf\left\{u: y(u) \leq D_f\right\} \leq t|\mathbf{b}\right) p(\mathbf{b}) d\mathbf{b}. \tag{5}$$

In practice, ML estimate of the lifetime distribution at time t, $\hat{F}(t)$, and ML estimate of the pth quantile, $\hat{t}_p$, can be computed by replacing $\boldsymbol{\Psi}_0$ with ML estimates of the parameters $\hat{\Psi} \equiv \left(\hat{\beta}, \hat{\mathbf{b}}, \hat{\sigma}^2\right)$. To estimate the lifetime distribution at normal use condition using (5) based upon the accelerated degradation data, incorporation of the effects of stress variables requires additional assumptions for a functional relationship between stress variables and the parameters ($\boldsymbol{\Psi}$)

in the degradation model. Such a relationship may arise from substantive knowledge in the area of application or may be suggested by examining the data. However, if there is no closed-form expression for $\hat{F}_T$, or if the inverse transformation with respect to t is overly complicated, derivation of lifetime distribution usually relies on Monte Carlo simulation (Meeker and Escobar 1998). The procedure to construct confidence intervals for the failure-time distribution is implemented via the parametric bootstrap method introduced in Bae and Kvam (2004).

2.2 Two-stage approach

First stage: Pseudo failure-time

The random-coefficients model is required to analyze the data through the single-stage approach. In the approach here, on the other hand, the degradation model D is simply used to estimate a degradation path and calculate the pseudo failure-time for each individual testing item. Meeker and Escobar (1998) used the two-stage approach, (they called the approach "failure-time analysis" therein), and compared it with a single-stage approach. Under the two-stage approach in this paper, pseudo failure-times are obtained for all of testing items by observing when a given item reaches a predetermined failure threshold level or extrapolating fitted degradation model to the threshold level. Therefore, accelerated degradation data will be transformed into complete failure data in our approach.

Recall the ADT model shown in (1). The equation can be simply rewritten without acceleration stress V_i as $y_{ij}(t) = D(t; \mathbf{\Theta}_{ij}) + \varepsilon_{ij}(t)$. Based on the predetermined critical level D_f, a pseudo failure-time T_{ij} is calculated by inverting the equation, $T_{ij} = D^{-1}(D_f; \mathbf{\Theta}_{ij})$.

Second stage: Lifetime distribution analysis with stress acceleration model

When the pseudo failure-times of accelerated degradation data are obtained, the ALT data analysis can be used to estimate the lifetime distribution $F(t)$. In order to predict lifetime distribution at normal use condition, a stress acceleration model that is mentioned in Section 2.1 should be combined with the lifetime distribution to extrapolate to other levels of the stress. Such model is called "ALT model" or "life-stress relationship".

For example, as the most commonly used in ALT data analysis, we consider the Weibull-Arrhenius model. The analysis is performed under the following assumptions:

A1. The Weibull distribution has the following cumulative density function

$$F(t; \eta_i, \gamma) = 1 - \exp\left(-\left(\frac{t}{n_i}\right)^{\gamma}\right), \quad t \geq 0, \tag{6}$$

where η_i is a scale parameter at stress level i and γ is a shape parameter. Both parameters have positive values and the shape parameter has a constant value across all the stress levels. Note that the probability density function is given by $f(t; \eta_i, \gamma) = \frac{\gamma}{\eta_i}\left(\frac{t}{\eta_i}\right)^{\gamma-1} \exp\left(-\left(\frac{t}{\eta_i}\right)^{\gamma}\right)$.

A2. The stress acceleration model is given by

$$\log(\eta_i) = \log[R_{AR}(V_i)] = \log(v_0) - E_a \frac{11605}{(V_i + 273.15)} = \alpha_0 + \alpha_1 x_i, \tag{7}$$

where α_0 and α_1 are parameters for log-linear Arrhenius model, and x_i is a transformed stress level.

The likelihood function of complete failure data is given by

$$L(\gamma, \alpha_0, \alpha_1 | t) = \prod_{i=1}^{l} \prod_{j=1}^{n_i} \frac{\gamma}{\eta_i^{\gamma}} t_{ij}^{\gamma-1} \exp\left(-\left(\frac{t}{\eta_i}\right)^{\gamma}\right), \tag{8}$$

where $x_0 = \alpha_0 + \alpha_1 x_i$. In RExADT, the ML estimates $\hat{\gamma}$, $\hat{\alpha}_0$, and $\hat{\alpha}_1$ of the parameter γ, α_0, and α_1 are obtained from `survreg` in package **survival**. The lifetime distribution at normal use condition via the estimated Weibull-Arrhenius model can be calculated as:

$$\hat{\eta}_0 = \hat{\alpha}_0 + \hat{\alpha}_1 x_0 \tag{9}$$

where $x_0 = 11605/(V_0 + 273.15)$ and the shape parameter at use condition is $\hat{\gamma}$. The inference for the parameters and the p-th quantiles, $t_p = F^{-1}(p; \hat{\eta}_0, \hat{\gamma})$, are evaluated by using a normal-approximation method. See Meeker and Escobar (1998) for a comprehensive guide and analysis for ALT data (Section 8.4 and Section 19). Bae et al. (2010) provided a step-by-step procedure for the two-stage approach to analyze degradation data of direct methanol fuel cells.

3. Interface and functions

3.1 Model setup

Before starting the program, we need a data table. The table can be set by selecting options and inserting numbers in the first spreadsheet, 0. Model Setup. As shown in Figure 1(a), the user can first set the number of accelerating levels and select the measurement unit. Then, acceleration levels and corresponding numbers of the samples need to be set. In addition, the user needs to enter a normal operating condition. When clicking the Generate a data table button, a blank table is created if there is no existing table in the second spreadsheet, 1. Data. Otherwise, the existing table is initialized in the same spreadsheet.

After the model specification is completed, the user needs to set the unit of time observing the data and the unit of the data in the top part of the spreadsheet, '1. Data'. Then, all degradation data need to be entered based on the form of the model. The user can insert all data by hand or open, save, and paste the data in the Excel program if the format of the data is fitted to the model. Figure 1(b) shows an example of an accelerated degradation testing data where testing products seem to degrade linearly. Clicking the Process button transfers the data to the R program and save it in R workspace. Variables in the Excel VBA program are also produced simultaneously.

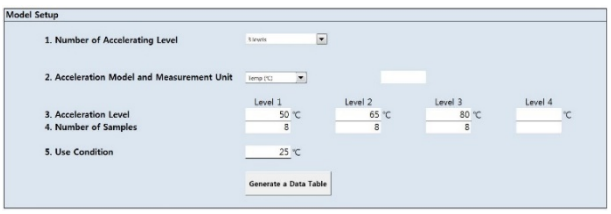
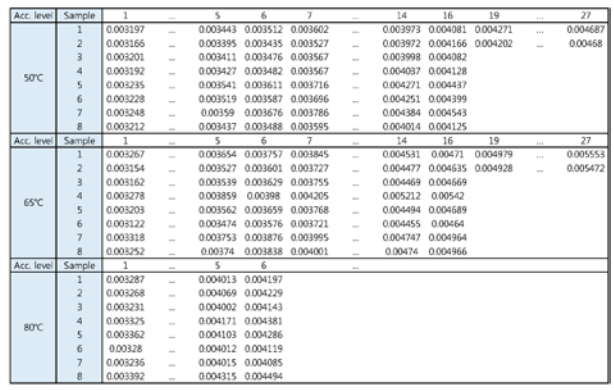


Figure 1.(a) Generating an input data table



Acc. level	Sample	1	...	5	6	7	...	14	16	19	...	27
50°C	1	0.003197	...	0.003443	0.003512	0.003602	...	0.003973	0.004081	0.004271	...	0.004687
	2	0.003166	...	0.003395	0.003435	0.003527	...	0.003972	0.004166	0.004202	...	0.00468
	3	0.003201	...	0.003411	0.003476	0.003567	...	0.003998	0.004082			
	4	0.003192	...	0.003427	0.003482	0.003567	...	0.004037	0.004128			
	5	0.003235	...	0.003541	0.003611	0.003716	...	0.004271	0.004437			
	6	0.003228	...	0.003519	0.003587	0.003696	...	0.004251	0.004399			
	7	0.003248	...	0.00359	0.003676	0.003786	...	0.004384	0.004543			
	8	0.003212	...	0.003437	0.003488	0.003595	...	0.004014	0.004125			
Acc. level	Sample	1	...	5	6	7	...	14	16	19	...	27
65°C	1	0.003267	...	0.003654	0.003757	0.003845	...	0.004531	0.00471	0.004979	...	0.005553
	2	0.003154	...	0.003527	0.003601	0.003727	...	0.004477	0.004635	0.004928	...	0.005472
	3	0.003162	...	0.003539	0.003629	0.003755	...	0.004469	0.004669			
	4	0.003278	...	0.003859	0.00398	0.004205	...	0.005212	0.00542			
	5	0.003203	...	0.003562	0.003659	0.003768	...	0.004494	0.004689			
	6	0.003122	...	0.003474	0.003576	0.003721	...	0.004455	0.00464			
	7	0.003318	...	0.003753	0.003876	0.003995	...	0.004747	0.004964			
	8	0.003252	...	0.00374	0.003838	0.004001	...	0.00474	0.004966			
Acc. level	Sample	1	...	5	6		...					
80°C	1	0.003287	...	0.004013	0.004197							
	2	0.003268	...	0.004069	0.004229							
	3	0.003231	...	0.004002	0.004143							
	4	0.003325	...	0.004171	0.004381							
	5	0.003362	...	0.004103	0.004286							
	6	0.00328	...	0.004012	0.004119							
	7	0.003236	...	0.004015	0.004085							
	8	0.003392	...	0.004315	0.004494							

Figure 1.(b) An example of entered degradation data.

3.2 Model fitting

Pressing the button, Model fitting, results in showing another window, Degradation Data Analysis. To fit the data, the user needs to select proper degradation model. Then, clicking the button Analysis allows the program to load a package (namely nlme) in R workspace. This provides specific information of the fitted model on the left side of the window, and shows scatter plots of the residuals from fitted degradation model to check characteristics of the data, such as homoscedasticity, normality, and independence. Figure 2(a) shows the model coefficients, slopes and intercepts, when fitting linear models and the same information is also reported in the spreadsheet, 2. Model Fitting. For example, the initial heat conductivity of sample 1 in Level 1 is 0.0031663, and it increases by 5.7528×10^{-5} every week. Figure 2(b) shows t-test results for the model coefficients. The t-test provides significance of estimated parameters and for the test, we set a null hypothesis that an estimated parameter equals to 0.

To check homoscedasticity, normality, and independence of the model, the right side of the window provides three types of graphs, residuals versus fitted values plot, Q-Q plot, and time versus fitted values plot, respectively. In Figure 3, the user may check three graphs according to accelerating levels. Moreover, the bottom part of summary of the estimated model provide Shapiro-Wilk's normality test results based on p-values as follows.

If the p-value is larger than a pre-specified significant level, then one can decide that normality assumption is satisfied.

3.3 Pseudo-failure data

After completion of '2. Model fitting', the user can calculate predicted lifetime (i.e., pseudo-failure time) for each sample in the spreadsheet '3. Pseudo-failure data'. This work can be simply done by two steps; (1) entering a threshold value for pseudo-failure, and (2) clicking the button, `Get Pseudo-failure data`. Then, the program derives coefficients of selected regression models and provides pseudo failure-times based on the coefficients. All of coefficients and failure-times are reported in the table of the spreadsheet '3. Pseudo-failure data'. If the number of accelerating levels is greater than 1, then the reliability analysis is needed to analyze and predict the lifetime distribution at normal use condition, and it is followed in the next Section. Otherwise, the lifetime analysis is completed at this step because this program is developed for analyzing ADT data. Note that if the user wants to analyze ADT data through the single-stage approach directly, it needs not to perform this step.

```
Summary of the model coefficients will be
displayed in the worksheet.

-------------------------------------------
>> Level 1 : 50 ℃
  (Intercept)            x
1 0.003166322 5.752783e-05
2 0.003116617 5.878245e-05
3 0.003128693 6.062477e-05
4 0.003100140 6.623056e-05
5 0.003139945 8.195728e-05
6 0.003122484 8.036146e-05
7 0.003152090 8.858386e-05
8 0.003123813 6.432075e-05

-------------------------------------------
>> Level 2 : 65 ℃
  (Intercept)            x
1 0.003232572 8.895577e-05
2 0.003103861 9.216855e-05
3 0.003051264 1.033061e-04
4 0.003156457 1.463318e-04
5 0.003089858 1.006044e-04
6 0.002996200 1.055703e-04
7 0.003213890 1.119476e-04
8 0.003183804 1.136772e-04

-------------------------------------------
>> Level 3 : 80 ℃
  (Intercept)            x
1 0.003095800 0.0001840571
2 0.003069933 0.0001968286
3 0.003048867 0.0001863714
4 0.003095867 0.0002125143
5 0.003158400 0.0001891714
6 0.003112400 0.0001744571
7 0.003033733 0.0001827429
8 0.003151600 0.0002271143
```

```
T-test results for the model coefficients.
-------------------------------------------
>> Level 1 : 50 ℃
Coefficients:
   (Intercept)
     Estimate   Std. Error  t value Pr(>|t|)
3 0.003128693 1.192141e-05 262.4433        0
...
1 0.003166322 9.140288e-06 346.4138        0

       Estimate   Std. Error  t value Pr(>|t|)
3 6.062477e-05 1.295091e-06 46.81120        0
...
1 5.752783e-05 6.499003e-07 88.51794        0

Residual standard error: 2.224141e-05 on 114
degrees of freedom
-------------------------------------------
>> Level 2 : 65 ℃
Coefficients:
   (Intercept)
     Estimate   Std. Error  t value Pr(>|t|)
6 0.002996200 2.106475e-05 142.2376        0
...
1 0.003232572 1.615061e-05 200.1518        0

       Estimate   Std. Error  t value Pr(>|t|)
6 1.055703e-04 2.288386e-06 46.13308        0
...
1 8.895577e-05 1.148354e-06 77.46374        0

Residual standard error: 3.929988e-05 on 114
degrees of freedom
-------------------------------------------
>> Level 3 : 80 ℃
Coefficients:
   (Intercept)
     Estimate   Std. Error  t value Pr(>|t|)
7 0.003033733 2.565984e-05 118.2289        0
...
8 0.003151600 2.565984e-05 122.8223        0
   x
       Estimate   Std. Error  t value Pr(>|t|)
7 0.0001827429 6.588838e-06 27.73522        0
...
8 0.0002271143 6.588838e-06 34.46955        0

Residual standard error: 2.756309e-05 on 32
degrees of freedom
```

(a) (b)

Figure 2: Degradation model fitting: (a) coefficients of the estimated model and (b) the summary of the estimated model

```
(Shapiro-Wilk's normality test)
Help : If P-value is less than the significance level
(e.g., 0.05 or 0.1), the residual is not normal.
>> Level 1 :  50 ℃
Test statistic :  0.94009
P-value :  0.00002
>> Level 2 :      65 ℃
Test statistic :  0.99287
P-value :  0.75671
>> Level 3 :  80  C
Test statistic :  0.97633
P-value :  0.4366
```

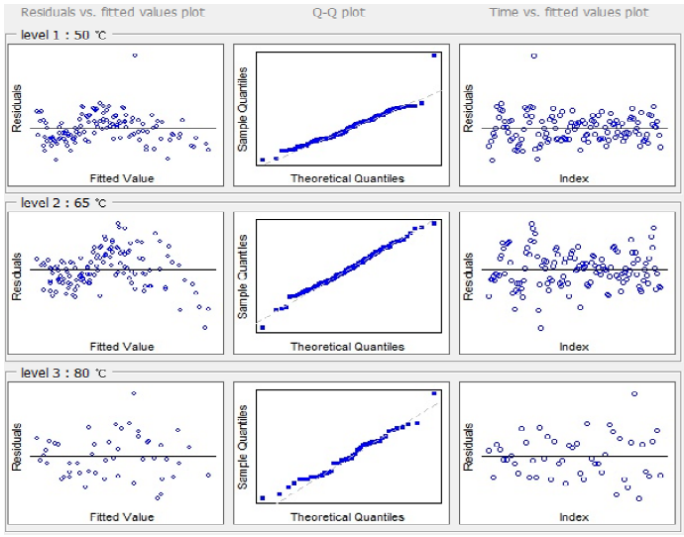


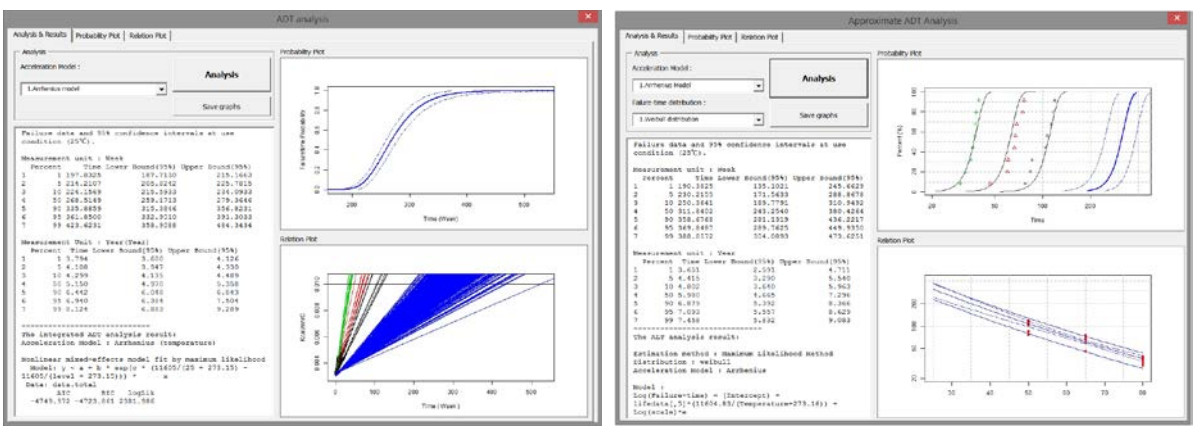
Figure 3: Scatter plots testing homoscedasticity, normality, and independence.

3.4 Reliability analysis

The previous three sections include the preparation steps. This section provides the reliability analysis which is a major function in RExADT. As described in Section 2, RExADT uses single-stage and two-stage approach to analyze ADT data. In the spreadsheet '4. Reliability Analysis', the user can choose "ADT Analysis (Single-stage approach)" or "Approximate ADT Analysis (Two-stage approach)". After selecting one of them, the user needs to enter the percentile of the lifetime distribution of interest, and choose a reference measurement unit. Then, clicking the button, Reliability Analysis, creates a new window according to user's selection. Figure 5 shows the respective analysis windows for each approach.

ADT analysis (Single-stage approach)

As described in Section 2, approximate ADT analysis (two-stage approach) is required to estimate both lifetime of each sample according to acceleration levels and lifetime under an operating condition in two steps. However, ADT analysis allows the program to directly estimate lifetimes under a normal operating condition from degradation data, thus it provides accurate estimate for lifetime distribution. The program primarily provides the ADT analysis using the Arrhenius model. Clicking the button, Analysis allows the program to preprocess the degradation data, to check fitness of the ADT model, and then show quantitative results on the left side and plots on the right side of the window. Similar to approximate ADT analysis, two tables on the top of the results provide the lifetime under the normal operating condition and its confidence interval with a significance level 5% according to a percentile the user specified. In ADT analysis, for example, the fifth percentile of the lifetime under the normal condition (i.e., 25 C) is 214.2107 weeks, and upper and lower bounds of its confidence interval with a significance level 5% are 163.0223 and 265.3990 weeks, respectively. The bottom part of the result also shows the fitness result of the ADT model based on the maximum likelihood method. As shown in Figure 4, a probability plot and a relation plot are provided to show reliability analysis.



(a) (b)

Figure 4: Result windows: (a) ADT analysis and (b) Approximate ADT analysis

Approximate ADT analysis (Two-stage approach)

The current version of the program is allowed to use the Arrhenius model as an acceleration model and the Weibull distribution as a failure-time distribution. Pressing the button, Analysis, enables the program to process the followings in the order: (1) preprocessing data that is obtained from the spreadsheet, 3. Pseudo-failure data, (2) checking fitness of the Arrhenius model and the Weibull distribution, then (3) showing the results.

Two tables on the top of the results show the lifetimes under the normal operating condition, and its confidence interval with a significance level 5%. For example, the fifth percentile of the lifetime under normal operating condition (i.e., 25 C) is 230.2155 weeks and upper and lower bounds of its confidence interval with a significance level 5% are 171.5633 and 288.8678 weeks, respectively. The bottom part of the result shows the fitness of pseudo ALT model based on the maximum likelihood method. As shown in Figure 5(b), a probability plot and a relation plot on the right side of the window enable the user to analyze product reliability easily.

4. Conclusion

Degradation data analysis can improve reliability analysis compared to traditional failure-time analysis (Lu, Meeker, and Escobar 1996) and also provide further information related to failure mechanisms for testing items. Despite its

overall advantage against the traditional failure-time analysis, ADT has been hardly expended to an application context due to the lack of appropriate tool supports. This paper provides an Excel add-in program, called RExADT, for analysis of accelerated degradation tests. RExADT is implemented in Visual Basic for Applications (VBA) and R based on the R-EXCEL environment, which provides a middleware between R and EXCEL and is developed by Baier and Neuwirth (2007). RExADT makes the users with diverse backgrounds to perform ADT analysis easier in a familiar working environment. Using RExADT, the users can enter, edit, and visualize ADT data in spreadsheets, as well as perform ADT analysis through Excel.

Although RExADT presented in this paper mainly uses the linear-Arrhenius ADT model for the single-stage approach and the linear-Weibull model for the two-stage approach, it can be customized further to provide a tailored data analysis depending on degradation data and degradation models. In addition, as aforementioned in Section 3.4, ADT models are nonlinear in nature, an initial parameters of optimization solver within the program might be customized with domain-specific knowledge according to the degradation data types.

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Include author bio(s) of 200 words or less.

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