Pricing Optimization and Seat Inventory Allocation for Intercity Passenger Train Services

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Abstract

In this research, a practical approach to estimating demand function for pricing optimization and seat inventory allocation of intercity passenger railway services was developed. Willingness-to-pay approach was used for estimating demand. Demand for the service was estimated from choice-based conjoint data using hierarchical Bayes estimation method. The proportion of customers willing to buy a certain service was estimated using market simulation. Monotonic cubic spline interpolation was used to estimate the price-response function. The size of maximum demand achievable was estimated from sales data. The approach was implemented to the largest intercity passenger train service in Indonesia. Two customer segments were assumed, i.e. business and leisure passengers, with four fare classes. Our first optimization problem sought to determine optimal capacity allocation for the existing fare classes. The second optimization problem aimed to determine optimal fare and the corresponding optimal capacity allocation for each fare class. Solution to the nonlinear pricing optimization problem was obtained using enumeration. Subsequently, the popular Expected Marginal Seat Revenue-a (EMSR-a) heuristic was used for the seat allocation inventory problem.

Keywords
Willingness-to-pay, choice-based conjoint, hierarchical Bayes, monotonic cubic spline, EMSR-a.

1. Backgrounds

Research on the implementation of revenue management (RM) in intercity passenger railway services is not as much as those in airlines industry. Ciancimino et al. (1999) developed a multi-leg single-fare RM model to determine optimal booking limits for passenger train services. The optimization problem was modeled using probabilistic nonlinear programming to incorporate demand uncertainty. Terabe and Ongprasert (2006) developed linear programming models to show that seat allocation can improve revenue while at the same time improve load factor and rejected request. You (2008) developed a railway seat allocation model using nonlinear programming. Bharill and Rangaraj (2008) developed a revenue maximization model for a premium segment of passenger train services which accommodate cancellation and overbooking. The model used information about demand elasticity and cancellation fraction from past data to come up with proposed cancellation fee that will maximize revenue. Qian and Shuai (2014) developed a dynamic programming model to solve seat allocation problem in a dedicated passenger line by considering passenger's preference order. In all of the above research, demand functions were assumed such that they fitted the optimization model, instead of being developed based on a robust consumer demand theory. Ciancimino et al. (1999) and You (2008) asummed probabilistic demand with known means and variances, while Terabe and Ongprasert (2006), and Bharill and Rangaraj (2008) assumed deterministic demand using historical past. Qian and Shuai (2014) used a different approach for demand by assuming it follows a Poisson process.

Estimating demand function and pricing optimization have been equally important in the implementation of revenue management (RM) initiatives (Phillips, 2005). The difficulty in estimating demand for RM implementation comes from the fact that demand is a function of price, the decision variable. Following this notion, researchers used different methods for estimating demand in transportation industry' RM, stemming from a simple linear model as in Weatherford (1997) and Chew, et al. (2008) to more sophisticated models as nested logit in Ongprasert (2006) and multinomial logit and latent class in Hetrakul and Cirillo (2014). Latent class estimation method has the capability of identifying and segmenting respondents with different preferences from choice data.

In this research hierarchical Bayes was employed for estimating respondents' preference from choice data. While latent class results in part-worth utilities at group level, hierarchical Bayes produces part-worth utilities at individual level. Having individual level utilities enables us to predict how each respondent makes choice in any given scenario, including a competitive one, through the use of market simulator. Using market simulation, the price-response curve for a given scenario can be derived. By combining it with information about market size, the demand function can be obtained.

The proposed approach was used in a case study, i.e. Argo Parahyangan passenger train service. Argo Parahyangan is a short-distance intercity passenger train serving the busiest line in Indonesia which operates between the capital city of Jakarta and the tourist destination city of Bandung. It is operated by PT.Kereta Api Indonesia (Persero), a state-owned train operator which monopolize the railway transportation in Indonesia. There are ten trips per day from Bandung to Jakarta and ten from Jakarta to Bandung. Two trips from each leg are facultative. It has three seating class, i.e. economy, business, and executive class. Executive class has the greatest number of seats, followed by economy, and business class. Business class seats are only available in some of the facultative trains. The tariff for the economy class is highly regulated and subject to approval by the government. In this research, it was assumed that each seating class is independent, and we focus on the executive class.

As in other intercity transportation modes, two customer segments, i.e. business passengers, and leisure passengers were assumed. Business passengers are usually less price sensitive, book later, less flexible to departure time, and less accepting of restrictions. On the other hand, leisure passengers tend to be more price-sensitive, book earlier, more flexible to departure time, and more accepting of restrictions. The executive class of Argo Parahyangan (hereinafter Argo Parahyangan) currently has four fare classes, i.e. Rp120k, Rp110k, Rp100k, and Rp90k.These fares are set based on judgment considering past demand and competition with other transportation modes. In setting fares, PTKAI has to comply with the regulation from the Ministry of Transportation which imposes lower and upper limit for tariff. The current capacity allocation for each fare class is set using judgment.

Using the derived demand functions for each segment, two optimizations were run; the first optimization aimed to determine optimal capacity allocation for current fare classes, and the second one was for determining optimal tariff for each fare-class and their corresponding optimal capacity allocation. The number of fare-class was assumed as it is now. Assuming that demand between different departure times are independent, a train with certain departure time was picked as an example and it was suggested that the same approach can be applied to other trains.

2. Methodology

In general, the methodology comprises three stages, i.e. estimating demand, determining optimal prices, and determining optimal capacity allocation to each fare class.

2.1. Demand Estimation

Willingness-to-pay approach which assumes that a potential buyer has a maximum willingness-to-pay for a certain product/service, and will buy if the price is less than this value, was used for demand estimation. Using this notion, demand for a product as a function of price, p, $d(p)$ can be formulated as follows (Phillips, 2005)

$$d(p) = D \int_{p}^{\infty} w(x)\, dx \tag{1}$$

where D is the maximum demand achievable and $w(x)$ is the willingness-to-pay distribution. Instead of estimating $w(x)$, choice-based conjoint (CBC) were used to estimate the value of $\int_{p}^{\infty} w(x)\, dx$. CBC is considered the best method for estimating demand in competitive settings (Jedidi and Jagpal, 2009). CBC with Hierarchical Bayes (HB) estimation method was employed and resulted in part-worth estimates for each respondent, based on which a market simulation was run to come up with the $\int_p^\infty w(x)\,dx$ values.

Input for HB is choice data, which are collected using CBC questionnaire. A CBC questionnaire consists of a number of choice tasks, each of which consists of a number of stimuli or product concepts with or without none option. A product concept is essentially a combination of levels of attributes that define the product. In this CBC study, four attributes were chosen, i.e. departure time, booking period, seating class, and price. The CBC parameters were then estimated from CBC data using HB method. HB estimation method comprises two levels of estimation, i.e. at upper level, estimation of individuals' part-worth, and at lower level, estimation of individuals' probabilities of choosing available alternatives (Sawtooth Software, Inc., 2009). The vector of individuals' part-worth, $\boldsymbol{\beta_i}$ are represented by a multivariate normal distribution,

$$\boldsymbol{\beta_i} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}) \tag{2}$$

where $\boldsymbol{\mu}$ is the vector of means of the distribution of individuals' part-worth, and $\boldsymbol{\Sigma}$ is the variance-covariance matrix of the distribution of part-worth across individual. The value of $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, and $\boldsymbol{\Sigma}$ are estimated using iterative process with the following algorithm:

1. Set initial value of $\boldsymbol{\beta_t}$, $\boldsymbol{\mu_t}$, and $\boldsymbol{\Sigma_t}$
2. Estimate $\boldsymbol{\mu_{t+1}}$ by drawing random vector from $N\left(\widehat{\boldsymbol{\beta_t}}, \boldsymbol{\Sigma_t}\right)$
3. Estimate $\boldsymbol{\Sigma_{t+1}}$ based on $\boldsymbol{\beta_t}$ and $\boldsymbol{\mu_{t+1}}$ using Cholesky decomposition of the inverse of the following matrix

$$\mathbf{H} = \mathrm{p}\mathbf{I} + \sum_{\mathrm{n}} (\boldsymbol{\mu}_{\mathbf{t+1}} - \boldsymbol{\beta}_{\mathbf{t}})(\boldsymbol{\mu}_{\mathbf{t+1}} - \boldsymbol{\beta}_{\mathbf{t}})^{-1} \tag{3}$$

4. Estimate $\boldsymbol{\beta_{t+1}}$ based on $\boldsymbol{\mu_{t+1}}$ and $\boldsymbol{\Sigma_{t+1}}$ using Metropolis-Hastings algorithm
5. Repeat until $\boldsymbol{\beta}$, $\boldsymbol{\mu}$, and $\boldsymbol{\Sigma}$ converge.

Given individual's part-worth, her probabilities of choosing a particular alternative, k, follows a multinomial logit model, as follows

$$\boldsymbol{p_k} = \frac{\mathrm{e}^{\mathbf{x}_\mathbf{k}'\boldsymbol{\beta}_\mathbf{i}}}{\sum_{\mathrm{j=1}}^{\mathrm{n}} \mathrm{e}^{\mathbf{x}_\mathbf{j}'\boldsymbol{\beta}_\mathbf{i}}} \tag{4}$$

where $\boldsymbol{p_k}$ is the probability of an individual choosing the kth concept in a particular choice task, and $\mathbf{x}_\mathbf{k}'$ and $\mathbf{x}_\mathbf{j}'$ is the vector of values describing the kth and jth alternative in the choice task, respectively.

In the next step, conjoint market simulator transform individuals' part-worth utilities into a more meaningful outcome, i.e. prediction on how respondents choose among specific market offerings. If respondents are representative to the market, this is like conducting a market poll on a number of product concepts within a competitive scenario.The market simulation was run using randomized first choice option. In this method, two random components are added to the utility value of product concept i for respondent j, U_{ij}. The first random component is product variability which usually has normal or Gumbel distribution, and the second random component is determined from fixed choice task data and assumes Gumbel distribution (Orme and Baker, 2000). Accordingly, we will have

$$\boldsymbol{U_{ij}} = \boldsymbol{X_i}\left(\boldsymbol{\beta} + \boldsymbol{E_p}\right) + \boldsymbol{E_f} \tag{5}$$

where E_p is the first random component and E_f is the second random component. Having utility value of each product from each respondent, the share of preference for each product concept can be estimated. By varying the price of the product of interest and holding other products' attributes constant, a number of data pairs of price and simulated share of preference were obtained. These data pairs are then used to estimate demand function.

Monotonic cubic spline interpolation (Wolberg and Alfy, 2002) was used to estimate demand function from simulated share of preference data. Suppose we have n pairs of observation, $\{(x_0, y_0), (x_1, y_1), \dots, (x_{n-1}, y_{n-1})\}$. This approach seeks a piecewise cubic polynomial function, $f(x) = y_k$ which consists of $n-1$ segments, where segment k, f_k is defined on the range $[x_k, x_{k+1}]$. To ensure continuity over our observations, constraints regarding the continuity of $f'(x_k)$ and $f''(x_k)$ are imposed. The kth polynomial segment, f_k, has the following form

$$f_k(x) = a_k(x - x_k)^3 + b_k(x - x_k)^2 + c_k(x - x_k) + d_k \tag{6}$$

where

$$a_k = \frac{1}{\Delta x_k^2}\left(-2\frac{\Delta y_k}{\Delta x_k} + y'_k + y'_{k+1}\right) \tag{7}$$

$$b_k = \frac{1}{\Delta x_k}\left(3\frac{\Delta y_k}{\Delta x_k} - 2y'_k - y'_{k+1}\right) \tag{8}$$

$$c_k = y'_k \tag{9}$$

$$d_k = y_k \tag{10}$$

Either of these two conditions will guarantee the monotonicity of $f_k(x)$

1. when $\alpha_k + \beta_k - 2 \le 0$, $f_k(x)$ is monotone $iff\ sign(y'_k) = sign(y'_{k+1}) = sign\left(\frac{\Delta y_k}{\Delta x_k}\right)$,
2. when $\alpha_k + \beta_k - 2 \le 0$, $f_k(x)$ is monotone $iff\ sign(y'_k) = sign(y'_{k+1}) = sign\left(\frac{\Delta y_k}{\Delta x_k}\right)$ and one of the following is satisfied: (a) $2\alpha_k + \beta_k - 3 \le 0$; (b) $\alpha_k + 2\beta_k - 3 \le 0$; (c) ${\alpha_k}^2 + \alpha_k(\beta_k - 6) + (\beta_k - 3)^2 < 0$;

where

$$\alpha_k = y'_k \frac{\Delta x_k}{\Delta y_k} \tag{11}$$

$$\beta_k = y'_{k+1} \frac{\Delta x_k}{\Delta y_k} \tag{12}$$

2.2. Pricing Optimization Model

Suppose a train with a specific origin-destination and time of departure has a fixed capacity of C. There are n customer segments and m fare classes. The demand function for customer segment i when the price is p_j is $d_i(p_j)$. The incremental cost is practically zero because this is a no-frills trip. Revenue from ancillary products was ignored and a fixed number of fare classes, m, which is the same as the current condition, was assumed. The fares are restricted by government regulation which imposes a lower, p_l, and upper limit, p_u for the tariff. Formulation of the corresponding revenue-maximization problem is as follows:

$$\max_{p,x} \sum_{i=1}^{n}\sum_{j=1}^{m} p_j x_{ij} \tag{13}$$

$$s.t.\ x_{ij} \le d_i(p_j), i = 1,2, \dots, n; j = 1,2, \dots m \tag{14}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij} \le C, i = 1,2, \dots, n; j = 1,2, \dots m \tag{15}$$

$$p_l \le p_j \le p_u, j = 1,2, \dots m \tag{16}$$

$$x_{ij} \ge 0, i = 1,2, \dots, n; j = 1,2, \dots m \tag{17}$$

where x_{ij} is the number of seats sold at the price of p_j to customer from segment i. This optimization problem has four types of constraint, i.e. demand constraint (Eq. 14), capacity constraint (Eq. 15), tariff regulation constraint (Eq. 16), and non-negativity constraint (Eq. 17).

2.3. Capacity Allocation

The capacity allocation problem used common notations where fare classes are represented with a number $(1,2,\ldots,n)$, in which lower number refers to higher-fare class. Capacity allocation can be indicated by booking limit, b or protection level, y. Those parameters are interrelated, i.e. $b_j = C - y_{j-1}$, where C is capacity and j represents fare classes. In general, capacity allocation problem seeks to optimally allocate capacity for lower-fare class when there is possible demand for higher-fare class. It is usually assumed that all demand for lower-fare class occur before demand for higher-fare class. Solution to a two-class capacity allocation problem can be simply explained as balancing the tradeoff between spoilage and dilution (Phillips, 2005). Spoilage is when the lower-fare passengers were turned away due to allocating too few seats for lower-fare class and it turns out that there is not enough higher-fare passengers to fill the empty seats. On the other hand, dilution is when too many seats were allocated for lower-fare class that higher-fare class passengers have to be turned away. A simple solution to a two-class problem is the following Littlewood's rule

$$C - b_l^* = y_h^* = F_h^{-1}\left(1 - \frac{p_l}{p_h}\right) \tag{18}$$

where b_l^* is the optimal booking limit for the lower-fare class, y_h^* is the optimal protection level for higher-fare class, $F_h(x)$ is the probability that the higher-fare demand is less than or equal to x, p_l is the price for lower-fare class, and p_h is the price for higher-fare class.

A multiple-fare class problem can be solved by successive application of Littlewood's rule from terminal period moving backward to the initial period. At the beginning of each period a booking limit for the lowest fare class available is set. When the booking limit for this class is added, one of the following could happen, i.e. either there will be additional passenger for the class or not. When an additional passenger comes in, there is a probability that this will supersede one potential passenger from higher-fare classes. The structure of the problem suggests the use of dynamic programming. Other popular method is using Expected Marginal Seat Revenue (EMSR) heuristics. In this research, the EMSR-a heuristic was used. This heuristic works by calculating protection levels for a certain class relative to all higher classes using Littlewood's rule, one at a time. The protection level for the corresponding class is obtained by summing up all those protection levels.

3. Results and Discussion

The levels of each attribute were set based on the current values existed in the market and those which were not currently available but were feasible to offer. The levels for price attribute were set conditional on seating class. Setting prices this way made prohibition unnecessary while maintained the questionnaire design orthogonal. Table 1 recaps the attributes and levels used in this research.

Table 1 Attributes and levels

Attributes	Departure time	Booking time	Seating class	Price	
				Economy	Executive
Levels	05.00 – 07.59	D-day	Economy	Rp45.000	Rp68.000
	08.00 – 10.59	D-1 to D-3	Executive	Rp55.000	Rp84.000
	11.00 – 13.59	D-4 to D-7		Rp65.000	Rp100.000
	14.00 – 16.59	D-8 to D-30		Rp75.000	Rp116.000
	17.00 – 19.59	Before D-30		Rp85.000	Rp132.000

With all attributes and levels, the number of possible product concept that can be made is $5 \times 5 \times 2 \times 5 = 250$. Putting all these concepts in the CBC questionnaire is impractical, if not impossible. CBC questionnaire is developed such that it consists of a balanced and orthogonal fractional factorial design of all possible level combination. Balanced means that the number of levels of each attribute occur in the questionnaire are the same. The design is orthogonal when each pair of levels appears equally across all pairs of attributes within the questionnaire. When there are many attributes and levels, manually designing a balanced and orthogonal fractional factorial design is very difficult. Sawtooth Software SSI Web was used to design the CBC questionnaire and administer the online survey. There were sixteen choice tasks in the CBC questionnaire developed in this research, where fourteen were random and two were fixed. Each choice task consisted of three stimuli and a dual-response none option. Although the use of dual-response none option implies more questions, it is considered better representing the real situation faced by the consumer in the market while, at the same time, results in estimated parameters which are not significantly different. Figure 1 depicts an example of choice task in our questionnaire.

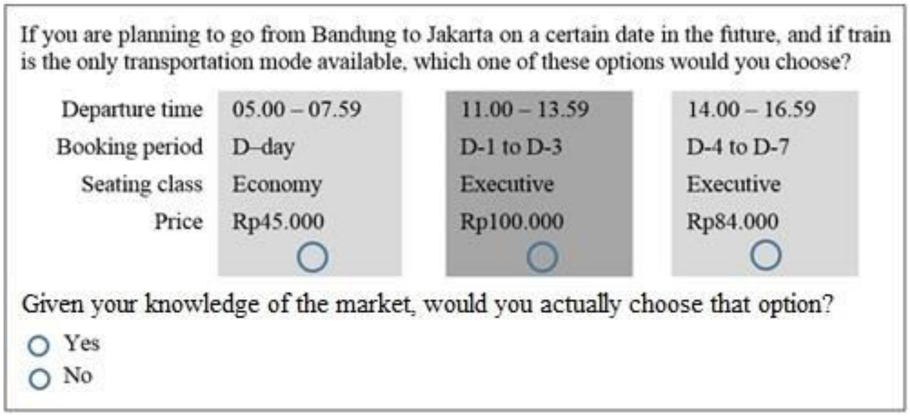

Figure 1 Example of choice task in our CBC questionnaire

Theoretically, a CBC study requires at least 300 respondents (Orme, 2010), but the educational license used in this research limits the number of respondent to 250 for each study. Since two customer segments were assumed, we collected a total of 500 respondents, 250 for each segment. From the CBC data, part-worth utilities for individual respondent were estimated using hierarchical Bayes method. Both estimations for business and leisure segment had converged.

The estimation for business segment resulted in the percent of certainty value of 0.909 which means that the model was 90.9 % fit. The root likelihood value of 0.890 means that the model was $0.890/0.33 = 2.67$ times better than a random guess. For leisure segment, the model is 85.2 % fit, and it is $0.817/0.33 = 2.45$ times better than a random guess. The validity of the CBC model was evaluated by comparing the actual and simulated share of preference of the fixed choice tasks from the CBC questionnaire. The validation process came up with a mean absolute error of 0.70% and 1.83% for business and leisure segment, respectively. For the choice task design, these correspond to mean relative error of 2.8% and 7.32%, respectively. This is a relatively accurate result despite the sample size.

Two sets of demand functions were derived in this research, one for each passenger segment. For each segment, separate demand functions for different departure times and different booking periods were defined. To derive each demand function, a market simulations was run using individuals' part-worth utilities as input. Referring to Equation 1, the output of market simulation is estimated share of preference, $\int_p^\infty w(x)\,dx$. Since there were five levels of price, the simulation came up with five pairs of price and share of preference data. Two data pairs were added, one indicating the satiating price and one indicating the share of preference when the price is zero. Using judgment, one data pair was added to ensure a 'reasonable' demand function. The 'reasonableness' refers to functions that resemble one of the theoretical demand functions, e.g. those in Huang et al. (2013).

These eight data points are then interpolated to get the price-response curve. Monotonic cubic spline interpolation was used to estimate the function of the price-response curve. This was consistent with one of the characteristics of demand function which is downward-sloping. Specifically, the freeware SRS1 Cubic Spline for Excel was used to estimate the curve. The software did not reveal the cubic spline functions that define the curve explicitly. Instead, it gave estimate for any given input value based on the spline functions. The demand functions were then obtained by combining these price-response functions with information about the size of achievable demand.

For illustration, demand function for one particular departure time, i.e. 05.00 — 07.59, was picked, and suggest that the same method can be used for other departure times. The corresponding demand functions for both segments are depicted in Figure 2.

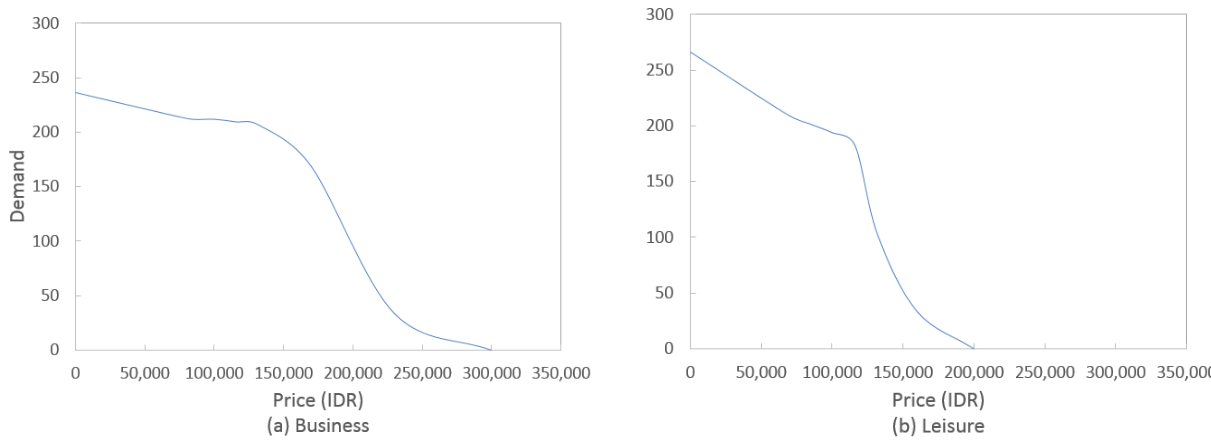

Figure 2 Demand functions for business and leisure passengers

The function of total demand was then obtained by aggregating these two demand functions. Figure 3 shows the aggregated demand function.

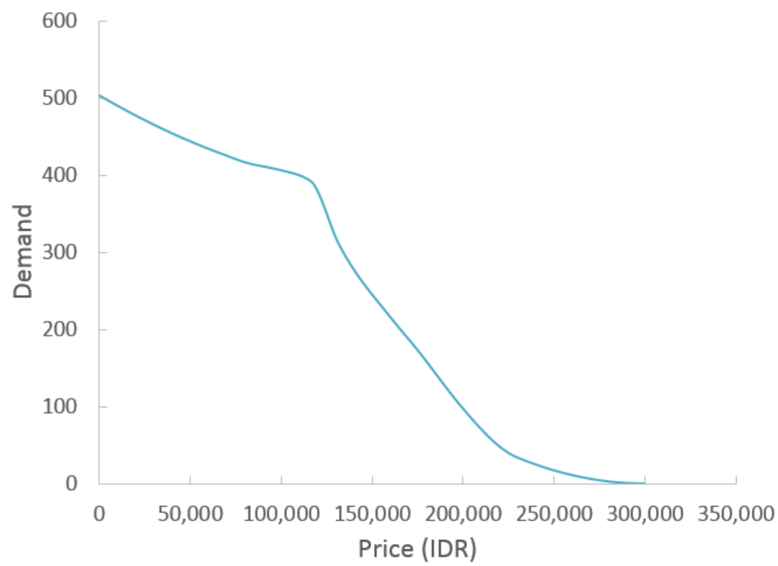

Figure 3 Demand function for total passengers

Based on this total demand function, the cumulative distribution function of demand for the EMSR-a heuristic, $F(x)$, was estimated, again without revealing its explicit mathematical formula. Estimating cumulative distribution function of demand requires assumption about probability distribution of price. A plausible assumption of truncated normally distributed price with mean of Rp150,000 and variance of $(\text{Rp}25{,}000)^2$ was made. This was to ensure that prices are positive and not greater than the satiating price, Rp300,000. The resulted cumulative distribution function of demand is depicted in Figure 4.

The protection levels and booking limits were then calculated based on this cumulative distribution function of demand using Equation 18. Table 2 presents the protection levels and the corresponding booking limits for current fares based on the EMSR-a heuristic. It can be observed from Table 2 that most capacity is allocated to the highest fare. This indicates that if the fares are not bounded above, the optimal fares should be higher.

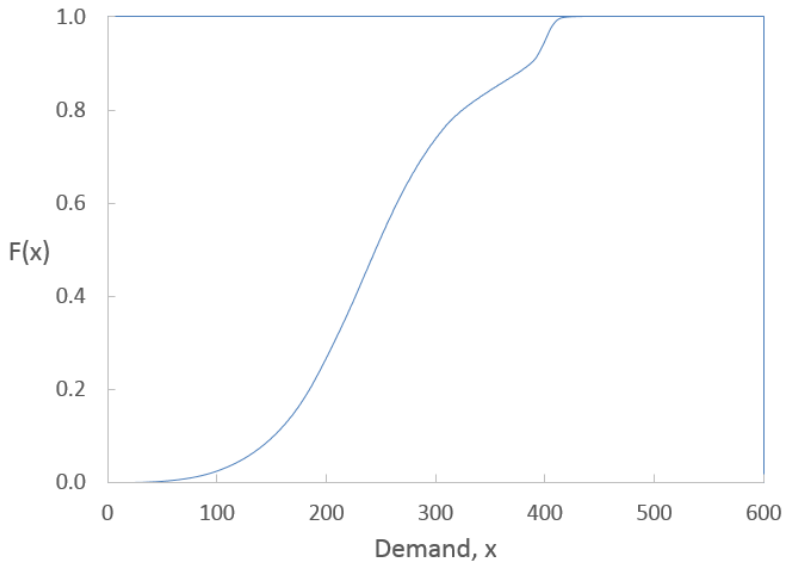

Figure 4 Cumulative distribution function of demand

Table 2 Protection levels and booking limits for current fares

Fare class	Fare	Protection level	Booking limit
1	Rp120,000	384	604
2	Rp110,000	400	220
3	Rp100,000	407	204
4	Rp90,000	604	197

At the next step, an optimization was run to determine optimal fares for this problem. Equation 13-17 were used with one modification, i.e. the value of p_j was restricted to values that are multiple of Rp5,000, which was the current practice in the company. The lower and upper bound for the fares, which were set by the regulation, were Rp80,000 and Rp140,000, respectively. By taking the number of class, m, the same as the current condition, which was four, the optimization problem was solved using enumeration. Solving the optimization problem analytically was not possible since the demand functions was not in an explicit formula. The model came up with optimal fares of Rp140,000, Rp130,000, Rp120,000, and Rp85,000 for fare class 1, 2, 3, and 4, respectively, with total revenue of Rp17,790,000. Comparison with the current condition could not be made since access to the financial data was not granted. But, since the fares were optimal, they should give greater revenue.

Based on these optimal fares, optimal seat allocation for each fare class was determined. Using the same method, optimal protection levels and booking limits as in Table 3 were obtained.

Table 3 Protection levels and booking limits for upper-bounded optimal fares

Fare class	Fare	Protection level	Booking limit
1	Rp140,000	283	604
2	Rp130,000	327	321
3	Rp120,000	393	277
4	Rp85,000	604	211

Similar to the result of seat allocation for the current fares, Table 3 indicates that without an upper bound, the optimal fares should be higher.

Motivated by this observation, another optimization was run to seek optimal fares without an upper bound on tariff. Using Equations 13, 14, 15, and 17, the optimization resulted in optimal fares of Rp225,000, Rp200,000, Rp120,000, and Rp85,000 with total revenue of Rp21,365,000. This was a 20% increase compared to the revenue with upper-bounded optimal fares. Table 4 shows the protection levels and booking limits for these unbounded optimal fares.

Table 4 Protection levels and booking limits for unbounded optimal fares

Fare class	Fare	Protection level	Booking limit
1	Rp225,000	157	604
2	Rp200,000	229	447
3	Rp120,000	393	375
4	Rp85,000	604	211

In March 2018, in addition to the existing four, the company launched a fare class of Rp200,000 with luxurious coach for Argo Parahyangan train. There are two trains in a day that have this new fare class, each of which has two coaches of this type. These coaches are always sold out. This confirmed the results of the optimization models in this research.

4. Conclusions and Future Works

This research developed optimization models for pricing and seat allocation of intercity passenger train services. An approach to estimating one of the most important component of the model, the demand function, based on consumer demand theory was proposed. The HB estimation method, which provides utility estimates at individual level, coupled with market simulation have been a practical and flexible yet accurate way for deriving demand functions. The practicality is due to the availability of software for designing the questionnaire, administering the online survey, estimating the utilities, simulating the market scenario, and estimating the demand function. The flexibility comes from the market simulation which can imitate any competitive scenario and estimate how the market will react accordingly. It is also claimed to be more accurate since the utilities are estimated at individual level and the demand function is interpolated straightforward as the simulation output suggests without assuming any a priori demand function. The disadvantage of using the approach proposed in this paper is when it comes to the pricing optimization. Since the approach does not result in an explicit demand function, analytical solution is impossible. To some extent, enumeration is possible, but for larger problem, this will be cumbersome.

The implementation of the approach in a case study of intercity passenger train service has shown the efficacy of the approach. From the protection levels and booking limits implied by the current fares, it could be inferred that the current fares were too low. The pricing optimization with imposing upper bound on tariff resulted in an increase in average fare, but the corresponding protection levels and booking limits suggested that the fares were still too low. Optimization without upper bound on tariff resulted in an estimated 20% increase in revenue.

Despite its efficacy, a few details on the implementation are still worth noting i.e. determining the appropriate satiating price, assumption regarding the probability distribution of price, and the size of the problem when the number of segment becomes a decision variable. This research assumed that the demand function was deterministic. In fact, the shares of preference from the market simulation were probabilistic. Hence, a more realistic approach to the pricing optimization problem would be assuming probabilistic demand. The research also assumed that there was no cancellation. Modeling approach to relaxations regarding these assumptions would be the future research agenda.

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