Robust hub location problem with uncertain inter hub flow discount factor

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Abstract

Hub location problem seeks to find the optimal location of hub facilities and allocate demand points to them for transferring of demands between origin and destination points. This paper is aimed to model the uncapacitated multiple allocation hub location problem in an uncertain environment. First, a deterministic model of uncapacitated multiple allocation hub location is introduced, then a robust optimization approach is used for dealing with uncertain parameters. The mathematical formulation of the considered problem is developed with uncertainty in demands, hub establishment fixed cost and inter hub flow discount factor (α). By means of an uncertainty budget, the level of conservatism is controlled. The counterpart models are compared with each other using well known CAB and AP data sets with different levels of uncertainty. The results show that with increasing of the uncertainty, more hubs will be established with presence of demands uncertainty, while it is decreased when establishment cost or inter hub flow discount factor has uncertainty nature.

Keywords
Hub location; Robust; Uncertainty; Hub establishment cost; Inter hub discount factor.

1. Introduction

Hub location is an important problem in the location literature and widely used in transportation, telecommunications and other applications. When the origin and destination points intend to send commodity to each other, direct path between origin and destination point is chosen for transfer commodity. In transportation problem when number of origin and destination nodes increase as a results a very large of arcs are created that complexity of the model. In hub location problem, one or two intermediate nodes are used to transfer commodity between origins and destinations, that reduces the number of arcs. This may decrease total transportation costs because of existing of the economy of scale property.

The first model in hub location problem was proposed by O'kelly (1987) that introduced the first quadratic mathematical model for hub location problem. In this model, only one hub used in each pathway. While, in the model proposed by Campbell (1994) the number of hubs between each origin and destination are determined based on hub establishment and routing costs. Contreras et al. (2011) proposed a two-stage stochastic programming model for multiple allocation hub location problem by considering uncertain demand and transportation cost. Makui *et al.* (2012) propesed robust optimization for multi-objective capaciteted p-hub location problem when demand and time required for process of commudity are uncertain. Alumur et al. (2012) considered a hub location problem with uncertain demand and hub establishment fixed cost and robust-stochastic model presented to deal with uncetainty related to demand and hub establishment fixed cost uncertainty. Also they proposed a two-stage stochastic programming model when demand is defined as uncertain parameter. Shahabi et al. (2014) proposed robust optimization for single and multiple allocation hub location problem with uncertain demand. They introduced a nonlinear programming that transfer to mixed integer conic quadratic program. The results show that for robust optimization more hub is required in comparison with deterministic model. Ghaffari-Nasab et al. (2015) proposed a robust optimization for capacitated single and multiple allocation hub location problem with uncertain demand. Meraklı et al. (2016) proposed a robust optimization for intermodal multiple allocation *p*-hub location problem. They used benders decomposition algorithm for large-scale problems and the effects of considering uncertainty in the model is examined. Habibzadeh Boukani et al. (2016) presented robust optimization model for single and multiple allocation hub location problem with uncertainty in hub establishment fixed cost and capacity of each hub. They showed that costs increases when uncertainties are not considered in the model. Zetina et al. (2017) proposed robust optimization for uncapacitated multiple allocation hub location problem by considering uncertain demand and transportation cost. Due to considering uncertainty in demand and transportation cost together the proposed model is NP-hard, therefore, branch and cut algorithm is used for solve this model. Martins de Sá et al. (2018) proposed robust optimization for multiple allocation hub location problem with uncertain demand and establishment hub fixed cost. Also, benders decomposition and hybrid heuristic approach are used for solving large scale problems.

In this paper, robust optimization problem is proposed for uncapacitated multiple hub location problem that demands, hub establishment fixed cost and inter hub flow discount factor (α) are considered as uncertain parameters and captured by intervals.

The remainder of this paper is organized as follows: In section 2, deterministic model of uncapacitated hub location is introduces and then this model updated for considering uncertainty with robust formulation. In section 3 computational results are reported for all models and the comparison between them are done. Section 4 summarizes the paper and propose directions for the future study.

2. Mathematical model

In this section, deterministic model and three robust counterparts are presented. The robust models are an extension of the deterministic model proposed by Hamacher et al. (2004).

2.1 Deterministic model

The sets, parameters and decision variables of deterministic model are defined as follows:

Set and Parameters:

N	Set of nodes
w_{ij}	Demand originated at node $i \in N$ and destined to node $j \in N$
f_k	Hub establishment fixed cost at node $k \in N$
d_{ij}	Distance between node $i \in N$ and node $j \in N$
χ	Collection cost per unit
α	Inter hub flow discount factor
δ	Transfer cost

Decision variables:

z_k	1 if a hub established at node $k \in N$, 0 otherwise
x_{ijkl}	The fraction of demand (w_{ij}) originated at node $i \in N$ and destined to node $j \in N$ routed via hub $k \in N$ to hub $l \in N$

The deterministic model of multiple hub location problem is as follows:

$$\min \sum_k f_k z_k + \sum_i \sum_k \sum_l \sum_j w_{ij} c_{ijkl} x_{ijkl} \tag{1}$$

Subjected to:

$$\sum_k \sum_l x_{ijkl} = 1 \qquad \forall i \in N, j \in N \qquad (2)$$

$$\sum_l x_{ijkl} + \sum_{l, l \neq k} x_{ijlk} \leq z_k \qquad \forall i \in N, j \in N, k \in N \qquad (3)$$

$$z_k \in \{0,1\} \qquad \forall k \in N \qquad (4)$$

$$x_{ijkl} \geq 0 \qquad \forall i \in N, j \in N, k \in N \qquad (5)$$

The objective function consists of hub establishment and transportation costs. Constraints 2 ensure that demands are fully transmitted. Constraints 3 prevent direction between non-hub nodes. Constraints 4 and 5 are the standard integrality and non-negativity constraints.

In the above model, it's assumed that all parameters are known in the planning time. While, in real condition some parameters are uncertain. These uncertainties are taken into account by robust optimization. In this paper the approach proposed by Bertsimas et al. (2003) is used to reformulate deterministic model to robust counterparts.

2.2 Uncertain demands (uhlp-d)

Demands are assumed to have an interval uncertainty and defined as $[w_{ij}^l, w_{ij}^l + w_{ij}^\Delta]$ where w_{ij}^l and $w_{ij}^\Delta \geq 0$ are nominal and deviation values, respectively. Γ_d denotes uncertainty budget that determine the maximum number of demand which are defined as uncertain parameter. The parameter s_w denote a subset of demand that are uncertain. With polyhedral uncertainty set, robust model can formulate as follows:

$$\min \sum_k f_k z_k + \sum_i \sum_k \sum_l \sum_j w_{ij}{}^l c_{ijkl} x_{ijkl} + \max_{s_w \subset w_{ij} : s_w \leq \Gamma_d} \left\{ \sum_i \sum_k \sum_l \sum_j w_{ij}{}^\Delta c_{ijkl} x_{ijkl} \right\}$$

Subjected to:

(2) -(5)

The first two terms of above formulation minimize total cost of hub location problem that equal to deterministic model and third term (min-max) is cost of transfer deviation due to demand uncertainty that maximum cost should be minimized. A binary variable $u_{ij} \in \{0,1\}$ is defined for reformulation of previous model. The new model is as follows:

$$\min \sum_k f_k z_k + \sum_i \sum_k \sum_l \sum_j w_{ij}{}^l c_{ijkl} x_{ijkl} + \max \left\{ \sum_i \sum_k \sum_l \sum_j w_{ij}{}^\Delta c_{ijkl} x_{ijkl} u_{ij} \right\}$$

Subjected to:

$$\sum_i \sum_j u_{ij} \leq \Gamma_d \qquad (6)$$

$$0 \leq u_{ij} \leq 1 \qquad (7)$$

(2)-(5)

because constraints (6) is totally unimodular, integrality variable of u_{ij} is discounted. μ and λ_{ij} denote the dual variables corresponding to constraints (6) and (7). The dual of previous model is as follows:

$$\min \sum_k f_k z_k + \sum_i \sum_k \sum_l \sum_j w_{ij}{}^l c_{ijkl} x_{ijkl} + \sum_i \sum_j \lambda_{ij} + \mu \Gamma_d$$

Subjected to:

$$\lambda_{ij} + \mu \geq \sum_k \sum_l w_{ij}{}^{\Delta} c_{ijkl} x_{ijkl} \qquad \forall i \in N, j \in N \qquad (8)$$
$$\lambda_{ij} \geq 0 \qquad \forall i \in N, j \in N$$
$$\mu \geq 0$$
(2)-(5)

2.3 Uncertain hub establishment fixed costs (uhlp-e)

Fix hub establishment costs are assumed to have an interval uncertainty and defined as $[f_k^l, f_k^l + f_k^{\Delta}]$ where f_k^l and $f_k^{\Delta} \geq 0$ are nominal and deviation values, respectively. Γ_f denotes uncertainty budget that determine the maximum number of hub establishment cost which are defined as uncertain parameter. The parameter s_f denote a subset of hub establishment cost that are uncertain. With polyhedral uncertainty set, robust model can formulate as follows:

$$\min \sum_k f_k^l z_k + \sum_i \sum_k \sum_l \sum_j w_{ij} c_{ijkl} x_{ijkl} + \max_{s_f \subset f_k : s_f \leq \Gamma_f} \left\{ \sum_k f_k^{\Delta} z_k \right\}$$

Subjected to:

(2)-(5)

The first two terms of the above formulation minimize total cost of hub location problem that equal to deterministic model and third term (min-max) is fixed cost of establishing hub deviation that maximum cost should be minimized. A binary variable $u_k \in \{0,1\}$ is defined for reformulation of previous model. The new model is as follows:

$$\min \sum_k f_k^l z_k + \sum_i \sum_k \sum_l \sum_j w_{ij} c_{ijkl} x_{ijkl} + \max \left\{ \sum_k f_k^{\Delta} z_k u_k \right\}$$

Subjected to:

$$\sum_k u_k \leq \Gamma_f \qquad (9)$$
$$0 \leq u_k \leq 1 \qquad (10)$$

(2)-(5)

because constraints (9) is totally unimodular, integrality variable of u_k is discounted. μ and λ_k denote the dual variables corresponding to constraints (9) and (10). The dual of previous model is as follows:

$$\min \sum_k f_k^l z_k + \sum_i \sum_k \sum_l \sum_j w_{ij} c_{ijkl} x_{ijkl} + \sum_k \lambda_k + \Gamma_f \mu$$

Subjected to:

$$\lambda_k + \mu \geq f_k z_k \qquad \forall k \in N \qquad (11)$$
$$\lambda_k \geq 0 \qquad \forall k \in N$$
$$\mu \geq 0$$
(2)-(5)

2.4 Uncertain inter hub flow discount factor (uhlp-α)

Inter hub flow discount factor are assumed to have an interval uncertainty and defined as $[\alpha_{kl}^l, \alpha_{kl}^l + \alpha_{kl}^{\Delta}]$ where α_{kl}^l and $\alpha_{kl}^{\Delta} \geq 0$ are nominal and deviation values, respectively. Γ_{α} denotes uncertainty budget that determine the maximum number of inter hub flow discount factor which are defined as uncertain parameter. The parameter s_α denote a subset of inter hub flow discount factor that are uncertain. With polyhedral uncertainty set, robust model can formulate as follows:

$$\min \sum_k f_k\, z_k + \sum_i \sum_k \sum_l \sum_j w_{ij} x_{ijkl} (\chi d_{ik} + \alpha^l_{kl} d_{kl} + \delta d_{lj}) + \max_{s_\alpha \subset \alpha_{kl} : s_\alpha \le \Gamma_\alpha} \left\{ \sum_i \sum_k \sum_l \sum_j w_{ij} x_{ijkl} (\alpha^\Delta_{kl} d_{kl}) \right\}$$

Subjected to:

(2)-(5)

The first two terms of the above formulation minimize total cost of hub location problem that equal to deterministic model and third term (min-max) is cost of transfer deviation due to inter hub flow discount factor uncertainty that maximum cost should be minimized. A binary variable $u_{kl} \in \{0,1\}$ is defined for reformulation of previous model. The new model as follows:

$$\min \sum_k f_k\, z_k + \sum_i \sum_k \sum_l \sum_j w_{ij} x_{ijkl} (\chi d_{ik} + \alpha^l_{kl} d_{kl} + \delta d_{lj}) + \max \left\{ \sum_i \sum_k \sum_l \sum_j w_{ij} x_{ijkl} (\alpha^\Delta_{kl} d_{kl})\, \mathrm{u}_{kl} \right\}$$

Subjected to:

$$\sum_k \sum_l u_{kl} \le \Gamma_\alpha \qquad (12)$$

$$0 \le u_{kl} \le 1 \qquad (13)$$

(2)-(5)

because constraints (12) is totally unimodular, integrality variable of u_{kl} is discounted. μ and λ_{kl} denote the dual variables corresponding to constraints (12) and (13). The dual of previous model is as follows:

$$\min \sum_k f_k\, z_k + \sum_i \sum_k \sum_l \sum_j w_{ij} x_{ijkl} (\chi d_{ik} + \alpha^l_{kl} d_{kl} + \delta d_{lj}) + \sum_k \sum_l \lambda_{kl} + \Gamma_\alpha \mu$$

Subjected to:

$$\lambda_{kl} + \mu \ge \sum_i \sum_j w_{ij} x_{ijkl} (\alpha^\Delta_{kl} d_{kl}) \qquad \forall k \in N, l \in N \qquad (14)$$

$$\lambda_{kl} \ge 0 \qquad \forall k \in N, l \in N$$

$$\mu \ge 0$$

(2)-(5)

3. Computational analysis

In this section numerical examples are used to show validity of proposed models. In section 3.1 the impact of uncertainty level on the optimal solution is analyzed and in section 3.2 the effect of the uncertainty budget on the optimal solution is examined. The importance of considering uncertainty is shown in section 3.3. For this computational analysis two different well known CAB and AP data sets are considered. Due to lack of hub establishment fixed cost in data sets, the following formulation is used to calculate the fixed cost of establishing each hub nodes that was proposed by Correia et al. (2018):

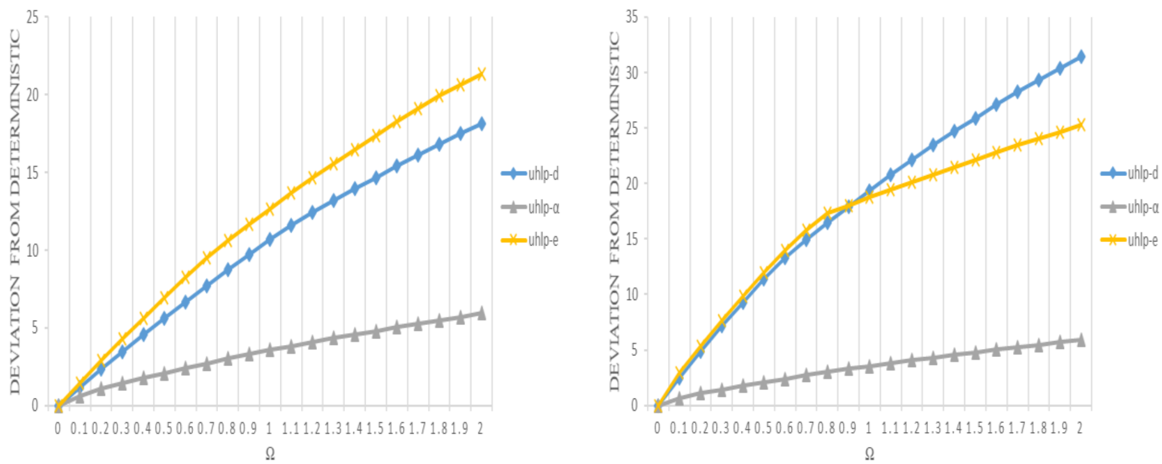
$$o_k = \sum_j w_{kj} \qquad \forall k \in N$$

$$f_k = 3500 \times \log(o_k) \qquad \forall k \in N$$

The proposed mathematical models are solved by using GAMS software, and run in an Intel Core i5 with 2.5 GHz CPU and 6 GB of RAM. it is assumed that $\chi = \delta = 1$ and for economic use of two hub for a path α is equal to 0.2. Parameter Ω is defined as the maximum possible variation of each parameter that the deviation parameter for example is defined as $w_{ij}^{\Delta} \sim U[0, \Omega w_{ij}^{l}]$.

3.1 Impact of uncertainty level on the optimal solution

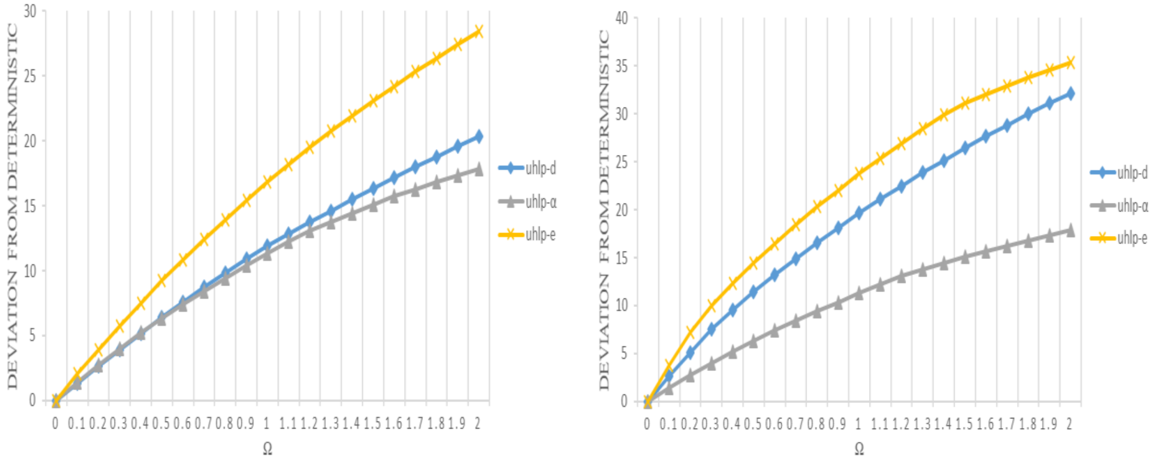
In this section, each of the robust counterparts is solved in different level of uncertainties and it's assumed that $\Omega \in \{0.1, 0.2, ..., 2\}$. The cost of each robust counterpart for any Ω is calculated and its deviation from the deterministic model is obtained and reported in figure 1 and 2. Figures 1 and 2 consider 20 nodes in AP and 25 nodes in CAB, respectively. Each of these robust counterparts is solved when the uncertainty budget is equal to 5% and 15%, which these results are shown in Figure 1, 2 (a) and 1, 2 (b), respectively. Figure 1(a) shows that uhlp-e and uhlp-d have more sensitivity to the level of uncertainty compared to uhlp-α and the cost deviation of uhlp-e is more than the other two robust counterparts. Figure 1(b) shows that when $\Omega = 1$, uhlp-e has more cost than uhlp-d, but when Ω is greater than 1, the cost of uhlp-d is more than uhlp-e. The uhlp-α has identical trend in Figure 1 (a) and (b), it means that the costs of it is not affected by different values of budget. The reason for this condition is that all parameters with uncertainty α are selected in the budget of 5% and the increase in the budget will not affect it. When the proper number of hubs is created, the parameters of the α are unique to the path between selected hubs. For example, if from 20 nodes, 3 nodes are selected as hubs in the model, the number of uncertain parameters are equal to 6.



(a) 5% budget (b) 15% budget

Figure 1. effect of uncertainty level on the optimal solution for 20 AP nodes

Figure 2 (a) have the same result as figure 1 (a) but in figure 2 (b) the deviation cost of uhlp-e model is more than the deviation cost of uhlp-d. In table 1 the optimal hub configuration for different levels of uncertainty for 20 AP nodes are reported. More hubs are established by increasing in the uncertainty in the presence of uncertain demand, while it is decreased when establishment cost or inter hub flow discount factor has uncertain nature.



(a) 5% budget (b) 15% budget

Figure 2. effect of uncertainty level on the optimal solution for 25 CAB nodes

Table 1. optimal hub configuration for different level of uncertainty for 20 AP nodes

Ω	Uhlp-d	Uhlp-α	Uhlp-e	Ω	Uhlp-d	Uhlp-α	Uhlp-e
0-0.1	2,9,18	2,9,18	2,9,18	0	2,9,18	2,9,18	2,9,18
0.2-0.7	2.9.18	8,18	2.9.18	0.1	2,9,18	2,9,18	8,18
0.8-1.1	2,9,18	8,18	2,14,17	0.2-0.5	2,9,18	8,18	8,18
1.2-1.7	2,8,15,18	8,18	2,14,17	0.6-0.7	2,8,15,18	8,18	8.18
1.8-2	2,8,15,18	8,18	3,12,14	0.8-2	2,8,15,18	8,18	13
		5% budget				15% budget	

3.2 Impact of the uncertainty budget on the optimal solution

In this section, each robust counterpart solved with different uncertainty budget that $\Gamma \in \{0.05s,...,1s\}$ and $s \in \{s_w, s_f, s_\alpha\}$.

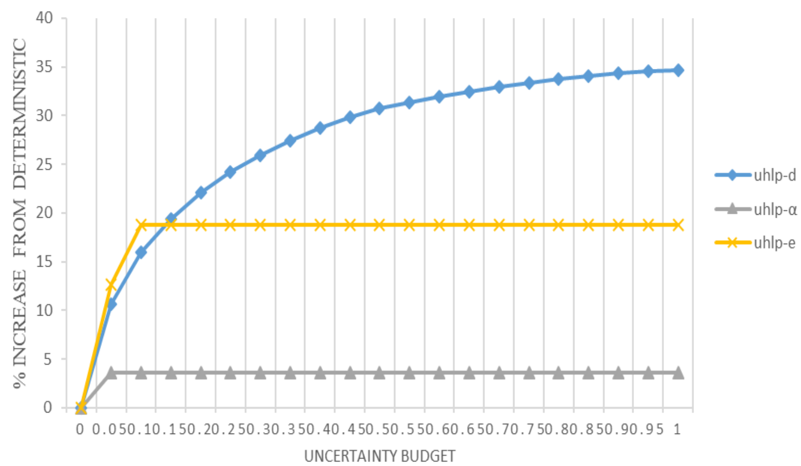


Figure 3. effect of uncertainty budget on the optimal solution for 20 AP nodes

Figure 3 shows that the cost of uhlp-d increases by increasing in uncertainty budget, because of selecting more uncertain demand parameters. For uhlp-α when uncertainty budget is more than 5% and for uhlp-e when uncertainty budget is more than 10% the cost dose not increase. This behavior is due to choosing of all parameters as uncertain ones in previous budgets.

3.3 The importance of considering uncertainty

In this section, each of the robust counterpart are solved by the uncertainty budget of 15% and Ω is equal to 2. Then, in the new robust model (named fixed robust model) the variables *z* and λ are considered as parameters and their values are equal to the values that are obtained by solving robust model. Also, the deterministic model is solved and the values of its decision variables, including *z* and *x*, are determined. Then, in the new deterministic model (named fixed deterministic) the variables *z* is considered as parameters and their values are equal to the values that are obtained by solving deterministic model. λ values are used in fixed determinist model. It's assumed that two new model (fixed robust and fixed deterministic) are face to uncertainty and solve with different value of $\Omega \in \{0.04, 0.08, ..., 1.96\}$ and it's results shown in figures 4, 5 and 6 . The amount of demand in new deterministic model is equal to its nominal value plus deviation value that obtained from solving robust model (λ). The amount of hub establishment fixed cost in fixed deterministic model is equal to its nominal value plus its deviation value for hubs that fixed at the model and the amount of α in fixed deterministic is equal to its nominal value (0.2) plus its deviation for arcs between hubs that fixed at the model.

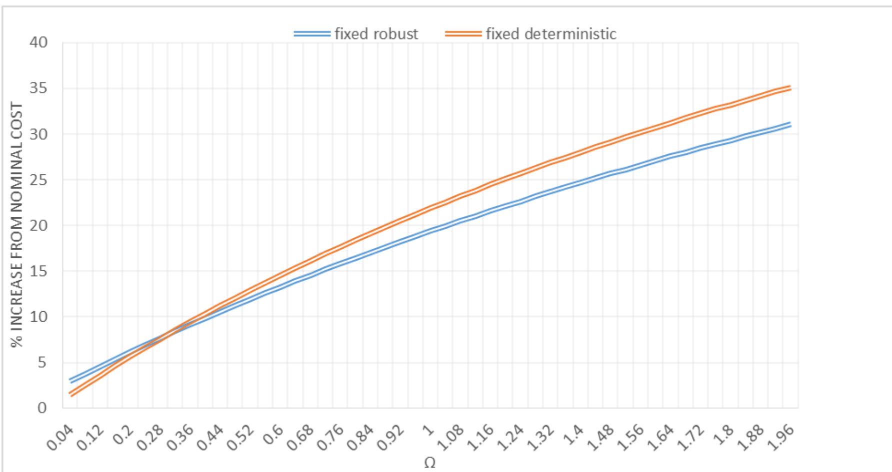


Figure 4. importance of considering uncertainty for 20 AP nodes and uncertainty in demands

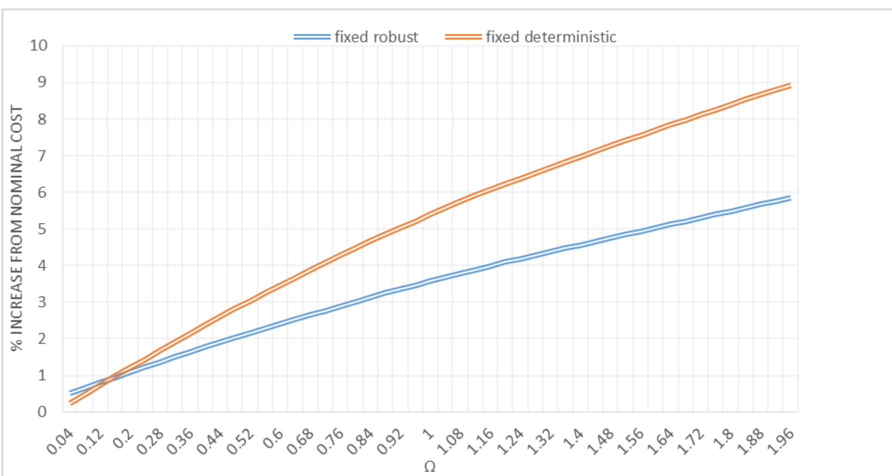


Figure 5. importance of considering uncertainty for 20 AP nodes and uncertainty in inter hub flow discount factor

Figure 4 shows the results for 20 AP nodes with uncertain demand. Until Ω is equal to 0.28 the cost of fixed deterministic is lower than fixed robust model. It means that deterministic decision is better than robust decision when Ω is smaller than 0.28, but when Ω increases, the fixed robust model have lower cost in comparison with fixed deterministic model. Figure 5 shows the results for 20 AP nodes with considering uncertainty in α. Until Ω is equal to 0.12 the cost of fixed deterministic is lower than fixed robust model. Figure 6 shows results for 20 AP nodes with hub establishment fixed cost uncertainty. Until Ω is equal to 0.44 the cost of fixed deterministic is lower than fixed robust model. By comparing between Figures 4, 5 and 6, the results show that considering the nature of robust for uncertain fixed cost is more important and more effective than others like demand and discount factor. It means that in case of uncertainty of fixed costs, the robust model should be definitely considered instead of deterministic form.

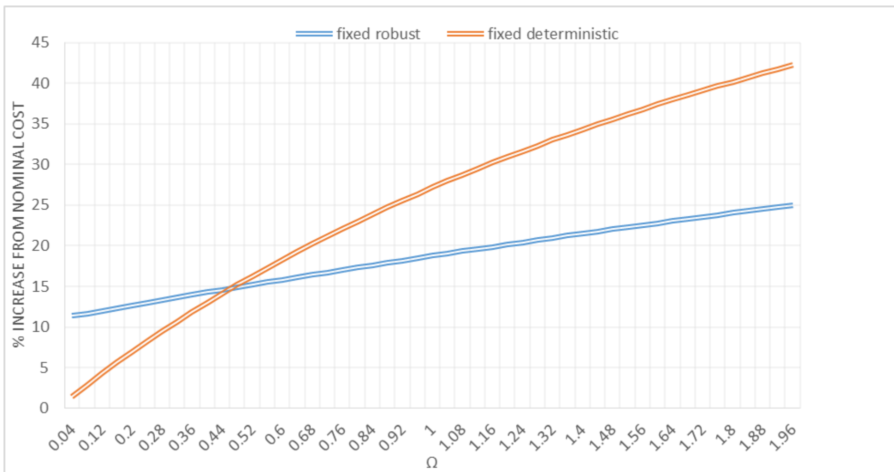


Figure 6. importance of considering uncertainty for 20 AP nodes and uncertainty in hub establishment fixed cost

4. Conclusion

In this paper, three robust multiple allocation hub location models are introduced. Demands, hub establishment fixed costs and inter hub flow discount factor (α) are considered as uncertain parameters. These models are solved by considering different levels of uncertainties and different uncertainty budgets. Results show that more hubs are needed in the presence of uncertain demands, while it is decreased when establishment cost or inter hub flow discount factor has an uncertain nature. Decision maker can set the level of uncertainty budget and can increase or decrease the level of risk. Level of risk is increased when uncertainty budget has value close to zero and conversely. If uncertainty is not considered in the model and later this appear at the model have a lot cost because of wrong decision at the first. It is interesting for future research to consider combination of uncertain parameter in the model instead of considering individual uncertain parameters. Using decomposition based algorithms to solve medium scale problems can be another direction for future studies.

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