Modeling and solving a bi-objective single-period problem within incremental discount in framework of multi-objective problems approach

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Abstract

Single-period problem which is called newsboy problem, is one of the commonplace problem in inventory control. Using inventory control models in each stage of industry cycle has become commonly to determine order quantities and commodity inventory. In this paper, optimizing a bi-objective, multi-product, multi-constraint, single-period problem is considered with incremental discount policy in purchasing commodity to find the order quantities which will be maximized both expected profit and minimized service level. Constraints are budget and warehouse capacity. In addition, decision variables are real and it is assumed that holding and shortage costs occur at the end of a period. Formulation of the problem is presented and shown to be a mixed integer nonlinear programming model. Furthermore, Multi-Objective Decision Making (MODM) approaches are utilized to solve the model with meta-heuristic algorithms. Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) are provided to find an approximate optimum solutions of the problem. After applying RSM method to calibrate the parameters of both algorithms, their performances in solving instances compare in terms of solution quality of both algorithms. In final, GAMS program is applied for validating the solutions.

Keywords

Inventory control; bi-objective; single-period; MODM approaches; meta-heuristic algorithms.

1. Introduction

Inventory control has found a great significance place with progress in technology and development in a large number of systems including industrial, productive, servicing and others. In general, 'newsboy problem' focuses on circumstances where demand for a commodity is random and unsold or unused ordered items become abolished at the end of the cycle. In one hand, buyer may incur a cost to dispose of them; on the other hand, if he initially decides to buy smaller amounts of these commodities; shortages may occur, causing loss of income. In this problem, commodity has the most considerable characteristic of a 'single-period' problem, and the chief purpose is to determine ordered-quantity in order to minimize costs and maximize profit. In real-world situations, many products have a limited period in selling, so the newsboy model is reflective of many real life circumstances and is often used to aid decision-making in fashion, sporting, service industries such as airlines, hotels, to name but a few to manage capacity or evaluate advanced booking of orders. Hence, the classical newsboy problem and its various extensions have been widely studied by many researchers as a substantial number of papers published since 1951.

The first efforts on newsboy problem were published by Arrow et al. (1951) and Morse and Kimball (1951). Hadley and Whitin (1963) proposed and solved a multi-products model in following years. After following decades with escalating interest in this problem, many articles were published in development of different extensions. Silver and Pyke (1998) and Khouja (1999) did complete studies for this problem; as Khouja (1999) published an article entitled "The single-period problem: literature review and suggestions for future research", which examined different extensions of newsboy problem and divided them into several categories. These extensions of newsboy problem that were introduced by Khouja include; (a) considering different objectives and utility functions; (b) applying different discount structures; (c) utilizing multi-product constraint models; and others.

The models to maximize the probability of achieving the profit as a target for newsboy problem were proposed by Lau (1980), Anvari (1987), A. Lau and H. Lau (1988), Silver and Pyke (1998), Das and Maiti (2007). Atkinson (1979), Anvari and Kusy (1990), and Chung (1990) also applied different effectiveness and risk-tolerance criteria for the first type of extension.

While the purchase cost per unit is often fixed, in classical newsboy problem, sometimes, vendor gives discounts to motivate the buyer to purchase earlier in order to decrease the inventory level. As a result, in the second type of extension, Anvari (1987), Pfeifer (1989), Anvari and Kusy (1990), and Chung (1990) analyzed sales discounts policy on quantities for their models. Furthermore, Khouja (1995) introduced a model which was perused the effects of discounts on demand. Chen and Ho (2011, 2013) proposed a model in fuzzy demands with discounts structures.

For the third type of extension, a multi-products problem with budget constraint was considered by A. Lau and H. Lau (1995, 1996), Khouja (1999), Vairaktarakis (2000), Abdel-Malek and Montanari (2005), and Abdel-Malek and Areeratchakul (2007). In addition, Shao and Ji (2006) analyzed in fuzzy modeling. Abdel-Malek et al. (2004) presented and exact solution for this type of extension which demand followed a uniform distribution.

In another type of extensions, Mostard and Teunter (2006) with assuming that sold products could be returned in a specific range of time and could be sold again if not abolished, considered the percentage of the returned products in a single-period model. Keren and Pliskin (2006) calculated the optimum order quantity in a risk-averse model. Chen and Chuang (2006) analyzed newsboy problem with shortage level as constraint. Abdel-Malek and Areeratchakul (2007) proposed the quadratic programming approach in a multi-product newsboy problem with budget, capacity, and order constraints. Moreover, Taleizadeh et al. (2009, 2010) developed a multi-product newsboy model with both incremental and total discounts in which both real and fuzzy costs. Then, they proposed a genetic algorithm to solve the obtained non-linear integer model. Furthermore, Tiwari et al. (2011) considered an unreliable newsboy problem with a forecast update. Murray et al. (2012) introduced a multi-product model to determine the order quantity and price setting.

In this paper, a multi-products newsboy problem would be developed in which demand follows a uniform distribution; warehouse capacity and budget have been considered as constraints and either the incremental discount policy is applied to purchase the items. The overall goal is to determine the optimal order quantity for each product that reaches the purposes of maximizing both of the expected profits and the minimized service level. The rest of the paper is organized as follows. Section 2 is allocated to define and model the problem along with its assumptions which are first introduced parameters and variables of the problem. Next, a single-product problem with incremental discount is modeled. At the end, the multi-product multi-constraint inventory control problem with incremental discount is formulated to solve. In section 3 and 4, MODM approaches and the solution procedures would be proposed. After tuning algorithm parameters in section 5, finally a numerical example is solved to demonstrate the applicability of proposed model analyze.

2. Problem definition and modeling

Commodity is the most significant character in a single period problem. In this paper, the chief goal such that constraints are satisfied would be maximizing both expected profit and minimized service level, simultaneously. Therefore, assumptions of the problem would be as below:

(1) Only one period is considered to be.
(2) Order opportunity is only once and only at the start of a period.
(3) Client demand for each product (j) follows a uniform distribution.
(4) Random variables are independents during demand for all products.
(5) Order quantity of each product should be real.
(6) There is no enforced constraint by the supplier to supply an order.
(7) Entire capacity of the warehouse is assigned to the products.
(8) Target intends to maximize both expected profit and minimized service level.
(9) Shortage is licensable and takes the lost sale condition.
(10) Shortage and holding costs are known and deployed at the end of the period.
(11) Discount for purchasing items is allowed and follows either the incremental discount policy.
(12) Since transportation and order-processing times are relatively very small compared with the cycle length, it is assumed that lead-time is equal to zero, which is common practice in newsboy problems.

Since random variables are independent, in order to maximize both objects in a period, demand for the products is ordered at the beginning of the period. In addition, costs associated with the inventory control system are holding and shortage, one needs to calculate expected inventory level and expected required storage space in each period. Firstly, parameters and variables of the model introduce;

2.1. Problem parameters and variables

Before developing the multi-constraint multi-product newsboy problem, Model parameters and variables are determined:

J : Number of products *(j=1,2,…,J)*.
I : Number of break points *(i=1,2,…,n)*.
X_j : Stochastic demand of j^{th} product.
Q_j : A decision variable exhibiting order quantity of j^{th} product.
$E(Q_j)$: Expected demand for j^{th} product.
$f_{X_j}(x_j)$: Probability mass function of j^{th} product.
P_j : Selling price of j^{th} product.
C_{P_j} : Expected selling cost of j^{th} product.
E_j : Expected revenue of j^{th} product.
IL_j : Expected inventory of j^{th} product at the end of a period.
h_j : Linear coefficient of holding cost of j^{th} product.
C_{h_j} : Expected holding cost of j^{th} product at the end of a period.
SH_j : Expected shortage cost of j^{th} product at the end of a period.
π_j : Linear coefficient of shortage cost of j^{th} product.
C_{π_j} : Expected shortage cost of j^{th} product at the end of a period.
q_{i_j} : i^{th} discount break point of j^{th} product.
C_{i_j} : Purchase cost of j^{th} product in i^{th} break point.
C_{ID_j} : Purchase cost of j^{th} product.
B : Total available budget.
f_j : Space required for each packet of j^{th} product.
F : Total available warehouse space.
Z_1 : Expected profit.
Z_2 : Expected service level.

In next section, a single-product problem is modeled and then it would be modified as a multi-product case.

2.2. Profit modeling – first objective

Revenue and costs (holding, shortage and purchase) equations which are involved to calculate the expected profit, should be introduced in a single-product problem. With considering total demand quantity is more than the order quantity in selling j^{th} product, the gained revenue is Q_j in a period; Otherwise, it is X_j. In other words:

$$\text{Sold quantity} = \begin{cases} Q_j & , X_j \geq Q_j \\ X_j & , X_j < Q_j \end{cases} \tag{1}$$

It is employed the probability mass function for j^{th} product to specify expected sold quantity and expected revenue of j^{th} product at the end of a period:

$$C_{P_j} = \int_{X_j=0}^{Q_j} X_j f_{X_j}(x_j)dX_j + \int_{X_j=Q_j}^{+\infty} Q_j f_{X_j}(x_j)dX_j \tag{2}$$

$$E_j = \int_{X_j=0}^{Q_j} P_j X_j f_{X_j}(x_j)dX_j + \int_{X_j=Q_j}^{+\infty} P_j Q_j f_{X_j}(x_j)dX_j \tag{3}$$

Holding, shortage and purchasing are costs of the problem which they are determined consecutively. For computing these costs, the end-point expected inventory must be specified at the start of the period. Whenever total demand is less than the order quantity, then inventory quantity is $Q_j - X_j$, else it will be zero at the end of a period. Hence:

$$\text{End-period inventory level of } j^{th} \text{ product} = \begin{cases} 0 & , X_j \geq Q_j \\ Q_j - X_j & , X_j < Q_j \end{cases} \tag{4}$$

Calculating expected inventory is mandatory for holding cost at the end of a period. The probability mass function of j^{th} product is used in order to determine the expected inventory, therefore holding cost would be obtained as;

$$IL_j = \int_{X_j=0}^{Q_j} (Q_j - X_j) f_{X_j}(x_j) dX_j \tag{5}$$

$$C_{h_j} = \int_{X_j=0}^{Q_j} h_j (Q_j - X_j) f_{X_j}(x_j) dX_j \tag{6}$$

Facing shortage during a period would miss sale opportunities. The expected shortage cost is calculated the same manner as modeling the holding cost at the end of a period. In this circumstance, shortage quantity will be $X_j - Q_j$, as total demand quantity is more than ordered quantity, else it will be zero. Therefore,

$$\text{shortage quantity of } j^{th} \text{ product} = \begin{cases} X_j - Q_j & , X_j > Q_j \\ 0 & , X_j \leq Q_j \end{cases} \tag{7}$$

Whereupon, expected shortage and shortage cost at the end of a period are generated by:

$$SH_j = \int_{X_j=Q_j}^{+\infty} (X_j - Q_j) f_{X_j}(x_j) dX_j \tag{8}$$

$$C_{\pi_j} = \int_{X_j=Q_j}^{+\infty} \pi_j (X_j - Q_j) f_{X_j}(x_j) dX_j \tag{9}$$

Purchasing cost of j^{th} product at the beginning of a period can be calculated applying incremental discount policy. Let incremental discount policy be,

$$C_{ID_j} = \begin{cases} C_{1_j} Q_j & , 0 < Q_j \leq q_{1_j} \\ C_{1_j} q_{1_j} + C_{2_j}(Q_j - q_{1_j}) & , q_{1_j} < Q_j \leq q_{2_j} \\ \vdots & \\ C_{1_j} q_{1_j} + C_{2_j}(Q_j - q_{1_j}) + \cdots + C_{n_j}(Q_j - q_{n-1_j}) & , Q_j \geq q_{n_j} \end{cases} \tag{10}$$

Where q_{i_j} and $C_{i_j} : i = 1, 2, \dots, n$ are discount point and the purchasing costs for each units of j^{th} product that corresponds to its i^{th} discount break point, respectively.

In order to include incremental discount policy in inventory model, Equation (11) is used to model the incremental discount policy; in which $V_{i_j} : i = 1, 2, \dots, n$ and $j = 1, 2, \dots, J$ are modeling variables to convert Equation (10) to Equation (11),

$$\begin{aligned} & C_{ID_j} = C_{1_j} V_{1_j} + C_{2_j} V_{2_j} + \cdots + C_{n_j} V_{n_j} \\ & Q_j = V_{1_j} + V_{2_j} + \cdots + V_{n_j} \\ & q_{1_j} \alpha_{2_j} \leq V_{1_j} \leq q_{1_j} \alpha_{1_j} \\ & (q_{2_j} - q_{1_j}) \alpha_{3_j} \leq V_{2_j} \leq (q_{2_j} - q_{1_j}) \alpha_{2_j} \\ & \quad \vdots \\ & (q_{n-1_j} - q_{n-2_j}) \alpha_{n_j} \leq V_{n-1_j} \leq (q_{n-1_j} - q_{n-2_j}) \alpha_{n-1_j} \\ & 0 \leq V_{n_j} \leq M \alpha_{n_j} \,,\; \alpha_{1_j} \geq \alpha_{2_j} \geq \cdots \geq \alpha_{n_j} \\ & \alpha_{i_j} = 0, 1 \,,\; V_{i_j} \geq 0 \,,\; \forall i : i = 1, 2, \dots, n \,,\; \forall j : j = 1, 2, \dots, J, M >> +\infty \end{aligned} \tag{11}$$

2.3. Service Level modeling – second objective

In second objective, minimized service level will be maximized. It is applied cumulative distribution function for j^{th} product in order to calculate service level.

$$SL_j = \{F(Q_j)\} = P\{X_j \leq Q_j\} = \int_0^{X_j=Q_j} f_{X_j}(x_j) dX_j \tag{12}$$

2.4. Constraints

Constraints of the problem are budget and warehouse capacity. Budget constraint is determined with incremental discount policy as follows,

$$\sum_{j=1}^{J}\sum_{i=1}^{n} C_{i_j} V_{i_j} < B \tag{13}$$

Since required space for each packet of j^{th} product is $f_j^{(m^2)}$, and total available warehouse space is $F^{(m^2)}$, warehouse space constraint becomes,

$$\int_0^{X_j=Q_j} (Q_j - X_j) f_j f_{X_j}(x_j) dX_j \le F \tag{14}$$

In short, the bi-objective newsboy problem with incremental discount for a single-product will become as,

$$\begin{aligned}
&Max \quad Z_1 = \int_{X_j=0}^{Q_j} P_j X_j f_{X_j}(x_j) dX_j + \int_{X_j=Q_j}^{+\infty} P_j Q_j f_{X_j}(x_j) dX_j - \int_{X_j=0}^{Q_j} \left(h_j(Q_j - X_j)\right) f_{X_j}(x_j) dX_j \\
&\qquad - \int_{X_j=Q_j}^{+\infty} \left(\pi_j (X_j - Q_j)\right) f_{X_j}(x_j) dX_j - \sum_{i=1}^{n} C_{i_j} V_{i_j} \\
&Max \quad Z_2 = Min \int_0^{X_j=Q_j} f_{X_j}(x_j)\, dX_j \\
&s.t.: \quad \sum_{i=1}^{n} C_{i_j} V_{i_j} < B \\
&\qquad \int_0^{X_j=Q_j} (Q_j - X_j) f_j f_{X_j}(x_j) dX_j \le F \\
&\qquad Q_j = V_{1_j} + V_{2_j} + \cdots + V_{n_j} \\
&\qquad q_{1_j} \alpha_{2_j} \le V_{1_j} \le q_{1_j} \alpha_{1_j} \\
&\qquad (q_{2_j} - q_{1_j}) \alpha_{3_j} \le V_{2_j} \le (q_{2_j} - q_{1_j}) \alpha_{2_j} \\
&\qquad \quad \vdots \\
&\qquad (q_{n-1_j} - q_{n-2_j}) \alpha_{n_j} \le V_{n-1_j} \le (q_{n-1_j} - q_{n-2_j}) \alpha_{n-1_j} \\
&\qquad 0 \le V_{n_j} \le M \alpha_{n_j}, \ \alpha_{1_j} \ge \alpha_{2_j} \ge \cdots \ge \alpha_{n_j} \\
&\qquad \alpha_{i_j} = 0,1, \ V_{i_j} \ge 0, \ \forall i : i = 1,2,\ldots,n, \ \forall j : j = 1,2,\ldots,J, M >> +\infty, \ Q_j, B, F \ge 0
\end{aligned} \tag{15}$$

2.5. Final Model

As a result, single product model in Equation (15), can be easily extended to a multi-product model in Equation (16),

$$\begin{aligned}
&Max \ Z_1 = \sum_{j=1}^{J} \int_{X_j=0}^{Q_j} P_j X_j f_{X_j}(x_j) dX_j + \sum_{j=1}^{J} \int_{X_j=Q_j}^{+\infty} P_j Q_j f_{X_j}(x_j) dX_j \\
&\qquad - \sum_{j=1}^{J} \int_{X_j=0}^{Q_j} \left(h_j(Q_j - X_j)\right) f_{X_j}(x_j) dX_j - \sum_{j=1}^{J} \int_{X_j=Q_j}^{+\infty} \left(\pi_j (X_j - Q_j)\right) f_{X_j}(x_j) dX_j - \sum_{j=1}^{J}\sum_{i=1}^{n} C_{i_j} V_{i_j} \\
&Max\, Z_2 = \left(\underset{j}{Min} \int_0^{X_j=Q_j} f_{X_j}(x_j)\, dX_j \right) \\
&s.t.: \quad \sum_{j=1}^{J}\sum_{i=1}^{n} C_{i_j} V_{i_j} < B \\
&\qquad \sum_{j=1}^{J} \int_0^{X_j=Q_j} (Q_j - X_j) f_j f_{X_j}(x_j) dX_j \le F \\
&\qquad Q_j = V_{1_j} + V_{2_j} + \cdots + V_{n_j} \\
&\qquad q_{1_j} \alpha_{2_j} \le V_{1_j} \le q_{1_j} \alpha_{1_j} \\
&\qquad (q_{2_j} - q_{1_j}) \alpha_{3_j} \le V_{2_j} \le (q_{2_j} - q_{1_j}) \alpha_{2_j} \\
&\qquad \quad \vdots \\
&\qquad (q_{n-1_j} - q_{n-2_j}) \alpha_{n_j} \le V_{n-1_j} \le (q_{n-1_j} - q_{n-2_j}) \alpha_{n-1_j} \\
&\qquad 0 \le V_{n_j} \le M \alpha_{n_j}, \ \alpha_{1_j} \ge \alpha_{2_j} \ge \cdots \ge \alpha_{n_j} \\
&\qquad \alpha_{i_j} = 0,1, \ V_{i_j} \ge 0, \ \forall i : i = 1,2,\ldots,n, \ \forall j : j = 1,2,\ldots,J, M >> +\infty, \ Q_j, B, F \ge 0
\end{aligned} \tag{16}$$

3. MODM approaches

Developed model in last section is a constrained bi-objective mixed integer nonlinear programming for newsboy problem. General configuration of mathematical model with n variables in vector $X = \{x_1, x_2, \cdots, x_n\}$ is;

$$\begin{aligned} & Max \;\; f_j(X) \qquad , j = 1, 2, \cdots, k \\ & s.t.: \;\; g_i(X) \geq 0 \; , i = 1, 2, \cdots, m \\ & \qquad X \in IR^n \end{aligned} \tag{17}$$

Where k and m represent the number of objective functions and constraints. An ideal solution for a problem modeled in (17) optimizes all the objective functions concurrently while all constraints are satisfied. Albeit, many problems in real world involve conflicting objectives such that a feasible solution cannot optimize all the objective functions simultaneously. Finally, decision makers seek an effective, preferred solution.

In addition to multi-objective optimization approaches that provide Pareto fronts, various methods are accessible in the literature to solve multi-objective programming models. Initially, the model is solved by individual optimization method which gains an effective solution for each objective, exclusively. If all effective solutions are the same in terms of all constraints, an ideal overall solution is obtained. Then problem is converted to a single-objective optimization using LP metrics (global criteria; Hwang and Masud,1979) and Goal Attainment methods which are construed in next subsections. Eventually, single objective optimization would be solved by two meta-heuristic algorithms.

3.1. LP metrics (global criteria) method

The aim of this method is to obtain a solution that minimizes digression (D) between k objective function values in a multi-objective model *($f_j(X)$: j=1,2,…,k)* and their corresponding ideal solution gained by individual optimization method *($f_j(X^*)$: j=1,2,…,k)* in maximization type. In other words, all the objectives must be converted into a maximization type in order to minimize '*D*' defined as,

$$Min\, D = \left(\sum_{j=1}^{k} \left(\frac{f_j(X^*) - f_j(X)}{f_j(X^*)} \right)^P \right)^{\frac{1}{P}} \tag{18}$$

Where P would be selected '1' in this paper.

3.2. Goal attainment method

This method has proposed by Gembiki (1975) and it tries to find a solution that minimizes the highest deviation (Z) between individual and overall objective functions values, where positive weights (w_j) are assigned such that $\sum_{j=1}^{k} w_j = 1$. In other words, the following mathematical problem is solved in this method,

$$\begin{aligned} & Min \;\; Z \\ & s.t.: g_i(x) \leq 0 \qquad\qquad ; i = 1, 2, \cdots, m \\ & \quad f_j(x) + w_j Z \geq f_j^* \;\; ; j = 1, 2, \cdots, k \;, Z : free \end{aligned} \tag{19}$$

In next section, meta-heuristic algorithms will be introduced to solve the model.

4. Solution Procedures

Since the model in Equation (16) is a mixed integer nonlinear programming, obtaining an analytical solution (if any exists) to the problem, especially in its large-scale form is difficult or even impossible to solve with exact methods (Kuk, 2004). As a result, stochastic research algorithms are used in this section to solve the model. However, as the model has two objectives, MODM approaches are first converted it to a single objective, and then single-objective evolutionary algorithms, such as GA and PSO are applied to solve the single-objective problem. Since no benchmark is available to assess its capability, performances of these algorithms are compared in terms of both solution quality and an attempt to get the verification. In next section, parameters of both algorithms are calibrated using RSM method.

4.1. Genetic Algorithm (GA)

The main information unit of any living organism is the gene, which is a part of a chromosome that determines specific traits such as eye-color, complexion, hair-color, and the like. The fundamental principle of the genetic algorithm, which was inspired by the concept of the survival of fittest, first was introduced by Holland (1975); however, the custom form of GA was presented by Goldberg in 1989 (Michaelraj and Shahabudeen, 2009). Since then, many researchers have applied and expanded this concept in different fields of study. In genetic algorithm, optimal solution is the conqueror of the genetic competition and any potential solution is assumed to be a creature determined by different parameters. These parameters are considered as genes of chromosomes that the nearest to the optimal solution would be selected as the better one. In practical applications, evolutionary process of species that reproduce and operate on a set of current solutions called a population. Populations of chromosomes are created randomly and number of these populations varies in each problem. Moreover, some hints about selecting proper number of population exist in different reports by Man et al. (1997). In each iteration, the crossover mechanism

generates new generation that combines genetic legacies of each parent. With some low probability, a portion of the new individuals, called offspring apply a random mutation. This process will pursue until a stopping criteria is satisfied. Then, best offspring is selected as a near optimum solution. The steps of GA utilized in this paper would be;
(1) Set the parameters (population size *Npop*, crossover probability P_c, mutation probability P_m, selection strategy, crossover operation, mutation operation, and number of generations as stopping criteria). Some of these parameters are calibrated using RSM method in next section.
(2) A chromosome is a *J*-dimensional vector that shows a possible solution (appropriate or non-appropriate) for order quantities of the products $(Q_j\text{s})$.
(3) A group of chromosomes is called population. Initial population containing *Npop* chromosomes is randomly generated.
(4) Genetic operators such as crossover and mutation generate the next population. Single-point crossover operator is used with the probability of P_c on each chromosome. Moreover, mutation is applied with probability P_m on offspring that are generated using crossover operation.
(5) Since chromosomes with better fitness should have a greater chance of selection than those with worse fitness, the roulette wheel selection method is used to evaluate new chromosomes whether a solution (represented by chromosomes) is appropriate or not. In this method, selection operates proportional to relative fitness of the chromosomes. Consequently, *Npop* chromosomes are chosen through parents and offspring according to their fitness function. It should be noted that after producing a new generation, both old and new generation are first sorted based on fitness function and surplus chromosomes are eliminated from the population.
(6) Generation process is repeated until a predetermined number of generation is performed as a stopping criteria.

4.2. Particle Swarm Optimization (PSO) Algorithm

Kennedy and Eberhart (1995) presented PSO as an optimization technique firstly in 1995. Similar to GA, PSO is a powerful population-based search algorithm. They were inspired by social organism behavior such as fish schooling, bird flocking and swarming theory to investigate the effect of a group species cooperation in order to reach their aims. Dynamics of bird flocking were studied for years and it was shown that it is possible to employ this behavior as an optimization tool. In a PSO system, multiple solutions candidate co-exist and cooperate simultaneously. Each solution candidate, called a 'particle', probes in problem search space, searching for optimal position to land. A predefined function named fitness function is utilized to analyze performance of each particle which is usually proportional to the objective function. Every particle, considers both of its own and neighboring particles experience to optimize its position as time passes through its exploration. Particle's experience is built up of tracking and memorizing the best position dealt (encountered). Consequently, PSO algorithm owns a memory (i.e. every particle memorize its best position achieved during entire process). In fact, PSO system attempts to balance exploration and exploitation by combining local search methods through self-experience with global search methods through neighboring experience. Assuming that search space is *J*-dimensional, the *J*-dimensional vector $X_{ij} = (x_{i1}, x_{i2}, ..., x_{iJ})$ represents i^{th} particle of the swarm and the particle which found the best fitness function heretofore which is called the best particle of the swarm is denoted by index g_j. $P_{ij} = (p_{i1}, p_{i2}, ..., p_{iJ})$ demonstrates the best previous position of i^{th} particle and the position change (velocity) of i^{th} particle is represented by $V_{ij} = (v_{i1}, v_{i2}, ..., v_{iJ})$. The particles are operated according to the following Equations,

$$v_{ij}(t+1) = w \cdot v_{ij}(t) + r_1 \cdot c_1 \cdot \left(p_{ij}(t) - x_{ij}(t)\right) + r_2 \cdot c_2 \cdot \left(g_j - x_{ij}(t)\right) \tag{20}$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \tag{21}$$

Where $i=1,2,\ldots, Npop$ is swarm's size, t is iteration number, and w is inertia weight. As experimental results confirm setting the inertia weight to a large value initially is beneficial to promote global exploration of search space, and then decrease it gradually to achieve refined solutions at final stage of the search (Eberhart and Shi, 2001). r_1 and r_2 are two random numbers uniformly distributed within the range $[0,1]$. c_1 and c_2 called cognitive and social parameter, respectively are two positive constants. Appropriate selection of w, c_1 and c_2 could result in a balance between global and local search. Equation (20) is employed to investigate particle's new velocity according to its previous velocity and the distances of its current position from its own best historical position and the best found position by any particle. Then particle probes toward a new position according to Equation (21). This process is repeated until stopping criteria is satisfied.

5. Result and discussion

Since quality of the solution obtained by any meta-heuristic algorithm such as GA and PSO depends on values of their parameters which are controllable factors, in this section, Response Surface Method (RSM) is employed to tune the parameters.

Originally, RSM was developed to model experimental responses (Box and Draper, 1987), and then migrated into the modelling of numerical experiments. This method is a mathematical and statistical technique for empirical model building. By careful design of experiments, objective is to optimize a response (output variable) which is influenced by several independent variables (input variables). An experiment is a series of tests, in which changes are made in input variables in order to identify the reasons for changes in output response.

Parameters that may have significant impacts on response are first selected for calibration. Then, using a trial and error procedure, the values that present proper fitness function are selected to implement the experiments. At the end, two levels are selected for the boundary of parameters to be considered in the experiments. In other words, four parameters each with two levels are considered in calibration process of both PSO and GA. Tables 1. shows the parameters along with their tuned. A numerical example with three break points of incremental discount and different size of products with basic data for thirty runs is considered for a typical calibration.

Comparing results reveals that GA verifies approximate optimal solutions obtained by PSO. It should be noted that best levels of parameters are dependent and alter when different problems along with different sizes are investigated. Moreover, all programming codes are written in MATLAB R2014a and parameters are tuned by Design Expert 7.0.0, where a Laptop with 2.50 GHz Intel Core i5-3210M CPU, and 4 GB of RAM memory is used for all calculations.

Table 1. Parameters along with tuned Parameters

MODM	LP-Metrics				Goal Attainment			
Algorithm	GA		PSO		GA		PSO	
Variables	Range	**Tuned**	Range	**Tuned**	Range	**Tuned**	Range	**Tuned**
Npop	100 ~ 250	**250**	100 ~ 300	**264**	150 ~ 350	**272**	150 ~ 300	**292**
Pc	0.5 ~ 0.9	**0.9**	0.4 ~ 1.2	**0.62**	0.55 ~ 0.9	**0.89**	0.4 ~ 1.15	**1.08**
Pm	0.01 ~ 0.1	**0.01**	0.75 ~ 1	**0.76**	0.01 ~ 0.34	**0.17**	0.75 ~ 1	**0.93**
iteration	500 ~ 1000	**809**	1.12 ~ 2.83	**2.04**	680 ~ 2000	**1509**	1.12 ~ 2.95	**1.44**

5.1. Numerical Examples

For better solution verification of both algorithms, thirty numerical problems with three break points of incremental discount are considered on which both parameter-tuned GA and PSO are applied that are listed in Table 1. Instances are employed for seven to fifteen products with their parameters that are randomly changing based on a Uniform distribution within the corresponding intervals which are exhibited in Table 2. Each problem is solved different times using each algorithm with each MODM approach. The best solution in terms of quality is chosen for comparison. Results for a 15-product instance fitting MODM approaches by using GA and PSO are depicted in Figure 1 and 2 which horizontal axial shows Number of Function Evaluation (NFE).

Table 2. Model Parameters

Variable	P_j	h_j	π_j	f_j	C_{i_j}	q_{i_j}
Distribution	$u \sim [30,70]$	$u \sim [1,15]$	$u \sim [4,15]$	$u \sim [2,15]$	$u \sim [10,50]$	$u \sim [0,100]$

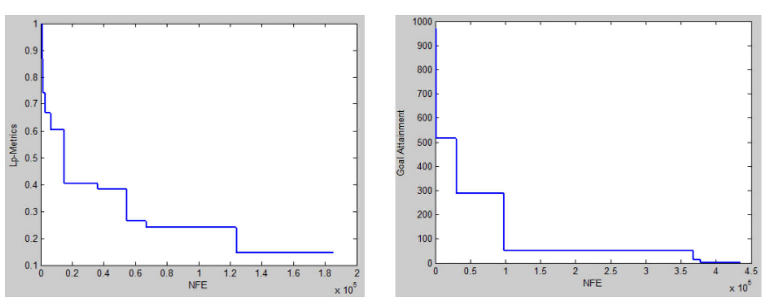

Figure 1. Results of MODM approaches using GA

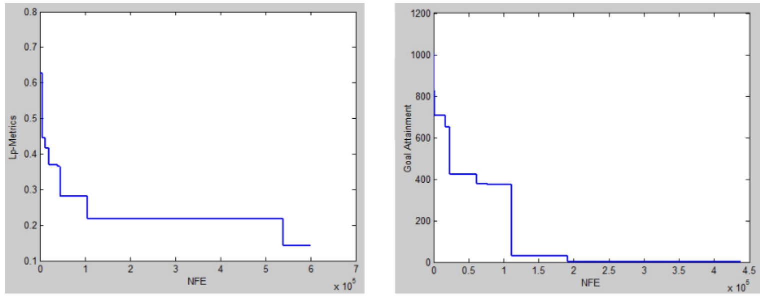

Figure 2. Results of MODM approaches using PSO

5.2. Statistical comparison

After running examples and collecting related data, obtained information is analyzed by statistical software SPSS statistics 21. Comparing results in Table 3 illustrates that between two MODM approaches via GA and PSO by utilizing statistical t-student test ($\alpha = 0.05$) for independent groups in both objectives are the same statistically, which means that there is no meaningful statistical differences between approaches and algorithms on calculations of both objectives. Moreover, in terms of solution quality (the approximate optimal order quantities), results obtained by PSO in Goal attainment method and obtained by GA in LP metrics method would be better than the results in GA within the Goal attainment method and in PSO within the LP metrics method for both objectives.

Table 3. Independent Group Test Information

Group	Algorithm	MODM	Objective	Runs	Mean	Std. Error Difference	*df*	t-student
1	GA	LP-metrics	1th	30	**1007.957**	58.68	58	0.174
	PSO				997.709			
2	GA	LP-metrics	2th	30	**0.259**	0.009	58	-0.9
	PSO				0.268			
3	GA	GoalAttainment	1th	30	1113.09	65.91	58	-0.327
	PSO				**1134.67**			
4	GA	GoalAttainment	2th	30	0.2385	0.0132	58	0.437
	PSO				**0.2327**			

5.3. Validation

Some numerical instances with both of MODM approaches are analyzed by GAMS 23.6 software which is an exact solution method to validate the results obtained from GA and PSO. Table 4. Shows that there is no significance discrepancy between both of objective functions in GAMS, GA and PSO. In addition, GA and PSO generate approximately optimized solution for different sizes while GAMS could usually solve the problems in lower sizes.

Table 4. Results validation

	1th objective						2th objective					
	LP metrics			Goal Attainment			LP metrics			Goal Attainment		
Prods	GA	PSO	GAMS	GA	PSO	GAMS	GA	PSO	GAMS	GA	PSO	GAMS
7	592.47	546.23	626.21	613.99	580.97	661.87	0.27	0.29	0.37	0.24	0.29	0.24
8	768.17	889.33	924.82	852.61	903.35	992.84	0.32	0.27	0.55	0.21	0.27	0.43
9	1044.5	1001.9	903.99	1227.4	1370.9	951.40	0.28	0.29	0.46	0.24	0.33	0.39
10	1061.9	1160.3	973.92	1229.2	1159.3	1015.8	0.23	0.26	0.47	0.22	0.32	0.41
11	1132.1	1037.3	1046.7	1215.1	1261.4	1105.1	0.24	0.26	0.38	0.23	0.18	0.26
12	828.15	849.81	888.11	1018.7	1054.9	950.19	0.28	0.25	0.34	0.22	0.19	0.25
13	1063.3	1307.3	-	1176.8	1336.6	-	0.265	0.22	-	0.24	0.21	-
14	1149.2	1033.1	-	1174.4	1201.2	-	0.22	0.21	-	0.23	0.12	-
15	1335.3	1293.7	-	1513.1	1632.2	-	0.22	0.26	-	0.16	0.21	-

6. Conclusion

In this research, a bi-objective single-period problem under incremental discount was investigated. The problem was modelled on multi-product with budget and warehouse capacity constraints. The aim of this paper was to determine the order quantities such that expected profit and minimized service level were maximized. The problem was formulated into a mixed integer nonlinear programming and it was solved using two MODM approaches via two meta-heuristic algorithms. In section 5, parameters of both algorithms were tuned using RSM method and then tuned algorithms were compared based on t-student test applying various numerical problems of different sizes in order to optimize both objectives which were provided to demonstrate the application and to compare the performance of both algorithms in both approaches. Moreover, the solution procedures were validated by GAMS program up to twelve products. It could be mentioned that this paper contributes in decision making of order quantities in the inventory control in a large number of industrial cycles.

For future work extensions, the followings are recommended

(1) Parameters of the model can be investigated in fuzzy form to bring the application closer to reality.

(2) It could be considered a parameter-tuned bi-objective meta-heuristic algorithm such as MOPSO, NSGA-II, etc to find Pareto solutions.

(3) In addition to mentioned constraints, further constraints such as transportation costs and total discount could be attended.

(4) Taguchi method can be employed to tune the parameters of algorithms as well.

References

Abdel-Malek, L., Montanari, R., and Morales, L. C., Exact, approximate, and generic iterative models for the multi-product newsboy problem with budget constraint. *International Journal of Production Economics*, vol. 91, no. 2, pp. 189-198, 2004.

Abdel-Malek, L. L., and Areeratchakul, N., A quadratic programming approach to the multi-product newsvendor problem with side constraints. *European Journal of Operational Research*, vol. 176, no. 3, pp. 1607-1619, 2007.

Abdel-Malek, L. L., and Montanari, R., An analysis of the multi-product newsboy problem with a budget constraint. *International Journal of Production Economics*, vol. 97, no. 3, pp. 296-307, 2005.

Anvari, M., and Kusy, M., Risk in inventory models: Review and implementation. *Engineering Costs and Production Economics*, vol. 19, pp. 267-272, 1990.

Anvari, M., Optimality criteria and risk in inventory models: The case of the newsboy problem. *Journal of the Operational Research Society*, vol. 38, no. 7, pp. 625-632, 1987.

Arrow, K. J., Harris, T., and Marschak, J., 1951. Optimal inventory policy. *Econometrica: Journal of the Econometric Society*, pp. 250-272, 1951.

Atkinson, A. A., Incentives, uncertainty, and risk in the newsboy problem. *Decision Sciences*, vol. 10, no. 3, pp. 341-357, 1979.

Box, G. E., and Draper, N. R., *Empirical model-building and response surfaces*. John Wiley & Sons, New Jersey, 1987.

Chen, M. S., and Chuang, C. C., An extension to newsboy problem with shortage-level constraint. *International Journal of Productivity Economy*, vol. 67, pp. 269–277, 2006.

Chen, S. P., and Ho, Y. H., Analysis of the newsboy problem with fuzzy demands and incremental discounts. *International Journal of Production Economics*, vol. 129, no. 1, pp. 169-177, 2011.

Chen, S. P., and Ho, Y. H., Optimal inventory policy for the fuzzy newsboy problem with quantity discounts. *Information Sciences*, vol. 228, pp. 75-89, 2013.

Chung, K. H., Risk in inventory models: The case of the newsboy problem—Optimality conditions. *Journal of the Operational Research Society*, vol. 41, no. 2, pp. 173-176, 1990.

Das, B., and Maiti, M., An application of bi-level newsboy problem in two substitutable items under capital cost. *Applied Mathematics and Computation*, vol. 190, no. 1, pp. 410-422, 2007.

Eberhart, R. C., and Shi, Y., Particle swarm optimization: Developments, applications and resources. *In: Proceedings of Congress on Evolutionary Computation, IEEE Service Center*. Piscataway New Jersey, pp. 81-86, 2001.

Gembicki, F. and Haimes, Y., Approach to performance and sensitivity multiobjective optimization: The goal attainment method. *IEEE Transactions on Automatic control*, vol. 20, no. 6, pp. 769-771, 1975.

Hadley, G., and Whitin, T. M., *Analysis of inventory systems*, 2nd, Cliffs, N.J., Prentice Hall, 1963.

Holland, J. H., *Adoption in neural and artificial systems*. The University of Michigan Press, Ann Arbor, 1975.

Hwang, C. L., and Masud, A. S. M., *Multiple objective decision making—methods and applications: a state-of-the-art survey (Vol. 164)*. Springer Science & Business Media, 1979.

Kennedy, J., and Eberhart, R., Particle swarm optimization. *In: Proceedings of IEEE international conference on neural networks. Perth WA Publishing, Australia*, pp. 1942‑1948, 1995.

Keren, B. and Pliskin, J. S., A benchmark solution for the risk-averse newsvendor problem. *European Journal of Operational Research*, vol. 174, no. 3, pp. 1643-1650, 2006.

Khouja, M., The newsboy problem under progressive multiple discounts. *European Journal of Operational Research*, vol. 84, no. 2, pp. 458-466, 1995.

Khouja, M., The single-period (news-vendor) problem: literature review and suggestions for future research. *Omega*, vol. 27, no. 5 pp. 537-553, 1999.

Kuk, G., Effectiveness of vendor-managed inventory in the electronics industry: determinants and outcomes. *Information & management*, vol. 41, no. 5, pp. 645-654, 2004.

Lau A. H. L., and Lau, H. S., Maximizing the probability of achieving a target profit level in a two-product newsboy problem. *Decision Sciences*, vol. 19, no. 2, pp. 392-408, 1988.

Lau, H. S. and Lau, A. H. L., The multi-product multi-constraint newsboy problem: Applications, formulation and solution. *Journal of operations management*, vol. 13, no. 2, pp. 153-162, 1995.

Lau, H. S. and Lau, A. H. L., The newsstand problem: A capacitated multiple-product single-period inventory problem. *European Journal of Operational Research*, vol. 94, no. 1, pp. 29-42, 1996.

Lau, H. S., The newsboy problem under alternative optimization objectives. *Journal of the Operational Research Society*, vol. 31, no. 6, pp. 525-535, 1980.

Man, K. F., Tang, K. S., Kwong, S., and Halang, W. A., *Genetic algorithms for control and signal processing*. Springer-Verlag, London, 1997.

Michaelraj, L. A., and Shahabudeen, P., Replenishment policies for sustainable business development in a continuous credit based vendor managed inventory distribution system. *Computers & Industrial Engineering*, vol. 56, no. 1, pp. 260-266, 2009.

Morse, M. P., and Kimball, G. E., *Method of Operation Research*. MIT Press, Cambridge MA, 1951.

Mostard, J., and Teunter, R., The newsboy problem with resalable returns: A single period model and case study. *European Journal of Operational Research*, vol. 169, no. 1, pp. 81-96, 2006.

Murray, C. C., Gosavi, A., and Talukdar, D., The multi-product price-setting newsvendor with resource capacity constraints. *International Journal of Production Economics*, vol. 138, no. 1, pp. 148-158, 2012.

Pfeifer, P. E., The airline discount fare allocation problem. *Decision Sciences*, vol. 20, no. 1, pp. 149-157, 1989.

Shao, Z., and Ji, X., Fuzzy multi-product constraint newsboy problem. *Applied Mathematics and Computation*, vol. 180, no. 1, pp. 7-15, 2006.

Silver, E. A., Pyke, D. F., and Peterson, R., *Inventory management and production planning and scheduling* (vol. 3, p. 30). New York: Wiley, 1998.

Taleizadeh, A. A., and Akhavan Niaki, S. T., A hybrid meta-heuristic method to optimize bi-objective single period newsboy problem with fuzzy cost and incremental discount. *Journal of Optimization in Industrial Engineering*, vol. 3, pp. 1-13, 2010.

Taleizadeh, A. A., Akhavan Niaki, S. T., and Hoseini, V., Optimizing the multi-product, multi-constraint, bi-objective newsboy problem with discount by a hybrid method of goal programming and genetic algorithm. *Engineering Optimization*, vol. 41, no. 5, pp. 437-457, 2009.

Tiwari, D., Patil, R., and Shah, J., Unreliable newsboy problem with a forecast update. *Operations Research Letters*, vol. 39, no. 4, pp. 278-282, 2011.

Vairaktarakis, G. L., Robust multi-item newsboy models with a budget constraint. *International Journal of Production Economics*, vol. 66, no. 3, pp. 213-226, 2000.

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