# A New formulation of Capacitated Plant Location Problem

**RRK Sharma and Ajay Jha**
Dept of Industrial and Management Engineering
Indian Institute of Technology, Kanpur - 208016, INDIA
rrks@iitk.ac.in, ajjha@iitk.ac.in

## Abstract

The capacitated plant location problem (CPLP) is a very well studied problem in literature. In this paper we give a new formulation of the same. We first give an intuitive justification for the addition of new constraints. We consider the extreme case when transportation costs are very high, compared to location cost then in that situation we will tend to locate many plants that will reduce the transportation costs. And on other hand, when transportation costs are very low compared to location cost, then may locate very few plants. We introduce a new formulation of CPLP that addresses the above mentioned situations.

**Key Words: CPLP, SPLP, New Formulation**.

## 1. Problem Description

Plant/ facility location is a key decision in supply chain network planning and a well studied problem in literature, where its variations are brought by considering the capacity of the plant to be located as un-capacitated/ capacitated in nature thus to fulfill the demands originated at market points. The installed capacity of a production unit is often constrained by various factors like land and construction cost, labor cost, raw material availability and other regulations applicable at that location. The monthly fixed costs (location costs) of the different locations and also the transportation costs have to be well estimated by the cross functional team comprising of Finance, Production and Supply Chain personnel. These problems are referred to as simple plant location problem (SPLP) and capacitated plant location problem (CPLP) in literature for which a variety of heuristic based approaches and exact solution methodology are available.

## 2. Problem Formulation

The important strategic decision in plant location problem is to set up the plants that minimize total cost (location plus transportation). Traditional capacitated facility location and transportation problem formulation is given below. The set of notations used in the models are:

$I$: denotes the set of possible sites where the plants can be located.

$i$: represents the element belonging to set I.

$f_i$: represents the fixed cost of installing a plant at the location i.

$S_i$: represents the production capacity of the plant at the location i.

$K$: denotes the set of existing markets to be served.

*k*: represents the element belonging to set K.

$D_k$: represents the market demand at the location k.

$C_{ik}$: represents the transportation Cost for one unit of good between locations i and k.

$y_i$: a binary variable that indicates whether a plant is to be installed at location i or not.

$X_{ik}$: an integer variable that indicates the quantity of goods delivered from the plant i to market k.

The figure 2.1 given below gives the pictorial representation of the model used in typical facility location and transportation problem.

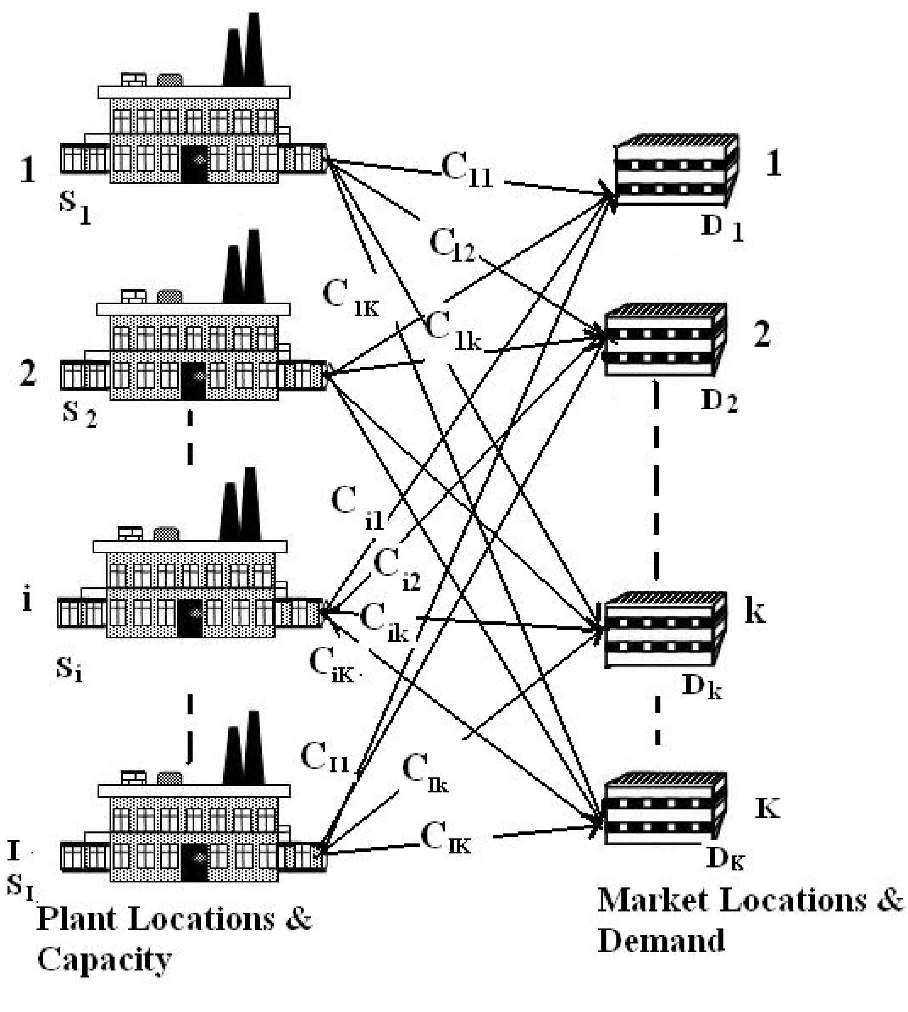


*Figure 2.1: Capacitated Plant Location Problem*

## 2.1 Simple Plant Location- Transportation Model

*Objective Function*

*Minimize* $Z = \sum_{i,k} C_{ik} * X_{ik} + \sum_i y_i * f_i$ ………….. *(1)*

*Subject to*

$\sum_{i=1}^{I} X_{ik} \geq D_k \quad \forall \; k$ ………… *(2)*

$\sum_{k=1}^{K} X_{ik} \leq y_i * S_i \quad \forall \; i$ ………… *(3)*

$X_{ik} \leq y_i * S_i \quad \forall \; i$ ………… *(4)*

$X_{ik} \leq y_i * D_k \quad \forall \; i,k$ ………… *(5)*

$X_{ik} \geq 0 \quad \forall \; i,k$ ………… *(6)*

$y_i \in \{0,1\} \quad \forall \, i$ ………… *(7)*

$\sum_{i=1}^{I} y_i * S_i \geq \sum_{k=1}^{K} D_k$ ………… *(8)*

The problem P: minimizing (1) subject to (2), (3), (6) & (7) is the traditional formulation of CPLP.

Suppose that there are two formulations A and B for the same problem. By excluding the integrality constraints (which force variables to take an integer value), we obtain the linear optimization relaxation. Let the feasible region of formulations be $P_A$ and $P_B$. When the region $P_B$ contains $P_A$, i.e., $P_A \subset P_B$, formulation A is stronger than formulation B (analogously, B is weaker than A). Also as $P_A$ is narrower than $P_B$, the upper bound obtained by the relaxation in a maximization problem (or the lower bound in minimization) of A is more close to the optimum of the integer problem.

Thus pose the following: minimize (1) subject to equations (2) – (8) as strong formulation of Problem 'P'. According to Sharma and Berry (2007) and Verma and Sharma (2011), the constraints (4), and (5) are strong constraints. Note that (8) is redundant both for the IP formulation and the LP relaxation of the CPLP, as it can be obtained from constraints (2) and (3). However, Cornuejols et al. (1991) show the theoretical and practical advantages of adding constraint (8) to strengthen Lagrangian relaxations.

## 2.2 Modified formulation as per extreme values of transportation costs

We create a set called SET-CAP by the following steps:

1) Start with $SET - CAP = \{ \quad \} = \varphi$
2) Then put smallest $S_i$ ($i \epsilon \; 1 \ldots . I$ ) to the set *SET-CAP*.
3) Keep on adding the next higher $S_i$ to the set *SET-CAP* till $\sum_{i=1}^{I'} S_i \geq \sum_{k=1}^{K} D_k$ , where $I' \leq I$.

Then above created set *SET-CAP* represents the maximum number of plants that can be opened for the case when transportation cost is a much smaller compared to other costs such as location cost. Let the cardinality of the set *SET-CAP* is = 'n'. This is related to the facility location variable as given by the equation 9 below.

$\sum_{i=1}^{I} y_i \leq n$ ………… *(9)*

However if the transportation cost is very large compared to location cost, then ideally one can open plants at all the locations. This is represented by the equation 10.

$\sum_{i=1}^{I} y_i \geq n$ ………… *(10)*

We do not know the situation whether equation (9) or equation (10) will hold true, but we wish to let exactly one of the equation out of equations (9) and (10)) to be the binding depending upon higher values of transportation or location costs. This is achieved by equations (11) and (12).

$\sum_{i=1}^{I} y_i - M * z_1 \leq n$ ………… *(11)*

$\sum_{i=1}^{I} y_i + (1 - z_1) * M \geq n$ ………… *(12)*

where $z_1$ is the binary variable $\epsilon \{0,1\}$ and M represents a very large positive number.

We hope that adding constraints (11) and (12) will serve as strong constraints to traditional formulation of CPLP (problem P). We are in the process of carrying out an empirical study to see the effect of constraints (11) and (12) on CPLP. We will get back soon once it gets done.

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## Biographies

**R. R. K. Sharma** is a HAG Scale Full Professor at Indian Institute of Technology, Kanpur India. Dr. Sharma did B.E. in Mechanical Engineering from Vishweshwariya Regional College of Engineering, Nagpur and Ph.D. in management from Indian Institute of Management, Ahmedabad. Prior to joining IIT Kanpur in 1989, he had worked for automotive companies like TATA MOTORS and TVS Suzuki. Dr. Sharma enjoys more than 30 years of career and has 104 journal and 2 book publications till date. His primary research interests include Simulation, Optimization, Scheduling, Location-allocation problems and MRP Systems. His secondary research interests are in the field of Strategy. He is member of Member, Board of Governors, of many Management Institutions and Editor of AIMS (All India Management Scholars International Society).

**Ajay Jha** is currently a fulltime research scholar at Indian Institute of Technology Kanpur. Mr. Ajay Jha holds a Bachelor of Technology degree in Mechanical Engineering from Harcourt Butler Technological Institute Kanpur and a Master of Technology in Industrial and Management Engineering from Indian Institute of Technology Kanpur. He has rich experience of production and marketing domains of over 10 years and also of teaching mechanical engineering course of 4 years. His research areas include Supply Chain Management and Strategy.