# An EOQ Inventory Model for Items Consisting of Components\*

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**Abstract:** A deterministic inventory model with constant demand and return on the base of economic order quantity (EOQ) is considered. We consider a firm that can manufacture new products and recover the value of a used product through remanufacturing with dismantling for components. The firm provides product at a constant demand rate. Product consists of two components. Each part is manufactured separately and placed in inventory, then two components are assembled. The dismantling operation yields two spare components. Products are returned according to the return rate, other products are disposed of. The returned product is dismantled for components, any part is inspected whether it is usable or not and then is placed in inventory. Both components are usable at the different rate. The usable components are then remanufactured or directly reused, other components are disposed of.

Keywords: reverse logistics, remanufacturing, EOQ.

# **1** Introduction

In recent years reverse logistics is receiving increasing attention from both academia and industry. There is increasing recognition that careful management can bring both environmental protection and lower costs: environmental and economic considerations have led to manufacturers taking their products back at the end of their lifetime. As a result reverse logistics process is now considered as a basis for generating real economic value, as well as support of environmental concerns.

Rogers and Tibben-Lembke [27] defined Reverse logistics as the process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal. Integration of forward and reverse supply chain resulted in origination of the concept of a closed-loop supply chain. The whole chain can be designed in such a way that it can service both forward and reverse processes efficiently.

One of the last most full review of quantitative modeling for inventory and production planning in closed-loop supply chain was made by Akcaly and Cetinkaya [1].

As an example of closed-loop supply chain Souza [32] considers Cummins, the original equipment manufacturer (OEM) of diesel engines based in Columbus, Indiana. Forward flows consist of new engines and/or engine components (such as a water pump or a turbocharger), and reverse flows consist of used products, and remanufactured products. For a diesel engine or part, remanufacturing consists of six different steps: full disassembly, thorough cleaning of each part, making a disposition decision for each part (remanufacture it or recycle it for materials recovery), refurbishing components to restore their functionality to that of a new part, reassembly, and testing. Remanufactured engines or components sell at a 35% discount relative to the corresponding new engine or module. Upon purchasing a Cummins product, customers receive a discount if they return their old product. Used products (also known as cores, or returns) are shipped from dealers to Cummins' depot for used

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products. At the depot, customers are given credit for returning the used product, and products are then shipped to one of two plants: engine remanufacturing (plant A), or module remanufacturing (plant B). Remanufactured engines are shipped from plant A to the main distribution center, for distribution to the dealers. Remanufactured components are shipped from plant B to either the distribution center, or to the engine remanufacturing plant A, depending on forecasts and current needs. Used components not suited for remanufacturing are sold to recyclers.

The next example considers product recovery activities at IBM. In its remanufacturing facility, IBM remanufactures or dismantles returned servers and storage systems. While remanufactured products are sold at a restricted price to meet demand for used products, components obtained through dismantling can be used to meet internal or external demand for spare components [12],[9].

The growth in automotive production has increased the number of end-of-life vehicles (ELVs) annually. ELV processing is similar worldwide (Ferrão and Amaral [10]) (Fig.1). For instance, there are many companies, that offer its services on dismantling cars, bikes and other vehicles for components. After a damaged car has been purchased at the auction, it is sent to warehouse of car components. Here the damaged car (or other vehicle) is dismantled for car components, then car components are packaged for shipping, after they are loaded into containers and sent to the port for their further shipping. The priority area of these companies is buying of damaged vehicles and their cut, dismantling for car components. Usually prices for damaged cars put up for sale at insurance car auctions are significantly lower in comparison to market prices. It should be noted that many car components in damaged cars remain in a good condition. By buying such a vehicle the buyer gets good quality car components at a low price.

Among all countries, the dismantling process in the United States is the most advanced. The United States has been studying and developing remanufacturing techniques for ELV components for more than 30 years. The United States has formed a large-scale system for decommissioning and remanufacturing ELV components, which is becoming the primary source of profits in ELV recycling industry. The remanufacturing of automotive components has become the largest remanufacturing industry in the United States in terms of number of companies, number of employees, and contribution to the economy. This industry caters to passenger vehicles, commercial vehicles, and special vehicles by remanufacturing components such as engines, transmission components, clutches, steering wheels, air conditioning compressors, starting dynamos, generators, wiper motors, water pumps, oil pumps, and brake booster pumps[34].



Figure 1: End-of-life vehicles processing [10].

Thus, in developed countries these products are prepared with remanufacturing policies and strategies in place. However, in developing countries, remanufacturing is still in the initial stages and is not a virtual application. Amelia et al. [2] cited the significant difference between EOL vehicle management strategies in developing and developed countries, noting that China, India, and Brazil are struggling with remanufacturing implementation. The paper [35] analyzes internal barriers met by automotive components remanufacturers if China and evaluates causal barriers by a proposed model framework [35]. The paper [11] provides a brief road map and insights into future research for

remanufacturing specifically in an Indian context.

In this paper an EOQ inventory model with remanufacturing and dismantling for components is considered. According to [1] inventory models are divided into two main categories: deterministic and stochastic according to modeling demand and return processes. The subject of this paper is deterministic inventory model on the base of EOQ.

The economic order quantity model (EOQ model), which was derived by Ford W. Harris in 1913, became basis for many reverse logistics models because of its simplicity and intelligibility. Paper [3] represents most detailed review devoted to the work on the EOQ problem.

Shrady was the first who applied EOQ model to reverse logistics processes, he introduced an EOQ model with instantaneous production and repair rates. A closed-form solution was developed [31]. In his work an efficient policy P(m,1) was established which is characterized by the fact that within each remanufacturing cycle a number m of remanufacturing lotes of equal size succeed exactly one manufacturing lot.

This work was extended by Nahmias and Rivera [22]. Mabini et al. [21] extended Shrady model to a multi-item case. Koh et al. [15] analyzed model similar to Shrady [31] with some differences.

Teunter [33] generalized the results of Schrady in a way that he examined different structures of a remanufacturing cycle. He considered different types of policy, alternating n manufacturing lotes with m recovery lotes, and concluded that the policy P(m,n), m > 1, n > 1 will never be optimal, if both m and n are simultaneously more than one, where m - quantity of remanufacturing lotes, n - quantity of manufacturing lotes, there are two possible policy: P(1,n) and P(m,1).

Choi et al. [4] generalized P(m,n) policy of Teunter by treating an ordered sequence of manufacturing and remanufacturing lotes within a cycle as a decision variable. Through the sensitivity analysis they found that only 0,2% out of the 8100000 tested problems has optimal solution in which both m and n are greater than one. Liu et al. [20] generated and solved 60,000 problems and found that only 0.19% of them have an optimal solution in P(m,n), which both m and n are greater than one. Konstantaras and Papachristos [17] evolved Teunter's work by developing an exact solution that leads to the optimal number of manufacturing and remanufacturing lotes and their corresponding lot sizes.

Richter became author of series of papers, where he considered EOQ-model from the point of view of waste disposal problem. Richter [23] proposed an EOQ-model which differs from Shrady, who assumed a continuous flow of used products to manufacturer. Richter [23] assumed the system of two shops: the first shop provided a product used by a second shop, the first shop manufactures new products and repairs products already used by the second shop and collected there according some rate, other products are disposed according disposal rate. At the end of time interval the collected items are brought back to the first shop. Richter [25] in his paper has examined the optimal inventory holding policy, if the waste disposal (return) rate is a decision variable. The result of this paper is that the optimal policy has an extremal property: either reuse all items without disposal or dispose off all items and produce new products. Also Richter proves that a policy of the type P(m,n) with m > 1 and n > 1 is never optimal. He also derives closed-form expressions for the optimal policy parameters. The analysis of repair and waste disposal model was continued in [24], [25], [26], [5].

In papers [6], [7] Dobos and Richter investigated a production/recycling system with constant demand that is satisfied by non-instantaneous production and recycling. The result of this paper is that it is optimal either to produce or to recycle all bought back items. In paper [8] Dobos and Richter extended their previous work by considering the quality of returned items.

El Saadany and Jaber [29] argued that such a pure policy of no waste disposal is

technologically infeasible and suggested demand-like function to depend on two decision variables which are the purchasing price and the acceptance quality level when analysing such systems.

Ahmed M.A. El Saadanya, Mohamad Y. Jaber, Maurice Bonney [30] regarded as unrealistic assumption that an item can be recovered indefinitely: material degrades in the process of recycling losing some of its mass and quality making the option of 'multiple recovery' somewhat infeasible. The paper [30] develops a model where an item is recovered a finite number of times.

Some authors extended the above mentioned models for varying assumptions, one option is to allow for backorders, where some customers are compensated for waiting for their delayed orders by either a reduction in price or some other forms of discount, which is a cost incurred by the supplying firm. This results in a backorder cost. Konstantaras and Scouri [18] considered two models with no shortage case and shortage case. Both models are considered for the case of variable setup numbers of equal sized lotes for production and remanufacturing processes. For these two models sufficient conditions for the optimal type of policy, referring to the parameters of the models, are proposed.

Saadany and Jaber in their work [14] extended the work of Richter [23] by assuming that demand for manufactured items is different from that for remanufactured (repaired)ones. This assumption results in lost sales situations where there are stock-out periods for manufactured and remanufactured items, i.e., demand for newly manufactured items is lost during remanufacturing cycles and vice versa. Konstantaras and Papachristos [16] extended the work of Richter [23] by allowing for planned backorders in remanufacturing and production, while keeping the other assumptions the same. The paper [13] extends the work of Jaber and El Saadany [14] for the full-backorder and partial-backorder cases, where recovered items (remanufacture or repaired) are perceived by customers to be of lower quality; i.e., not as-good-as new.

In paper Konstantaras, Scouri and Jaber[19], which extends the work of [15], a combined inspection and sorting process is introduced with a fixed setup cost and a unit variable cost. This paper assumes that remanufactured and new purchased items are sold in a primary market, while refurbished units are sold in a secondary market. In this paper two types of policy are developed. The first case considers a policy of a single inspection and sorting and a single recovery (remanufacturing and refurbishing) lot, and multiple lotes of new items P(1,n), the second case considers a policy of multiple lotes of recovery and of inspection and sorting, and a single lot of new items P(m,1).

In paper Saadany and Jaber [28] the extended the EOQ production, repair and waste disposal model [23] was modified to show that ignoring the first time interval results in an unnecessary residual inventory and consequently an over estimation of the holding costs. Also paper [28] accounts for switching costs (e.g., production loss, deterioration in quality, additional labor). When shifting from producing (performing) one product (job) to another in the same facility, the facility may incur additional costs refereed to as switching costs switching costs when alternating between production and repair runs. Hence paper [28] accounts also for the switching costs which integrate production loss, deterioration in quality, additional labor, etc.

The main peculiarity of the paper [28] is accounting of switching costs (e.g., production loss, deterioration in quality, additional labor). Our paper generalizes the approach of Saadany and Jaber[28]. In the model three types of costs are considered. First, the EOQ non related cost, which is independent on the numbers of lots and lot sizes (the manufacturing cost), the EOQ related cost, which depends on dynamics of the inventory, the lot sizes and numbers of lots (the holding cost), and the EOQ related cost, which depends on the numbers of lots and production scheduling (the switching cost). We study an EOQ inventory model with remanufacturing and dismantling for components. We consider a firm that can manufacture new products and recover the value of a used product through remanufacturing with dismantling for components. The firm provides product, which consists of two components are assembled. Also each part can be directly reused, if it is in good technical

#### condition.

To determine the optimal production and remanufacturing policy means to find the optimal numbers of production and remanufacturing lots of each part for the minimization of the total cost. The optimal policy is found depending on the parameters of the model. For the solution of the problem a theorem was proved that provides solution to the certain class of deterministic inventory models with constant demand and return. The theorem can be used for the complete solution of some of above mentioned models.

This paper is organized as follows: the first section contains introduction, in the second section the formulation and analysis of an EOQ inventory model with remanufacturing and dismantling for components is considered, in the third section the theorem is derived which gives closed-form expressions for the optimal policy parameters and can be used for other various remanufacturing problems, the fourth section contains the numerical example, the last section of the paper is conclusion.

# **2 Model formulation**

We consider a firm that receives recoverable product from the market. The firm can manufacture new products and recover the value of a used product or return through remanufacturing with dismantling for components. The firm provides product at a constant demand rate of d items per time unit. Product consists of two components, denoted as part 1 and part 2. Each part is manufactured separately and placed in inventory (SS1 - serviceable stock inventory for part 1, SS2 serviceable stock inventory for part 2), then two components are assembled with the cost  $c_{A}$  and are sold in a market. Products are returned to the firm according the rate  $\beta$ , other products are immediately disposed of at the rate  $\alpha = 1 - \beta$ . The dismantling operation costs  $c_D$ . Returned product is dismantled for components, any part is inspected whether it is usable or not, and then is placed in inventory (RS1 – inventory for returned stock of part 1, RS2 – inventory for returned stock of part 2). Part 1 is not usable at the rate  $q_1$  and should be remanufactured, the rest  $\beta_1 - q_1$  are as good as new and directly reused, part 2 isn't usable at the rate  $q_2$ . The Fig. 1 represents the integrated closed-loop supply chain inventory system. The sequence of production activities is the following: in any time cycle [0,T] demand for part 1 and part 2 is satisfied firstly through usable components, then through remanufacturing of used components and at last manufacturing of new components. All activities in the model are supposed to be instantaneous and lot-for-lot. The production activities of each part are evaluated on separate production lines.



Figure 2: The integrated closed-loop supply chain inventory system

# Assumptions

This paper assumes:

(1) infinite manufacturing and recovery rates,

(2) remanufactured items are as good as new,

(3)demand is known, constant and independent,

(4) lead time is zero,

(5) a case of a single product, which consist of two components

(6) no shortages are allowed,

(7) unlimited storage capacity is available, and

(8) infinite planning horizon.

Notations:

 $c_M^1$  – part 1 unit manufacturing cost

 $c_M^2$  – part 2 unit manufacturing cost

 $c_R^1$  – part 1 unit remanufacturing cost

 $c_R^2$  – part 2 unit remanufacturing cost

 $c_D$  – dismantling operation cost

 $c_A$  – assembling operation cost

d – constant demand rate

[0,T] – time cycle interval

 $\beta$  – percentage of returned items

 $\alpha = 1 - \beta$  – disposal rate

 $q_1$  – percentage of not usable returned components of type 1

 $q_2$  – percentage of not usable returned components of type 2

 $Q_1^n$  – manufacturing lot size for part 1

 $Q_2^n$  – manufacturing lot size for part 2

 $Q_1^m$  – lot size for directly reused part 1

 $Q_2^m$  – lot size for directly reused part 2

- $Q_3^m$  remanufacturing lot size for part 1
- $Q_4^m$  remanufacturing lot size for part 2
- $H_1$  holding cost for SS1 per item per time unit
- $H_2$  holding cost for SS2 per item per time unit
- $h_1$  holding cost for RS1 per item per time unit
- $h_2$  holding cost for RS2 per item per time unit
- $R_1$  fixed inspection cost for lot of usable part 1
- $R_2$  fixed inspection cost for lot of usable part 2
- $R_3$  fixed inspection cost and remanufacturing setup cost for part 1
- $R_4$  fixed inspection cost and remanufacturing setup cost for part 2
- $S_1$  manufacturing setup cost for part 1
- $S_2$  manufacturing setup cost for part 2

 $P_1$  – the total switching costs for part 1, which include machine start-up, when remanufacturing is started and machine adjustment, when remanufacturing is switched to manufacturing

 $P_2$  – the total switching costs for part 2, which include machine start-up, when remanufacturing is started and machine adjustment, when remanufacturing is switched to manufacturing.

In this model setup and switching cost are differentiated. The setup cost incurred every time a manufacturing or remanufacturing of next lot is started, and the switching costs are incurred when the activity is changed, for example, remanufacturing is changed to remanufacturing and on the contrary. If a machine is shut down in the middle of a production run of one item, a setup cost is incurred when production is resumed, but no switching costs are incurred. Switching costs are defined as the costs incurred whenever two consecutive jobs do not share the same features. Switching costs may include cleaning cost, machine adjustment/fine tuning cost, changing product family, changing production supplies, equipment start-up/shutdown.

This paper assumes demand is supplied by dT of part 1 and dT of part 2 per time interval [0,T], which are assembled together. The quantity of dT of first components are accomplished through  $\alpha_1 dT$  of newly manufactured items in  $n_1$  lots of size  $Q_1^n$ ,  $(\beta_1 - q_1)dT$  of directly reused items in  $m_1$  lots of size  $Q_1^m$  and  $q_1 dT$  of remanufactured items in  $m_3$  lots of size  $Q_3^m$ . Similarly dT of second components are accomplished through  $\alpha_2 dT$  of newly manufactured items in  $n_2$  lots of size  $Q_2^m$ ,  $(\beta_2 - q_2)dT$  of directly reused items in  $m_2$  lots of size  $Q_2^m$  and  $q_2 dT$  of remanufactured items in  $m_2$  lots of size  $Q_2^m$  and  $q_2 dT$  of remanufactured items in  $m_4$  lots of size  $Q_4^m$ . The following system of equations is fulfilled:

$$n_{1}Q_{1}^{n} = \alpha_{1}dT$$

$$n_{2}Q_{2}^{n} = \alpha_{2}dT$$

$$m_{1}Q_{1}^{m} = (\beta_{1} - q_{1})dT$$

$$m_{2}Q_{2}^{m} = (\beta_{2} - q_{2})dT$$

$$m_{3}Q_{3}^{m} = q_{1}dT$$

$$m_{4}Q_{4}^{m} = q_{2}dT$$

$$\alpha_{1} + q_{1} + (\beta_{1} - q_{1}) = 1$$

$$\alpha_{1} + q_{2} + (\beta_{1} - q_{2}) = 1$$
(1)

We divide all costs of the firm over [0,T] into three groups:

(1) EOQ non related cost, which doesn't depend on the numbers of lots and lot sizes at all, i.e. manufacturing cost, remanufacturing cost, assemble and dismantle cost. It is assumed that EOQ non related costs in the model are proportional to the quantity or product dT, i.e. T.

(2) EOQ related cost, which depends on dynamics of the inventories, lot sizes and numbers of lots, i.e. holding cost.

(3) EOQ related cost, which depends on the numbers of lots and production scheduling, i.e. setup cost, switching cost [28].

Let us denote the total cost of the firm over [0,T] by

$$TC = TC(T, m_1, m_2, m_3, m_4, n_1, n_2, Q_1^m, Q_2^m, Q_3^m, Q_4^m, Q_1^n, Q_2^n),$$
(2)

Using (1) the lot-sizes can be defined by the following formulas:

$$Q_{1}^{n} = \frac{\alpha_{1}dT}{n_{1}}$$

$$Q_{2}^{n} = \frac{\alpha_{2}dT}{n_{2}}$$

$$Q_{1}^{m} = \frac{(\beta_{1} - q_{1})dT}{m_{1}}$$

$$Q_{2}^{m} = \frac{(\beta_{2} - q_{2})dT}{m_{2}}$$

$$Q_{3}^{m} = \frac{q_{1}dT}{m_{3}}$$

$$Q_{4}^{m} = \frac{q_{2}dT}{m_{4}}$$
(3)

Taking into account (3), the variables  $Q_i^m, Q_j^n, i = 1..4, j = 1, 2$  can be excluded from (2):

$$TC = TC(T, m_1, m_2, m_3, m_4, n_1, n_2),$$
(4)

The total cost per [0,T] is the sum of manufacturing cost,  $\alpha_1 c_M^1 dT$  of part 1,  $\alpha_2 c_M^2 dT$  of part 2, remanufacturing cost,  $q_1 C_R^1 dT$  of part 1,  $q_2 C_R^2 dT$  of part 2, dismantle and assemble cost,  $(c_D + c_A) dT$ , fixed setup, switching, inspection cost of part 1,  $G_1(m_1, m_3, n_1)$ , and part 2  $G_2(m_2, m_4, n_2)$ , holding cost of part 1,  $H_1(m_1, m_3, n_1)$ , and part 2  $H_2(m_2, m_4, n_2)$ , and is given as:

$$TC(T, m_1, m_2, m_3, m_4, n_1, n_2) = ((\alpha_1 c_M^1 + \alpha_2 c_M^2) + q_1 C_R^1 + q_2 C_R^2 + (c_D + c_A))dT + G_1(m_1, m_3, n_1) + G_2(m_2, m_4, n_2) + H_1(T, m_1, m_3, n_1) + H_2(T, m_2, m_4, n_2)$$

Eoq related costs of type (3), i.e. setup costs and switching costs equals the sum of the total switching cost  $P_1$ , which include machine start-up, when remanufacturing is started, and machine adjustment, when remanufacturing is switched to manufacturing, fixed inspection cost for  $m_1$  lots of usable part 1,  $R_1m_1$ , fixed inspection cost and remanufacturing setup cost for  $m_3$  lots of part 1,  $R_3m_3$ , and manufacturing setup cost for  $n_1$  lots of part 1,  $S_1n_1$  (the setup and switching cost expression for the second part is obtained by the similar way):

$$G_1(m_1, m_3, n_1) = P_1 + R_1 m_1 + R_3 m_3 + S_1 n_1$$
  

$$G_2(m_2, m_4, n_2) = P_2 + R_2 m_2 + R_4 m_4 + S_2 n_2$$

The behavior of RS and SS inventories for part 1 and part 2 is also similar. The behavior of RS1 and SS1 inventories is represented on the Fig. 3.

The holding costs function for first part inventories RS1,SS1 are given by  $H_1(T, m_1, m_3, n_1)$ , for the second part inventories RS2,SS2 by  $H_2(T, m_2, m_4, n_2)$ :

$$\begin{split} H_1(T,m_1,m_3,n_1) &= T^2 H_1(m_1,m_3,n_1) \\ H_2(T,m_2,m_4,n_2) &= T^2 H_2(m_2,m_4,n_2) \end{split}, \\ H_1(m_1,m_3,n_1) &= \frac{d}{2} \Biggl[ h_1 \alpha \beta + \frac{1}{n_1} H_1 \alpha^2 + \frac{1}{m_1} (H_1 - h_1) (\beta - q_1)^2 + \frac{1}{m_3} (H_1 q_1^2 + h_1 \beta q_1 + h_1 q_1 (\beta - q_1)) \Biggr] \\ H_2(m_2,m_4,n_2) &= \frac{d}{2} \Biggl[ h_2 \alpha \beta + \frac{1}{n_2} H_2 \alpha^2 + \frac{1}{m_2} (H_2 - h_2) (\beta - q_2)^2 + \frac{1}{m_4} (H_2 q_2^2 + h_2 \beta q_2 + h_2 q_2 (\beta - q_2)) \Biggr]. \end{split}$$



Figure 3: The behavior of Serviceable Stock and Recoverable Stock inventories for part 1

Denote the total setup cost and total holding cost by:

$$G(m_1, m_2, m_3, m_4, n_1, n_2) = G_1(m_1, m_3, n_1) + G_2(m_2, m_4, n_2)$$
  
$$H(m_1, m_2, m_3, m_4, n_1, n_2) = H_1(m_1, m_3, n_1) + H_2(m_2, m_4, n_2)$$

The unit time cost function is obtained by dividing by T the total cost function:

$$ATC(T, m_1, m_2, m_3, m_4, n_1, n_2) = ((\alpha_1 c_M^1 + \alpha_2 c_M^2) + q_1 C_R^1 + q_2 C_R^2 + (c_D + c_A))d + \frac{G(m_1, m_2, m_3, m_4, n_1, n_2)}{T} + T(H(m_1, m_2, m_3, m_4, n_1, n_2)).$$
(5)

It can be obtained from (5) by differentiating that the length of the optimal time cycle is equal

$$T = \sqrt{\frac{G(m_1, m_2, m_3, m_4, n_1, n_2)}{H(m_1, m_2, m_3, m_4, n_1, n_2)}}$$

and the corresponding cost equals

$$ATC(T, m_1, m_2, m_3, m_4, n_1, n_2) = ((\alpha_1 c_M^1 + \alpha_2 c_M^2) + q_1 C_R^1 + q_2 C_R^2 + (c_D + c_A))d + + 2\sqrt{G(m_1, m_2, m_3, m_4, n_1, n_2) \cdot H(m_1, m_2, m_3, m_4, n_1, n_2)}.$$
(6)

Let the radicand of the root (6) be denoted by

$$L(m_1, m_2, m_3, m_4, n_1, n_2) = G(m_1, m_2, m_3, m_4, n_1, n_2) \cdot H(m_1, m_2, m_3, m_4, n_1, n_2),$$
(7)

where

$$G(m_1, m_2, m_3, m_4, n_1, n_2) = P_1 + P_2 + \sum_{j=1}^4 R_j m_j + \sum_{i=1}^2 S_i n_i$$
$$H(m_1, m_2, m_3, m_4, n_1, n_2) = h_1 + \sum_{j=1}^4 \frac{h_2^j}{m_j} + \sum_{i=1}^2 \frac{h_3^i}{n_i}.$$

The coefficients  $h_i^j$ , i = 1, 2, 3, j = 1..4 equals:

$$h_{1} = \frac{d}{2} (\alpha_{1}\beta_{1}h_{1} + \alpha_{2}\beta_{2}h_{2})$$

$$h_{2}^{1} = \frac{d}{2} (H_{1} - h_{1})(\beta_{1} - q_{1})^{2}$$

$$h_{2}^{2} = \frac{d}{2} (H_{2} - h_{2})(\beta_{2} - q_{2})^{2}$$

$$h_{2}^{3} = \frac{d}{2} (H_{1}q_{1}^{2} + h_{1}\beta_{1}q_{1} + h_{1}q_{1}(\beta_{1} - q_{1}))$$

$$h_{2}^{4} = \frac{d}{2} (H_{2}q_{2}^{2} + h_{2}\beta_{2}q_{2} + h_{2}q_{2}(\beta_{2} - q_{2}))$$

$$h_{3}^{1} = \frac{d}{2} H_{1}\alpha_{1}^{2}$$

$$h_{3}^{2} = \frac{d}{2} H_{2}\alpha_{2}^{2}.$$
(8)

To determine the optimal policy means to find the optimal numbers  $m_1, m_2, m_3, m_4, n_1, n_2$  for the minimum total unit time cost (6). The problem of determining the optimal numbers of lots takes the form of a nonlinear integer optimization problem:

$$\min_{\substack{(m_1,m_2,m_3,m_4,n_1,n_2)}} ATC(T,m_1,m_2,m_3,m_4,n_1,n_2) = \\ = ((\alpha_1 c_M^1 + \alpha_2 c_M^2) + q_1 C_R^1 + q_2 C_R^2 + (c_D + c_A))d + 2\sqrt{L(m_1,m_2,m_3,m_4,n_1,n_2)}, \qquad (9) \\ m_j, n_i \in \{1,2,\ldots\},$$

Where  $L(m_1, m_2, m_3, m_4, n_1, n_2)$  is defined by (7).

### **3** Solution of the model

Instead of solving the problem (9) the function L(m,n) can be minimized subject to  $m_j \ge 1, n_i \ge 1$ , i.e., the following two-dimensional nonlinear integer optimization problem is relevant:

$$\min_{(m,n)} L(m,n) = \min_{(m,n)} \left( P + \sum_{j=1}^{l} R_{j} m_{j} + \sum_{i=1}^{k} S_{i} n_{i} \right) \cdot \left( h_{1} + \sum_{j=1}^{l} \frac{h_{2}^{j}}{m_{j}} + \sum_{i=1}^{k} \frac{h_{3}^{i}}{n_{i}} \right),$$

$$m = (m_{1}, m_{2}, \dots, m_{l}), n = (n_{1}, n_{2}, \dots, n_{k})$$

$$m_{i}, n_{i} \in \{1, 2, \dots\}$$
(10)

For the solution of the problem (10), consider the following two-dimensional nonlinear integer optimization problem:

$$\min_{\substack{(x_1, x_2, \dots, x_n)}} K(x_1, x_2, \dots, x_n) = \min_{\substack{(x_1, x_2, \dots, x_n)}} (b_0 + \sum_{i=1}^i b_i x_i) \cdot (a_0 + \sum_{i=1}^n \frac{a_i}{x_i}),$$

$$x_i \in \{1, 2, \dots\}, i = 1, 2, \dots n.$$
(11)

First, let us consider the following continuous auxiliary problem:

$$\min_{\substack{(x_1, x_2, \dots, x_n)}} K(x_1, x_2, \dots, x_n) = \min_{\substack{(x_1, x_2, \dots, x_n)}} (b_0 + \sum_{i=1}^i b_i x_i) \cdot (a_0 + \sum_{i=1}^n \frac{a_i}{x_i}),$$

$$x_i \ge 1, i = 1, 2, \dots, n.$$
(12)

By analysing the first partial derivatives, we can prove the following lemma:

**Lemma 1.** If  $x_i > 0, i = 1, 2, ..., n$ , there are n curves of local minima (12) with respect to  $x_j$ :

$$X_{j}(x_{1}, x_{2}, \dots, x_{j-1}, x_{j+1}, \dots, x_{n}) = \sqrt{\frac{a_{j}(b_{0} + \sum_{i=1, i \neq j}^{n} b_{i}x_{i})}{b_{j}(a_{0} + \sum_{i=1, i \neq j}^{n} \frac{a_{i}}{x_{i}})}},$$
(13)

and the point of the local minimum

$$x_{j}^{*} = \sqrt{\frac{a_{j}b_{0}}{a_{0}b_{j}}}, \ i = 1, 2, \dots, n.$$
 (14)

Let us denote the radicands of the expressions (14) by

$$A_{i} = \frac{a_{i}b_{0}}{a_{0}b_{i}}, \quad i = 1, 2, \dots, n.$$
(15)

Without loss of generality, it is supposed that  $A_1 < A_2 < ... < A_n$ . We denote:

$$B_{i}(j) = \frac{a_{i}(b_{0} + \sum_{k=1}^{j} b_{k})}{b_{i}(a_{0} + \sum_{k=1}^{j} a_{k})}, \quad i = 1, 2, \dots, n.$$
(16)

Then the optimal solution for the continuous problem (15) is provided by the following theorem.

**Theorem 1.** The optimal solution to the problem (12) has the following structure depending on the value of the parameters  $A_i, B_i(j)$ :

- 1. If  $A_i \ge 1, i = 1, 2, ..., n$ , then  $x_i = \sqrt{A_i}, i = 1, 2, ..., n$ .
- 2. If  $A_1 < 1$ , then consider  $B_2(1), B_3(2), \dots, B_j(j-1), \dots, B_n(n-1)$ ; if  $B_j(j-1) < 1$  and  $B_{j+1}(j) \ge 1$  then  $x_i = 1, i = 1, \dots, j, x_i = \sqrt{B_i(j)}, i = j+1, \dots, n$ .
- 3. If  $B_n(n-1) < 1$ , then  $x_i = 1, i = 1, ..., n$ .

This theorem gives the solution for problem (11) for any initial parameters. Also theorem can be used for solution more complicated models. The next section contains numerical illustrations.

### 4 Numerical examples

Example 1

This section presents numerical example to illustrate the behavior of the model. Let the parameters of the model be fixed:

$$d = 10000, H_1 = 15, H_2 = 12, h_1 = 10, h_2 = 7,$$
  

$$R_1 = 30, R_2 = 35, R_3 = 40, R_4 = 45, S_1 = 50, S_2 = 60, P_1 + P_2 = 100,$$
  

$$\alpha_2 = 0, 5, \beta_2 = 0, 5, q_1 = 0, 2, q_2 = 0, 3.$$

We consider different values of parameter  $\alpha_1 \in [0,05,0,75]$  with the step 0,05 and the function  $L(\cdot)$  (7) as the function of  $\alpha_1$ . If  $\alpha_1$  changes in [0,05,0,75], then  $\beta_1 = 1 - \alpha_1$  changes in [0,25,0,95] and  $\beta_1 - q_1$  changes in [0,05,0,75]. So the interval [0,05,0,75] covers all the range of possible values of  $\alpha_1$  with the step 0,05. On the Fig. 4 the total average cost  $L(\alpha_1)$  depending on depending on the disposal rate of part 1  $\alpha_1$  is represented. It is obvious from the Figure 4, that the more disposal rate  $\alpha_1$ , the more total average cost  $L(\alpha_1)$ . It is logic consequence of the fact that the more percentage of final product is accomplished through manufacturing, which is more expensive alternative of remanufacturing. The table 1 represents values of  $L(\alpha_1)$  at different  $\alpha_1$  with the step 0,05, optimal lot numbers  $(m_1^*, m_2^*, m_3^*, m_4^*, n_1^*, n_2^*)$ , and optimal lot sizes  $(Q_1^m, Q_2^m, Q_3^m, Q_4^m, Q_1^n, Q_2^n)$ 



Figure 4: Total average total cost  $L(\alpha_1)$  depending on the disposal rate of part 1  $\alpha_1$ 

$\alpha_{_{1}}$	$L(\alpha_1)$	$(m_1^*, m_2^*, m_3^*, m_4^*, n_1^*, n_2^*)$	$(Q_1^m, Q_2^m, Q_3^m, Q_4^m, Q_1^n, Q_2^n)$	Т
0,05	9 531	(3,1,3,2,1,2)	(317,254,85,190,63,317)	0,127
0,1	9 750	(2,1,3,2,1,2)	(413,236,79,177,118,295)	0,118
0,15	9 918	(2,1,2,2,1,2)	(351,216,108,162,162,270)	0,108
0,2	10 095	(2,1,2,2,1,2)	(318,212,106,159,212,265)	0,106
0,25	10 288	(2,1,2,2,1,2)	(286,208,104,156,260,260)	0,104
0,3	10 461	(1,1,1,1,1,1)	(344,138,138,206,206,344)	0,069
0,35	10 538	(1,1,1,1,1,1)	(307,137,137,205,239,342)	0,068
0,4	10 632	(1,1,1,1,1,1)	(271,135,135,203,271,339)	0,068
0,45	10 741	(1,1,1,1,1,1)	(235,134,134,201,302,335)	0,067
0,5	10 866	(1,1,1,1,1,1)	(199,133,133,199,331,331)	0,066
0,55	10 925	(1,1,1,1,2,1)	(188,150,150,225,206,375)	0,075
0,6	10 941	(1,1,1,1,2,1)	(150,150,150,225,225,375)	0,075
0,65	10 962	(1,1,1,1,2,1)	(112,150,150,224,243,374)	0,075
0,7	10 988	(1,1,1,1,2,1)	(75,149,149,224,261,373)	0,075
0,75	11 018	(1,1,1,1,2,1)	(37,149,149,223,279,372)	0,074

Table 1: The results of example 1.

#### Example 2

Let the parameters of the model be fixed:

$$d = 10000, H_1 = 20, H_2 = 20, h_1 = 7, h_2 = 19,$$
  

$$R_1 = 30, R_2 = 35, R_3 = 40, R_4 = 100, S_1 = 40, S_2 = 60, P_1 + P_2 = 100$$
  

$$\alpha_1 = 0, 5, \beta_1 = 0, 5, \alpha_2 = 0, 1, \beta_2 = 0, 9, q_2 = 0, 8.$$

We consider different values of parameter  $q_1 \in [0,02,0,48]$  and the function  $L(\cdot)$  (7) as the the function of  $q_1$ . The question is, how the change of the percentage of remanufactured items changes the total cost. On the Fig. 5 the total average cost  $L(q_1)$  depending on the percentage of remanufactured items  $q_1$  is represented. The table 2 represents values of  $L(q_1)$  at different  $q_1$  with the step 0,02, optimal lot numbers  $(m_1^*, m_2^*, m_3^*, m_4^*, n_1^*, n_2^*)$ , and optimal lot sizes  $(Q_1^m, Q_2^m, Q_3^m, Q_4^m, Q_1^n, Q_2^n)$ .



Figure 5: Total average cost  $L(q_1)$  depending on the percentage of remanufactured items  $q_1$ If  $q_1$  changes in [0,02,0,48], then  $\beta_1 - q_1$  also changes in [0,02,0,48]. So the interval [0,02,0,48] covers all the range of possible values of  $q_1$ . We can see from the figure 5, that the dependency is rather complex. For instances, the "fall" at  $q_1 = 0,36$  can be observed: the optimal policy (1,1,1,3,2,1), which was invariable from  $q_1 = 0,04$  to 0,36, was changed to (1,1,3,4,3,1). The total cost increased at the values of  $q_1$  from 0,04 to 0,36. However the policy (1,1,3,4,3,1) remained invariable from  $q_1 = 0,36$  to 0,48 and the total cost also increased. The minimum value of total cost 15 094 was reached at  $q_1 = 0,02$  under the policy (3,1,1,4,3,1).

$q_1$	$L(q_1)$	$(m_1^*, m_2^*, m_3^*, m_4^*, n_1^*, n_2^*)$	$(Q_1^m, Q_2^m, Q_3^m, Q_4^m, Q_1^n, Q_2^n)$	Т
0,02	15 094	(3,1,1,4,3,1)	(179,112,22,224,187,112)	0,112
0,04	15 471	(1,1,1,3,2,1)	(384,83,33,222,208,83)	0,083
0,06	15 443	(1,1,1,3,2,1)	(368,84,50,223,209,84)	0,084
0,08	15 423	(1,1,1,3,2,1)	(351,84,67,223,209,84)	0,084
0,1	15 412	(1,1,1,3,2,1)	(335,84,84,223,209,84)	0,084
0,12	15 410	(1,1,1,3,2,1)	(318,84,100,223,209,84)	0,084
0,14	15 416	(1,1,1,3,2,1)	(301,84,117,223,209,84)	0,084
0,16	15 431	(1,1,1,3,2,1)	(284,84,134,223,209,84)	0,084
0,18	15 455	(1,1,1,3,2,1)	(267,83,150,223,209,83)	0,083
0,2	15 487	(1,1,1,3,2,1)	(250,83,167,222,208,83)	0,083
0,22	15 528	(1,1,1,3,2,1)	(233,83,183,222,208,83)	0,083
0,24	15 578	(1,1,1,3,2,1)	(215,83,199,221,207,83)	0,083
0,26	15 635	(1,1,1,3,2,1)	(198,83,215,220,206,83)	0,083
0,28	15 702	(1,1,1,3,2,1)	(181,82,230,219,205,82)	0,082
0,3	15 776	(1,1,1,3,2,1)	(164,82,245,218,204,82)	0,082
0,32	15 859	(1,1,1,3,2,1)	(146,81,260,217,203,81)	0,081
0,34	15 949	(1,1,1,3,2,1)	(129,81,275,216,202,81)	0,081
0,36	15 559	(1,1,3,4,3,1)	(156, 111, 133, 222, 185, 111)	0,111
0,38	15 583	(1,1,3,4,3,1)	(133,111,141,222,185,111)	0,111
0,4	15 615	(1,1,3,4,3,1)	(111,111,148,222,185,111)	0,111
0,42	15 654	(1,1,3,4,3,1)	(88,111,155,221,184,111)	0,111
0,44	15 701	(1,1,3,4,3,1)	(66,110,162,220,184,110)	0,110

$q_1$	$L(q_1)$	$(m_1^*, m_2^*, m_3^*, m_4^*, n_1^*, n_2^*)$	$(Q_1^m, Q_2^m, Q_3^m, Q_4^m, Q_1^n, Q_2^n)$	Т		
0,46	15 755	(1,1,3,4,3,1)	(44,110,168,220,183,110)	0,110		
0,48	15 817	(1,1,3,4,3,1)	(22,109,175,219,182,109)	0,109		
	Table 2: The results of example 2					

Table 2: The results of example 2.

## 5 Summary and conclusions

Inventory policies for joint remanufacturing and manufacturing have recently received much attention. Most efforts, though, were related to (optimal) policy structures and numerical optimization, rather than closed form expressions for calculating near optimal policy parameters. The focus of this paper is on the latter. We consider a case of a firm that can manufacture new products and recover the value of a used product through remanufacturing with dismantling for components. The cost structure consists of setup costs, switching cost, holding costs and eoq non related costs. Theorem from the section 3 presents simple, closed form formulae for approximating the optimal policy parameters under a cost minimization objective.

The numerical examples were considered, which demonstrated the alteration of the optimal policy under different values of the initial parameter ceteris paribus. It also demostrates that the dependency of total average cost from other parameters of the model can be rather complex, which is the consequence of integer optimal numbers of optimal policy.

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