Optimal weighting method for fuzzy opinions

Mohamed El Alaoui and Hussain Ben-Azza
Department of industrial and production engineering
Ecole Nationale Supérieure des Arts et Métiers, Moulay Ismail University
Meknes, 50000, Morocco
mohamedelalaoui208@gmail.com, hbenazza@yahoo.fr

Khalid El Yassini
IA Laboratory, Faculty of Sciences Meknes
Moulay Ismail University
Meknes, 50000, Morocco
khalid.elyassini@gmail.com

Abstract

We propose a new method for group decision making by using trapezoidal fuzzy numbers to describe decisions. Optimality is achieved by minimizing the sum of weighed incoherencies between individual opinions and the consensus. When the experts' opinions are not equally important, the method is adapted to the heterogeneous situation. In order to promote coherence or hierarchical weights, a parametrization is proposed.

Keywords
Fuzzy number, fuzzy consensus, decision making.

1. Introduction

In a group decision making context, the main challenge is to attribute the correct weighting for each decision in order to achieve the group consensus. To overcome the subjectivity of the human reasoning, we use trapezoidal fuzzy numbers to model the experts' opinions. Several methods have been proposed [1-3] to obtain consensus from fuzzy opinions. Hsu [1] presented the similarity aggregation method (SAM) in which fuzzy opinions are aggregated depending on their similarities. Since SAM needs an intersection between the supports of each pair of fuzzy numbers representing the fuzzy opinions, Lu et al. [2] presented the coherence aggregation method (CAM). They specified that in addition to similarity, the opinions should be aggregated in function of both dissimilarity and similarity. The combination of the two is called coherence.

Lee presented the optimal aggregated method (OAM) [3] in which he aimed to achieve consensus by minimizing the dissimilarity between the fuzzy opinions and the aggregated consensus. We note that both the distances used by CAM and OAM are depending on the aggregated decisions. In other words, in a situation with 3 Decision Makers (DMs) the distance between good and fair is not the same if the third opinion is very good or bad, because of the normalizing term employed in each method. The distance presented in our work, unlike the distances proposed in OAM or CAM, does not need to recalculate the distances between each couple of DMs opinions when adding, deleting or modifying one opinion.

Herrera et al. [4] classified group decision making problems into a homogenous group and a heterogeneous group. For the first group, the opinions are treated equally while for the second group the importance of each DM is taken into consideration. In this work, we replace the similarity given in [3] by the one used in [5], to achieve consensus by minimizing the incoherence between each decision and the aggregated consensus. In a second approach, we treat how to include hierarchical weights into the process of decision making, and finally we introduce two parameters to promote one aspect over the other, that are hierarchical weights and coherence.

This paper is organized as follows. Section 2 defines the mathematical background needed and the similarity/coherence used. Section 3 introduces the proposed method for homogeneous group case while Section 4 considers the method to heterogeneous group and it treats the proposed parameterization. Finally, a conclusion is presented at Section 5.

2. Preliminaries

A fuzzy number [6] is a fuzzy set defined by its membership function $\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0,1]$ We restrict ourselves to trapezoidal fuzzy numbers (cf. Fig. 1) given by 4-tuples (a, b, c, d) where $a \leqslant b \leqslant c \leqslant d$ and represented by:

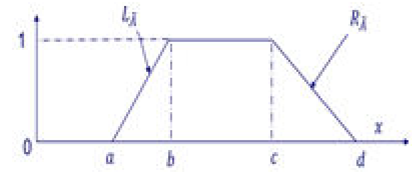

Figure 1. Trapezoidal fuzzy number.

where

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} = \mu_{L_{\tilde{A}}}(x) & if\ a \leqslant x \leqslant b \\ 1 & if\ b \leqslant x \leqslant c \\ \dfrac{x-d}{c-d} = \mu_{R_{\tilde{A}}}(x) & if\ c \leqslant x \leqslant d \\ 0 & else. \end{cases}$$

Weighting methods can be classed into two categories [6]: objective weighting methods that are based on a mathematical method, which includes the proposed, and subjective weighting methods. Different DMs express various opinions one from the others, hence unanimity is rarely achieved [7], in that sense the consensus is achieved by minimizing the sum of weighted incoherencies between the consensus and the individual opinions. The incoherencies, dissimilarities, are proposed as the dual of the similarity defined in [5] as follows:

$$r_p(\tilde{R}_i, \tilde{R}_j) = \frac{1}{p+1} * \{p + S_w(\tilde{R}_i, \tilde{R}_j)\} * \{1 - D_1(\tilde{R}_i, \tilde{R}_j)\} \tag{1}$$

where $\tilde{R}_1$ and $\tilde{R}_2$ are two trapezoidal fuzzy numbers defined by 4-tuples $\tilde{R}_i = (a_i, b_i, c_i, d_i)$ [4].

The similarity $S_w(\tilde{R}_i, \tilde{R}_j)$ introduced in [1], is defined by:

$$S_w(\tilde{R}_i, \tilde{R}_j) = \frac{\int_x \left(\min\left\{\mu_{\tilde{R}_i}(x), \mu_{\tilde{R}_j}(x)\right\}\right)^2 dx}{\int_x \left(\max\left\{\mu_{\tilde{R}_i}(x), \mu_{\tilde{R}_j}(x)\right\}\right)^2 dx}$$

The distance proposed in [5] $D_1(\tilde{R}_i, \tilde{R}_j)$ is calculated as:

$$D_1 = \begin{cases} A, & if\ \ max\ (b_1, b_2) < min(c_1, c_2) \\ A + B, & else, \end{cases}$$

where

$$A = \frac{1}{2}\int_{a_1}^{d_1} \mu_1 - \min(\mu_1, \mu_2) + \frac{1}{2}\int_{a_2}^{d_2} \mu_2 - \min(\mu_1, \mu_2)$$

$$B = \int_{min\,(c_1, c_2)}^{max\,(b_1, b_2)} 1 - max(\mu_1, \mu_2)^2.$$

3. Homogeneous groups

Let $r_p(\tilde{R}_1, \tilde{R}_2)$ be the fuzzy coherence between the i^th^ and the j^th^ opinions presented in the previous section. Similarly to OAM, we are going to minimize the incoherence defined by $c - r_p(\tilde{R}_1, \tilde{R}_2)$ where c is a real number $c > 1$. The problem is formulated as follows:

$$\min_{M \times IR^4} \sum_{i=1}^{n} w_i^m * \left(c - r_p(\tilde{R}_i, \tilde{R})\right), \tag{2}$$

where $M = \left\{ \begin{matrix} W = (w_1, w_2, \dots, w_n), w_i \geq 0, \\ \sum_{i=1}^{n} w_i = 1, \end{matrix} \right\}$, and m is an integer $m > 1$.

To solve this optimization problem, the iterative method of OAM is used [3].

To able comparison with OAM, we consider the following examples:

Example 1: Considering a group decision problem evaluated by three experts. The opinions are modeled by trapezoidal fuzzy numbers as follows:

$$\tilde{R}_1 = (1, 2, 3, 4), \tilde{R}_2 = (1.5, 2.5, 3.5, 5), \tilde{R}_3 = (2, 2.5, 4, 6),$$

Let $p = 2$, $m = 2$, $c = 1.5$ and the initial consensus $W^{(0)} = (1, 0, 0)$.

The aggregation process using (Eq.2) and the iterative scheme in OAM results in the following weights $w_1 = 0.17406598$ $w_2 = 0.59641758$ and $w_3 = 0.22951644$ (Tab. 1). Hence the weights corresponding to each decision favor the most coherent opinion which is the second one in this case. We mention also that the gap between the most coherent opinion's weight 0.59641758 and the least coherent one 0.17406598, is more significant than OAM. The OAM results in the following weights: 0.3296387, 0.3401198 and 0.3302414, conserving the same weights' ranking with less significant distinction between coherent and incoherent opinions.

Table 1. Results of example 1

Starting weights	1	Final weights	0.17406597
	0		0.59641759
	0		0.22951644
Starting aggregation	1	Final aggregation	1.52550639
	2		2.46546660
	3		3.52550639
	4		5.05101278
Number of iterations	21		

Example 2
It is assumed that all the conditions remain identical to those of the preceding example, with the exception of the starting weights $W^{(0)} = (0, 1, 0)$. The aggregation produces the same result as Example 1 even if starting from different points (Tab. 2).

The hesitancy represents the decision uncertainty, which is represented by the area under its membership function, which is computed as follows: $H(\tilde{R}) = \int_{-\infty}^{+\infty} \mu_{\tilde{R}}(x)dx$
We note that our method results in the smallest value of hesitancy compared to the methods in [1-3]. In this example, our aggregated opinion hesitancy equals 2.2928, while it is 2.341 by SAM [1], 2.322 by CAM [2] and 2.3321 by OAM [3].

Table 2. Results of example 2

Starting weights	0	Final weights	0.17406598
	1		0.59641758
	0		0.22951644
Starting aggregation	1	Final aggregation	1.52550639
	2.5		2.46546660
	3		3.52550639
	5		5.05101278
Number of iterations	16		

Example 3
Preserving the same parameters and let the fuzzy opinions be $\tilde{R}_1 = (1, 2, 3, 4), \tilde{R}_2 = (1, 2, 3, 4), \tilde{R}_3 = (2, 2.5, 4, 6)$, with the starting weights are $W^{(0)} = (0, 0, 1)$.

The aggregation results in $w_1 = w_2 = 0{,}44667693$ and $w_3 = 0{,}10664614$ which emphasizes the gap between the majority and the incoherent vote (Tab. 3). Using the OAM method, the result is 0.3382948; 0.3382948 and 0.3234104.

Table 3. Results of example 3

Starting weights	0	Final weights	0.17406598
	0		0.59641758
	1		0.22951644
Starting aggregation	2	Final aggregation	1.52550639
	2.5		2.46546660
	4		3.52550639
	6		5.05101278
Number of iterations	23		

4. Heterogeneous groups

In a decision making group, not all the DMs have the same importance, which will be represented by a real number between 0 and 1, $0 \leq e_i \leq 1$ reflecting the individual importance of each DM where the sum of individual importance equals 1; $\sum_{i=1}^{n} e_i = 1$. In such situation, the problem is reformulated as follows:

$$\min_{M \times IR^4} \sum_{i=1}^{n} w_i^m * \left(c - e_i * r_p\left(\vec{R}_i, \vec{R}\right)\right), \tag{3}$$

with the same constraints as the problem (Eq. 2).

Example 4

Same as Example 1, with the following hierarchical weights $e_1 = 0.4$; $e_2 = 0.3$ and $e_3 = 0.3$ the aggregation results in the following weights $w_1 = 0.33960108$ $w_2 = 0.33775916$ and $w_3 = 0.32263976$ (Tab. 4).

The influence of the hierarchical weights is clear on final weights when comparing the current example with example 1. In order to control the influence of each aspect, coherence and hierarchical importance, we introduce two parameters *t* and *s* as powers r_p^t, e_i^s. The aim is to increase the difference between coherent and incoherent opinions and to reduce the impact of hierarchical weights. For example let 0.5 be the minimal value considered for a coherent opinion with the consensus, the gap between two coherences that worth 0.8 and 0.7 is 0.1 while it is 0.15 = 0.82 - 0.72 when choosing $t = 2$, which emphasizes the role of coherence. In order to minimize the role of hierarchical importance, we can chose a value of $s < 1$. The problem becomes

$$\min_{M \times IR^4} \sum_{i=1}^{n} w_i^m * \left(c - e_i^s * r_p^t\left(\vec{R}_i, \vec{R}\right)\right),$$

with the same constraints as (Eq. 2)

The following table illustrates the result of the aggregation method for different values of t and s.

Table 4. Effect of the parameters on the aggregation results

cases	weights	result
t=1 *s*=1	0.339601080 0.337759157 0.322639763	1.483160035 2.327096513 3.483160035 4.966320071
t=1 *s*=0.25	0.314752739 0.371458608 0.313788653	1.499096956 2.352361964 3.499096956 4.998193912
t=4 *s*=1	0.320293151 0.360087452 0.319619397	1.499355362 2.346612615 3.499355362

		4.998710724
t=4 *s*=0.25	0.277322053 0.428937890 0.293740057	1.513502710 2.389239209 3.513502710 5.027005421
t =100 *s*=0.01	0.330479610 0.339040781 0.330479609	1.500000000 2.336198853 3.500000000 5.000000000

The previous table shows the impact of the parameters on the aggregation result. The parameter t emphasizes the effect of coherence when taken greater than 1, while s reduces the effect of hierarchical importance when taken smaller than 1. However the last line of Tab. 4 shows that there is an optimum to not out pass.

5. Conclusion

In this work, we presented an approach to decision making by introducing new distance, consistency/similarity. The general idea is that the more a DM opinion is consistent, the larger its weight will be, taking into account the hierarchical weights. The homogenous situation compared to OAM, permits distinguishing consistent opinions with inconsistent ones, but needs more iterations. In the heterogeneous situation, we introduced two parameters in order to promote the desired aspect: coherence or DMs importance.

References

Hsu, H. M. and Chen, C. T., Aggregation of fuzzy opinions under group decision making, Fuzzy Sets and Systems, vol 79, no. 3, pp. 279–285, 1996.

Lu, C., Lan, J. and Wang, J., Aggregation of Fuzzy Opinions Under Group Decision-Making Based on Similarity and Distance, Journal of Systems Science and Complexity, vol. 19, no. 1, pp. 63-71, 2006.

Lee, H. S., Optimal consensus of fuzzy opinions under group decision making environment, Fuzzy Sets and Systems, vol. 132, no. 3, pp. 303-315, 2002.

Herrera-Viedma, E., Cabrerizo, F. J., Kacprzyk, J., and Pedrycz, W., A review of soft consensus models in a fuzzy environment, Information Fusion, vol. 17, pp. 4-13, 2014.

El Alaoui, M., Ben-azza, H., and Zahi, A., New Multi-criteria Decision-Making Based on Fuzzy Similarity, Distance and Ranking, Proceedings of the Third International Afro-European Conference for Industrial Advancement — AECIA 2016, pp. 138-148, Marrakech, Morocco, November 21-23, 2016.

Skalna, I., Rębiasz, B., Gaweł, B., Basiura, B., Duda, J., Opiła, J., and Pełech-Pilichowski, T., Advanced in fuzzy decision-making, Studies in fuzziness and soft computing, Springer, New York, 2015.

Wang, J. J., Jing, Y. Y., Zhang, C. F., & Zhao, J. H., Review on multi-criteria decision aid in sustainable energy decision-making. Renewable and Sustainable Energy Reviews, vol. 13, pp. 2263–2278, 2009.

Fedrizzi, M., Pasi, P., Fuzzy Logic Approaches to Consensus Modelling in Group Decision Making, Intelligent Decision and Policy Making Support Systems, Studies in Computational Intelligence, vol. 117, pp. 19-38, 2008.

Biographies

Mohamed El Alaoui is a PhD Student, at Department of Production and Industrial Engineering department at the Ecole National Supérieure des Arts et Metiers (ENSAM), Meknes-Morocco. He is studying multiobjective optimization using fuzzy logic. He is an industrial engineer from ENSAM.

Hussain Ben-azza is a professor Department of Production and Industrial Engineering department at the Ecole National Supérieure des Arts et Metiers (ENSAM) of Moulay Ismail University, Meknèes – Morocco. His research expertise concerns Mathematics and Computer Science.

Khalid El Yassini holds a PhD in mathematics obtained in 2000 with a specialty in Operations Research from University of Sherbrooke in Canada where he obtained a MSc in mathematics in 1994. Previously, he had a degree in applied mathematics (specialty Statistics) at Abdelmalek Essaadi University in Northern Morocco in 1991.

During his academic career, he has taught in several Canadian and Moroccan institutions such as University of Sherbrooke, Univesité du Québec à Rimouski, St-Boniface University at Winnipeg-Canada, Moulay Ismail University, International University of Rabat or Royal Military Academy. He has taught various courses in mathematics, computer science, operations research and logistics. His main current activities focus on mathematical programming including linear programming and multiobjective optimization, artificial intelligence, information systems, security in computer networks, telecommunications, intelligent systems (Smart Cities & Smart Vehicles), logistics engineering, and green logistics with application in hospital field or in medication domain.