Vehicle Routing Problem for Blood Mobile Collection System with Stochastic Supply

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Abstract

The mobile collection system of blood products is considered in this study. Blood centers often use bloodmobiles that park near crowded places where donors can donate blood directly. We propose the use of additional vehicles, called shuttles, that pick up the collected blood by the bloodmobiles. Hence, bloodmobiles can continue their tours without having to return to the blood center. The system manager must decide the set of sites to visit by the bloodmobiles among a group of potential sites, and to determine the tours of the vehicles responsible for this operation. In this paper, the blood mobile collection system is modelled as a vehicle routing problem with profits. The objective is to minimize the total routing, wastage and shortage costs. Each collection site has a random potential blood quantity that is modeled as a stochastic profit which can be collected by a vehicle when it visits this site. A Two-Stage Stochastic Model with recourse is developed to represent the problem using a scenario-based approach. The fast-forward selection algorithm is implemented to reduce the set of scenarios. Experiments are performed considering Poisson distributed profits.

Keywords
Blood supply chain, blood products, Stochastic Programming, scenario reduction, blood collection process.

1. Introduction

Blood has different functions including the transportation of oxygen and nutrients to the lungs and tissues [1]. The main transfusable blood components are: red blood cells, platelets, plasma and cryoprecipitated AHF condition [2]. These components can be mechanically separated from a unit of Whole Blood (WB), or can be obtained using apheresis, an automated procedure that can filter only the desired components from the blood of donors [3]. WB and blood components are necessary for cancer treatments, surgeries, organ transplants, among others. The Blood Supply Chain (BSC) controls the flow of blood products from donors to patients. The aim of the BSC is to ensure that blood units are available when they are required by a person. The BSC consists of five echelons: donors, mobile donation sites, Blood Centers (BC), hospitals and patients. BC and hospitals are responsible for the collection, testing, processing, storage, distribution and transfusion processes.

Donations are the only source because blood cannot be produced in a synthetic way. Donations are obtained from donor at a BC or through bloodmobiles which park near universities, companies or crowded places. The 2013 AABB Blood Survey estimates that 13.6 million Whole Blood (WB) and red blood cells units were collected in US, a significant decline of 12.1% than 2011. On the contrary, the transfusion of platelets was 15.4% more in 2013 than 2011 [4]. On the one hand, considering the scarcity of blood products and the costs associated with collecting, processing and transporting, the wastage of this resource is undesirable. On the other hand, shortage is also undesirable since it may cause deaths and many patients suffering from ill-health [5]. For these reasons, optimizing process in the BSC is considered of vital importance.

The Vehicle Routing Problem with Profits (VRPP) is considered to model the decisions of selecting the mobile donation sites and routing blood vehicles. VRPP is a variant of Vehicle Routing Problem (VRP) introduced by Dantzing and Ramser in 1959 [6]. The VRP can be defined as the problem of designing optimal delivery or collection routes from one or several depots to a number of geographically scattered customers, subject to constraints [7]. The key characteristic of VRPP is that, contrary to what happens for the classical VRP, it is not mandatory to visit the whole set of customers [8]. The first decision to take in the VRPP is which customers to visit. The second one is the routes of the vehicle fleet to serve the selected customers. In general, a profit is associated with each customer that makes such a customer more or less attractive [8]. Any route or set of routes can be measured both in terms of costs and in terms of profit. Basic problems of this class with only one route are known in the literature as the Traveling Salesman Problem with Profits (TSPP).

The number of donation units over a time horizon is uncertain. Considering the supply as an uncertain parameter allows to make better decisions than defining it as deterministic. Stochastic Programming is used to find an optimal decision in environments with random parameters in which their probability distributions are known [9]. One way to model stochastic problems is through *probabilistic* (also called *chance*) constraints [10]. Consider A, x, and b are $m \times n$ matrix, n-vector, and m-vector, respectively. Let $Ax \geq b$ be a deterministic linear constraint in which x is the decision variable vector. Assuming uncertainty for matrix A, then $P(Ax \geq b) \geq \alpha$ is a probabilistic linear constraint saying that $Ax \geq b$ should be satisfied with a pre-specified probability $\alpha \in (0,1)$ [9]. For modeling stochastic problems, the two-stage and multi-stage programs with recourse are also used. In two-stage stochastic linear programming problems, we have a set of decisions x to be taken without full information on some random events ξ. Later, full information is received on the realization of vector ξ. Then, corrective actions y are taken. The linear programming equivalent for the two-stage problem is developed considering that ξ has a finite number of *scenarios* ξ_k with respective probabilities $p_k, k = 1, .., K$ [10]. The two-stage stochastic programs can be extended to a multi-stage setting. In the multi-stage setting, the uncertain data $\xi_1, \ldots, \xi_T$ is revealed gradually over time, in T periods, and decisions should be adapted to this process [10].

The rest of the document is structured as follows. In the next section, a brief review of existing literature related to decision-making in BSC as well as modelling approaches dedicated to stochastic studies is presented. Section 3 defines the problem and present the model formulation. In section 4, a scenario reduction algorithm is stated. Numerical results are given in section 5. Finally, concluding remarks and opportunities for future work are presented in section 6.

2. Literature review

Some authors have conducted literature reviews on BSC. Beliën et al. [3] present a review of papers published until 2010. In this document, Beliën et al. [3] classify the material according to product type, solution method, supply chain echelon, performance measures and practical implementation/study cases considered in articles. Osorio et al. [11] developed a literature review paper of the BSC, which covers papers published up to 2014. Osorio classifies the reviewed documents according to the echelons considered in the supply chain and the processes. In general, the authors mentioned above, expose that main problems in BSC are the location and routing of mobile donation sites and the inventory management.

The location of mobile donation sites is studied in the literature by Chaiwuttisak et al. [12], Alfonso et al. [13], Zahiri et al. [14], Ramezanian et al. [15] and Arvan et al. [16]. Zahiri is the only author who considers both the supply and demand as stochastic parameters. In this study, Zahiri develops a Mathematical Programming Model using Chance-Constrained Programming. The problem of routing mobile donation sites, considering deterministic parameters, has been studied by authors such as Mobasher et al. [17], Sahinyazan et al. [18], Ganesh et al. [19], Ghandforoush et at. [20], Gunpinar et al. [21] and Hemmelmayr et al. [22]. Sahinyazan formulates the problem as a VRPP and develops heuristics to find near-solutions. Gunpinar uses Mixed Integer Linear Programming to model the exposed problem and the remaining authors develop heuristics or metaheuristics additionally as approximation methods.

Rabbani et al. [23] and Zahiri et al. [24] study the problem of location and routing mobile donation sites. Zahiri considers the donation quantity and the demand as stochastic parameters and formulated the problem as a Mixed Integer Linear Program. Rabbani considers the number of donors as a fuzzy parameter and develops a Fuzzy Mathematical Programming Model to obtain solutions for small instances, and for larger instances the authors propose a Simulated Annealing (SA).

To our knowledge, there are few articles that include uncertain parameters in the BSC optimization. This paper deals with the VRP with stochastic profits and proposes a mathematical model to determine where to locate mobile collection sites in order to minimize shortage and wastage levels. The next section details the developed model.

3. Model formulation

3.1. Description of the mobile collection system and notation

Bloodmobiles have the necessary equipment and staff for the blood donation procedure. From a set of potential sites, the BC determines the location of bloodmobiles on each period of a planning horizon. Blood collection at the selected site is a whole day activity including the traveling, set-up and collection times [18]. That is, a bloodmobile cannot visit multiple locations in a day. Each site has a potential donation or supply associated with the day the collection activities are performed on that location. Due to the perishability of WB, the blood units need to be transported to the BC for testing and processing within a maximum of 24 hours after its collection [25]. In a regular configuration of the mobile collection system, bloodmobiles should return to the BC with the collected blood at the end of each period. Some authors in the literature propose also including a vehicle called shuttle, in addition to the regular bloodmobile [18], [20], [23], [26]. The main function of the shuttles is to transport the collected WB from bloodmobiles to the BC at the end of each day. With the help of shuttles, the bloodmobile only returns to BC at the end of the planning horizon or the day when the bloodmobile has no scheduled collection activities at any location. Another activity related to shuttles is to supply bloodmobiles with the necessary inputs for the next period collection activities. The system manager must decide: (1) potential sites to be visited by the bloodmobiles and (2) the tours of bloodmobiles and shuttles to collect a desired amount of blood minimizing the shortage and wastage levels in the BSC. The quantities of blood products to be collected in a period by bloodmobiles are determined by the inventory models of BC and hospitals.

The Vehicle Routing Problem with Profits (VRPP) adapted to blood mobile donation system is defined as follows. Let $G = (V, A)$ be a complete weighted and directed graph where $V = \{1, \dots, N\}$ is a set of N nodes and A is a set of arcs. Nodes $2, \dots, N-1$ are potential sites to visit, whereas nodes 1 and N correspond to starting and end points of the paths to build respectively. A fleet of K identical bloodmobiles and a fleet of F identical shuttles are available at node 1. Let $H = \{1, \dots, T\}$ be the planning horizon where periods are indexed by t. Let a nonnegative blood collection potential p_i be associated with each node $i \in V$, with $p_1 = 0$ and $p_N = 0$, and c_{ij} be associated with the distance of each arc $(i, j) \in A$. Each bloodmobile and shuttle have a collection capacity QB and QS respectively. Let d_t represent the desired value for blood to be collected in the period t. If the total collection potential of nodes visited in the period t is greater than d_t, the bloodmobiles only collect a quantity equal to d_t. A wastage cost μ is generated for each unit not collected at the visited nodes. Conversely, if the amount collected is less than d_t , a shortage cost σ is caused for each missing unit. The goal is to determine the tours of blood mobiles and shuttles in the planning horizon, to minimize the total cost. This problem can be formulated as an integer programming model with the following decision variables:

b_{ijkt} = 1 if the bloodmobile k travels from node i to node j in the period t, 0 otherwise.
x_{ijft} = 1 if the shuttle f travels from node i to node j in the period t, 0 otherwise.
y_{it} = Collected units at node i in the period t.
z_{ift} = Collected units up to node i by the shuttle f in the period t.

w_{it} = Units, associated with wastage, not collected at node i in the period t.
l_t = Units of unsatisfied demand, associated with shortage, in the period t.

$$\text{Min}\left(\sum_{i=1}^{N-1}\sum_{j=2}^{N}\sum_{k=1}^{K}\sum_{t=1}^{T} c_{ij}\cdot b_{ijkt} + \sum_{i=1}^{N-1}\sum_{j=2}^{N}\sum_{f=1}^{F}\sum_{t=1}^{T} c_{ij}\cdot x_{ijft} + \sigma\sum_{t=1}^{T-1} l_t + \mu\sum_{i=2}^{N-1}\sum_{t=1}^{T-1} w_{it}\right) \tag{1}$$

The objective function (1) is to minimize the total cost subject to,

$$\sum_{j=2}^{N} b_{1jk1} = 1; \qquad \forall k \in \{1, \dots, K\} \tag{2}$$

$$\sum_{i=1}^{N-1}\sum_{j=2}^{N}\sum_{k=1}^{K} b_{ijkt} \le K; \qquad \forall t \in \{1, \dots, T-1\} \tag{3}$$

$$\sum_{i=1}^{N-1} b_{ijk(T-1)} = b_{jNkT}; \qquad \forall j \in V \setminus \{1, N\} \tag{4}$$

$$\sum_{i=1}^{N-1} b_{ijkt} = \sum_{i=2}^{N} b_{jik(t+1)}; \qquad \forall j \in V \setminus \{1, N\}, \quad \forall k \in \{1, \dots, K\}, \quad \forall t \in \{1, \dots, T-1\} \tag{5}$$

$$\sum_{i=1}^{N-1} b_{iNkt} = \sum_{i=2}^{N} b_{1ik(t+1)}; \qquad \forall k \in \{1, \dots, K\}, \quad \forall t \in \{1, \dots, T-1\} \tag{6}$$

$$\sum_{i=1}^{N-1}\sum_{k=1}^{K}\sum_{t=1}^{T-1} b_{ijkt} \le 1; \qquad \forall j \in V \setminus \{1, N\} \tag{7}$$

$$y_{it} = \left(\sum_{j=1}^{N-1}\sum_{k=1}^{K} p_i \cdot b_{jikt}\right) - w_{ij} \le QB; \qquad \forall i \in V \setminus \{1, N\}, \quad \forall t \in \{1, \dots, T-1\} \tag{8}$$

$$\sum_{i=2}^{N-1} y_{it} = d_t - l_t; \qquad \forall t \in \{1, \dots, T-1\} \tag{9}$$

$$\sum_{j=1}^{N-1}\sum_{f=1}^{F} x_{jift} = \sum_{j=1}^{N-1}\sum_{k=1}^{K} b_{jikt} - \sum_{k=1}^{K} b_{iNk(t+1)}; \qquad \forall i \in V \setminus \{1, N\}, \quad \forall t \in \{1, \dots, T-1\} \tag{10}$$

$$\sum_{j=2}^{N-1} x_{1jft} = \sum_{i=2}^{N-1} x_{iNft} \le 1; \qquad \forall f \in \{1, \dots, F\}, \quad \forall t \in \{1, \dots, T-1\} \tag{11}$$

$$\sum_{i=1}^{N-1}\sum_{j=2}^{N-1} x_{ijft} \le K \cdot \left(\sum_{j=2}^{N-1} x_{1jft}\right); \qquad \forall f \in \{1, \dots, F\}, \quad \forall t \in \{1, \dots, T-1\} \tag{12}$$

$$\sum_{i=1}^{N-1} x_{ijft} = \sum_{i=2}^{N} x_{jift}; \qquad \forall j \in V \setminus \{1, N\}, \quad \forall f \in \{1, \dots, F\}, \quad \forall t \in \{1, \dots, T-1\} \tag{13}$$

$$z_{ift} + y_{jt} - M \cdot (1 - x_{ijft}) \le z_{jft}; \quad \forall i \in V \setminus \{N\}, \ \forall j \in V \setminus \{1, N\}, \ \forall f \in \{1, \dots, F\}, \ \forall t \in \{1, \dots, T-1\} \tag{14}$$

$$z_{ift} \le QS; \qquad \forall i \in V \setminus \{1, N\}, \quad \forall f \in \{1, \dots, F\}, \quad \forall t \in \{1, \dots, T-1\} \tag{15}$$

$$\sum_{i \in U}\sum_{j \in U} x_{ijft} \le |U| - 1; \quad \forall f \in \{1, \dots, F\}, \forall t \in \{1, \dots, T-1\}, \forall |U| \subseteq V, 2 \le |U| \le N-2 \tag{16}$$

$$b_{ijkt} \in \{0,1\}; \quad \forall i \in V, \quad \forall j \in V, \quad \forall k \in \{1, \dots, K\}, \quad \forall t \in \{1, \dots, T\} \tag{17}$$
$$x_{ijft} \in \{0,1\}; \quad \forall i \in V, \quad \forall j \in V, \quad \forall f \in \{1, \dots, F\}, \quad \forall t \in \{1, \dots, T\} \tag{18}$$
$$y_{it}, w_{it} \in \mathbb{R}^+; \quad \forall i \in V, \quad \forall t \in \{1, \dots, T\} \tag{19}$$
$$z_{ift} \in \mathbb{R}^+; \quad \forall i \in V, \quad \forall f \in \{1, \dots, F\}, \quad \forall t \in \{1, \dots, T\} \tag{20}$$
$$l_t \in \mathbb{R}^+; \quad \forall t \in \{1, \dots, T\} \tag{21}$$

Constraints (2) guarantee that the bloodmobiles are located at the BC at the beginning of the planning horizon. Constraints (3) state that the maximum number of sites to visit in a period is the number of available bloodmobiles. All bloodmobiles must return to the BC at the end of the planning horizon because of constraints (4). Constraints (5) and (6) guarantee flow conservation of each bloodmobile tour. Each potential site is visited at most once in the planning horizon because of constraints (7). Constraints (8) state that the collected blood units cannot exceed the capacity of bloodmobiles. Constraints (9) are used to define the demand satisfaction and to compute the shortage levels. With constraint (10), it is defined if a site needs a visit of a shuttle. Constraints (11) and (12) guarantee that if a bloodmobile leaves the BC, it must end at node N. Constraints (13) guarantee flow conservation of each shuttle tour. With constraints (14), it is defined the collected units by the shuttles. Constraints (15) state that the capacity of shuttles cannot be exceeded. Constraints (16) prevent subtours. Finally, constraints (17) - (21) fix the nature of variables.

3.2. Two-stage stochastic programming model for the mobile collection system

The uncertainty in the number of collected blood units, or profits, in a potential site is formulated using a Two-Stage Programming Model. We follow the notation proposed by Shapiro et al. [10]. Two-stage stochastic linear programming problems are of the form,

$$\begin{aligned} &\min_{x \in \mathbb{R}^n} c^\intercal x + \mathbb{E}[Q(x, \xi)] \\ &\text{s.t.} \quad Ax = b, \quad x \geq 0, \end{aligned} \tag{22}$$

where $Q(x, \xi)$ is the optimal value of the second-stage problem,

$$\begin{aligned} &\min_{y \in \mathbb{R}^n} q^\intercal y \\ &\text{s.t.} \quad Tx + Wy = h, \quad y \geq 0. \end{aligned} \tag{23}$$

Here $\xi \coloneqq (q, h, T, W)$ are the data of the second stage problem. Some or all elements of vector ξ are viewed as random. A set of decisions must be taken without full information on random events. These decisions are called *first-stage decisions* and are represented by the vector x. Later, full information is received on the realization of vector ξ. Then, *second-stage decisions* or corrective actions y are taken. In this model, the *fist-stage* decisions are to determine the bloodmobiles and shuttles used each period and their tours. In the *second-stage* the profit realizations are reveled, and recourse cost are imposed based on corrective actions. These corrective actions are associated with the not collected units from the site and the units of unsatisfied demand. The recourse costs are denoted as waste and shortage costs.

The expected value function $\phi(x) \coloneqq \mathbb{E}[Q(x, \xi)]$ is taken with respect to the probability distribution of random vector ξ. To proceed with numerical calculations, it is considered that ξ_i represents the profit of each site i, $\forall i \in V$, with a specified probability distribution and a finitely number of realizations. According to the realizations of ξ_i, the scenario set Ω is considered, where $\omega = (\xi_1^\omega, \dots, \xi_N^\omega)$, $\forall \omega \in \{1, \dots, \Omega\}$ and their associated joint discrete probabilities $\mathbb{P}(\omega) = \prod_{i=1}^{N} \mathbb{P}(\xi_i^\omega), \forall \omega \in \{1, \dots, \Omega\}$. In the model, the probability associated with each scenario is represented by $prob_\omega$. This Stochastic Programming Model consider the definition of parameters and variables given in section 2.1., except for the parameter p_i, replaced by ξ_i^ω and defined as the available units to be collected at site $i \in V \setminus \{1, N\}$ for the scenario $\omega \in \{1, \dots, \Omega\}$. The variables y_{it}, z_{ift}, w_{it} and l_t associated to *second-stage* decisions are replaced by,

y_{it}^ω	=	Collected units at node i in the period t for the scenario ω.
z_{ift}^ω	=	Collected units up to node i by the shuttle f in the period t for the scenario ω.
w_{it}^ω	=	Units not collected at node i in the period t for the scenario ω.
l_t^ω	=	Units of unsatisfied demand in the period t for the scenario ω.

The objective function (1) is replaced by,

$$\text{Min}\left[\sum_{i=1}^{N-1}\sum_{j=2}^{N}\sum_{k=1}^{K}\sum_{t=1}^{T} c_{ij}\cdot b_{ijkt} + \sum_{i=1}^{N-1}\sum_{j=2}^{N}\sum_{f=1}^{F}\sum_{t=1}^{T} c_{ij}\cdot x_{ijft} + \sum_{\omega=1}^{\Omega} prob_{\omega}\left(\sigma\sum_{t=1}^{T-1} l_t^{\omega} + \mu\sum_{i=2}^{N-1}\sum_{t=1}^{T-1} w_{it}^{\omega}\right)\right] \quad (24)$$

The aim of the two-stage model is to find the tours of bloodmobiles in the planning horizon and shuttles in each period such that the *first-stage* cost, corrected by the expected *second-stage* waste and shortage costs, is minimized subject to constraints (2) – (7), (10) – (13), (16) – (18) and equations (8), (9), (14), (15) and (19) – (21) are replaced by the following constraints respectively,

$$y_{it}^{\omega} = \left(\sum_{j=1}^{N-1}\sum_{k=1}^{K} \xi_i^{\omega}\cdot b_{jikt}\right) - w_{ij}^{\omega} \le QB; \qquad \forall i \in V\setminus\{1,N\}, \quad \forall t \in \{1,\dots,T-1\}, \quad \forall \omega \in \{1,\dots,\Omega\} \quad (25)$$

$$\sum_{i=2}^{N-1} y_{it}^{\omega} = d_t - l_t^{\omega}; \qquad \forall t \in \{1,\dots,T-1\}, \quad \forall \omega \in \{1,\dots,\Omega\} \quad (26)$$

$$z_{ift}^{\omega} + y_{jt}^{\omega} - M\cdot\left(1 - x_{ijft}\right) \le z_{jft}^{\omega}; \quad \forall i \in V\setminus\{N\}, \forall j \in V\setminus\{1,N\}, \forall f \in \{1,\dots,F\}, \forall t \in \{1,\dots,T-1\}, \forall \omega \in \{1,\dots,\Omega\} \quad (27)$$

$$z_{ift}^{\omega} \le QS; \qquad \forall i \in V\setminus\{1,N\}, \quad \forall f \in \{1,\dots,F\}, \quad \forall t \in \{1,\dots,T-1\}, \quad \forall \omega \in \{1,\dots,\Omega\} \quad (28)$$

$$y_{it}^{\omega}, w_{it}^{\omega} \in \mathbb{R}^{+}; \quad \forall i \in V, \quad \forall t \in \{1,\dots,T\}, \quad \forall \omega \in \{1,\dots,\Omega\} \quad (29)$$

$$z_{ift}^{\omega} \in \mathbb{R}^{+}; \quad \forall i \in V, \quad \forall f \in \{1,\dots,F\}, \quad \forall t \in \{1,\dots,T\}, \quad \forall \omega \in \{1,\dots,\Omega\} \quad (30)$$

$$l_t^{\omega} \in \mathbb{R}^{+}; \quad \forall t \in \{1,\dots,T\}, \quad \forall \omega \in \{1,\dots,\Omega\} \quad (31)$$

4. The scenario reduction

In order to proceed with numerical calculations in the two-stage stochastic programming model, it is necessary to make a discretization of the random profit. Such discrete approximations allow the construction of the scenario set. The dimension of the model grows exponentially with the number of involved scenarios [27]. For this reason, the reduction of the scenario set is an indispensable problem to consider in this study.

Finding the optimal scenario set with a fixed number of scenarios to eliminate is a hard-combinatorial optimization problem [27]. Heitsch and Römisch [28] have developed two heuristics based on fast forward selection (FFS) and simultaneous backward reduction (SBR) techniques, to handle this problem. Some considerations are presented below to introduce the algorithm implemented in this study for the scenario reduction.

Let the probability distribution Q be discrete with many scenarios $\omega_i \in \Omega$, weights $q_i > 0$, $i = 1,\dots,|\Omega|$ and $\sum_{i=1}^{|\Omega|} q_i = 1$. Let $n < |\Omega|, J \subset \{1,\dots,|\Omega|\}$, where J represents the index set of eliminated scenarios, with $\#J = |\Omega| - n$ and consider the probability measure $\tilde{Q}$ having scenarios $\omega_j, j \in \{1,\dots,|\Omega|\}\setminus J$. The new probabilistic weights $\tilde{q}_j$ are assigned to each scenario $\omega_j, j \in \{1,\dots,|\Omega|\}\setminus J$ according to the minimal distance,

$$D_K\left(Q,\tilde{Q}\right) = \sum_{i\in J} q_i \cdot \min_{j\notin J} c(\omega_i,\omega_j)$$

D_K represents the Monge-Kantorovich distance between two finite discrete probability distributions Q and $\tilde{Q}$ where $c\left(\omega_i,\omega_j\right) = \left\|\omega_i - \omega_j\right\|_n$ measure the distance between scenario realizations and $\|\cdot\|$ denotes the norm on $\mathbb{R}^n$. Moreover, the minimum is attained with the probability $\tilde{q}_j$ of the preserved scenarios ω_j of $\tilde{Q}$, $j \notin J$, and is given by the following *optimal redistribution rule* exposed by Heitsch and Römisch [28],

$$\tilde{q}_j = q_j + \sum_{i \in J(i)} q_i, \tag{32}$$

Where $J(i) = \{i \in J: j = j(i)\}$ and, respectively, $j(i) \in \arg\min_{j \notin J} c(\omega_i, \omega_j), \forall i \in J$. The optimal redistribution rule exposes that the new probability of a preserved scenario is equal to the sum of its initial probability and of all probabilities of deleted scenarios that are closest to it considering the distance measure c.

Heitsch and Römisch [28] state that the complexity $f_{|\Omega|}(n)$ of FFS increases with increasing n and it is maximal when $n = |\Omega|$. On the contrary, the complexity $b_{|\Omega|}(n)$ of SBR increases with decreasing n and it is minimal when $n = |\Omega|$. Thus, the use of FFS is recommendable if the number of remaining scenarios n satisfies the condition $f_{|\Omega|}(n) \leq b_{|\Omega|}(n)$. The authors also state that the number n_* such that $f_{|\Omega|}(n_*) = b_{|\Omega|}(n_*)$ is $n_* \approx \frac{|\Omega|}{4}$ for large $|\Omega|$. In this study, $n \ll \frac{|\Omega|}{4}$ and we decide to implement the FFS algorithm instead of SBR. This decision is supported in more detail in section 5.

With the notation exposed above, the FFS algorithm is given by,

Algorithm 1. Fast forward selection. Source: Heitsch and Römisch [28]	
Step 1:	$c_{ku}^{[1]} := c(\omega_i, \omega_j); \forall k, u \in \{1, \dots, \|\Omega\|\}$
	$z_u^{[1]} := \sum_{k=1, k \neq u}^{\|\Omega\|} q_k c_{ku}^{[1]}; u \in \{1, \dots, \|\Omega\|\}$
	$u_1 \in \arg\min_{u \in \{1,\dots,N\}} z_u^{[1]}, J^{[1]} := \{1, \dots, \|\Omega\|\} \setminus \{u_1\}$
Step i:	$c_{ku}^{[i]} := \min\{c_{ku}^{[i-1]}, c_{ku_{i-1}}^{[i-1]}\}; \forall k, u \in J^{[i-1]}$
	$z_u^{[i]} := \sum_{k \in J^{[i-1]} \setminus \{u\}}^{\|\Omega\|} q_k c_{ku}^{[i]}; u \in J^{[i-1]}$
	$u_i \in \arg\min_{u \in J^{[i-1]}} z_u^{[i]}, J^{[i]} := J^{[i-1]} \setminus \{u_i\}$
Step n+1:	*Optimal redistribution by* (32).

The FFS algorithm selects, in a recursive way, the scenarios that will be not deleted. The closest scenario to the others considering the distance c is selected in the first step as the scenario to preserve. In the following steps, new scenarios to conserved are selected. The condition is the minimization of the distance between the selected scenarios and eliminated ones.

5. Results

The aim of this section is to report on numerical experience on testing the mathematical models and the algorithm described in section 4. We consider a mobile donation system consisting of a set of 16 nodes with 14 potential donation sites. The coordinates and blood collection potential p_i associated with each node $i \in V$ are obtained from instances proposed by Chao et al. [29]. The Euclidean distance is used to calculate the distance c_{ij} of each arc $(i, j) \in A$. It is assumed that the Blood Center has a fleet of six identical bloodmobiles and a fleet of two identical shuttles. The capacity of bloodmobiles and shuttles is 23 and 68 blood units respectively. The planning horizon defined is 3 periods to face the demand of 2 days. The wastage and shortage costs generated are \$10 and \$1000 to guarantee that shortage is a more undesirable situation than wastage.

MATLAB R2015a was used to generate the set of scenarios and to implement the algorithm 1. The deterministic model and the two-stage stochastic programming model have been implemented using IBM ILOG CPLEX Optimization Studio, version 12.7.1. Computational experiments have been carried out on a computer with processor Intel(R) Core(TM) i5 – 7200U CPU, 2.5GHz and 8GB RAM memory.

We propose two realizations for the profits of potential donation sites. These profits are assumed to follow a Poisson distribution and independent of each other. With these two realizations for the 14 sites, $|\Omega| = 2^{14} = 16{,}384$ scenarios. For the scenario set reduction, let us consider the Euclidean norm as a distance measure between two finite scenarios $\omega_s = (\xi_1^s, \dots, \xi_J^s)$ and $\omega_t = (\xi_1^t, \dots, \xi_J^t)$, i.e.,

$$c(\omega_s, \omega_t) = \|\xi_i^s - \xi_i^t\|_2 = \sqrt{(\xi_1^s - \xi_1^t)^2 + (\xi_2^s - \xi_2^t)^2 + \dots + \left(\xi_J^s - \xi_J^t\right)^2}$$

Where $\omega_s, \omega_t \in \Omega$.

The number of scenarios to preserve n is determined empirically as $n = 10, 15, 20, 25, 30, 35, 40, 45, 50, 100, 150,$ 200, where $n \ll 4{,}096$. With this last consideration, it is justified the implementation of FFS algorithm instead of SBR exposed in the previous section.

Figures 1 and 2 present the results for the different size of preserved scenario subset in terms of CPU time and objective value respectively. Table 1 summarizes the details on performance for the deterministic programming model, the two-stage stochastic programming model and the FFS algorithm.

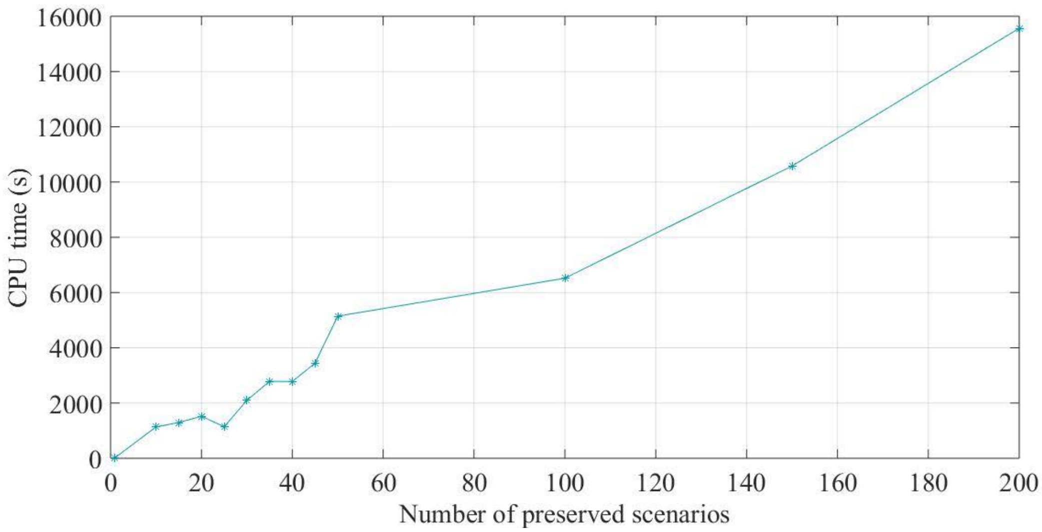

Figure 1. CPU times according to n.

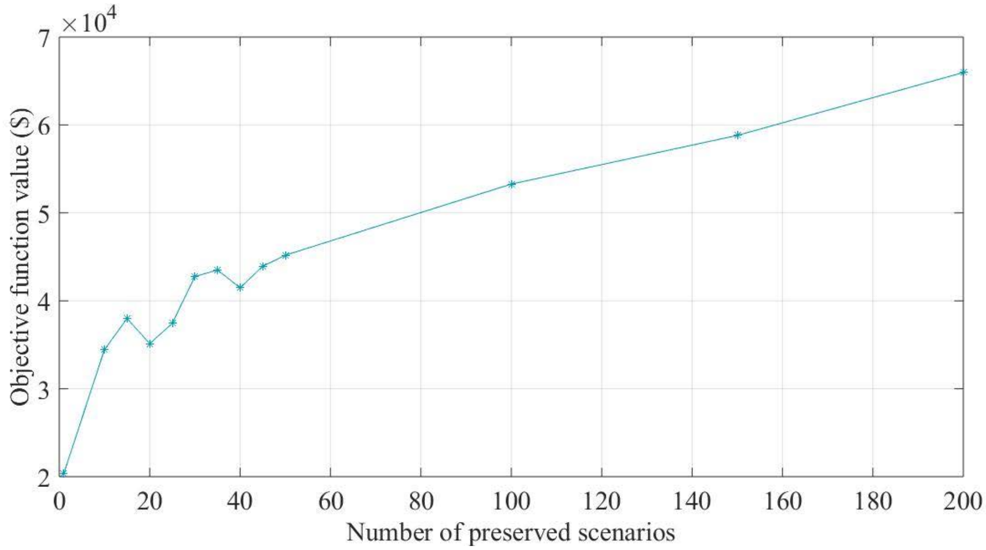

Figure 2. Value of the objective function according to n.

Table 1. Performance of models and FFS algorithm.

n	Problem features	Value	n	Problem features	Value
1	Objective function ($)	20,346	10	Objective function ($)	34,456.43
	CPU time (s) for generation	-		CPU time (s) for generation	1.95
	CPU time (s) for FFS	-		CPU time (s) for FFS	1,080.82
	CPU time (s) for MILP	21.13		CPU time (s) for MILP	54.09
	Total CPU time (s)			Total CPU time (s)	1,136.86
15	Objective function ($)	37,993.61	20	Objective function ($)	35,129.57
	CPU time (s) for generation	2.32		CPU time (s) for generation	1.79
	CPU time (s) for FFS	1,205.94		CPU time (s) for FFS	1,476.09
	CPU time (s) for MILP	86.02		CPU time (s) for MILP	44.27
	Total CPU time (s)	1,294.28		Total CPU time (s)	1,522.15
25	Objective function ($)	37,447.51	30	Objective function ($)	42,756.52
	CPU time (s) for generation	1.09		CPU time (s) for generation	1.13
	CPU time (s) for FFS	1,074.6		CPU time (s) for FFS	2,002.54
	CPU time (s) for MILP	64.59		CPU time (s) for MILP	93.38
	Total CPU time (s)	1,140.28		Total CPU time (s)	2,097.05
35	Objective function ($)	43,515.42	40	Objective function ($)	41,474.22
	CPU time (s) for generation	2.57		CPU time (s) for generation	1.23
	CPU time (s) for FFS	2,717.54		CPU time (s) for FFS	2,666.06
	CPU time (s) for MILP	67.73		CPU time (s) for MILP	111.03
	Total CPU time (s)	2,787.84		Total CPU time (s)	2,778.32
45	Objective function ($)	43,930.97	50	Objective function ($)	45,189.26
	CPU time (s) for generation	1.16		CPU time (s) for generation	1.21
	CPU time (s) for FFS	3,231.88		CPU time (s) for FFS	4,944.71
	CPU time (s) for MILP	210.69		CPU time (s) for MILP	198.67
	Total CPU time (s)	3,443.73		Total CPU time (s)	5,144.59
100	Objective function ($)	53,277.27	150	Objective function ($)	58,812,72
	CPU time (s) for generation	1.15		CPU time (s) for generation	1.16
	CPU time (s) for FFS	6,214.70		CPU time (s) for FFS	9,349.44
	CPU time (s) for MILP	303.69		CPU time (s) for MILP	1,230.69
	Total CPU time (s)	6,519.54		Total CPU time (s)	10,581.29
200	Objective function ($)	65,964.05			
	CPU time (s) for generation	1.47			
	CPU time (s) for FFS	14,784.98			
	CPU time (s) for MILP	779.31			
	Total CPU time (s)	15,565.76			

6. Conclusions

This paper presents a study to optimize the decisions in the mobile collection system of blood products with two types of vehicles: bloodmobiles and shuttles. The challenge of optimizing process in the Blood Supply Chain and its vital importance have motivated us to start this research. This paper adapts the Vehicle Routing Problem with Profits with the objective of minimizing the total routing, wastage and shortage costs. Each collection site has a random blood collection potential following a Poisson distribution function that is modeled as a stochastic profit collected by a vehicle when it visits a site. A Two-Stage Stochastic Model with recourse is developed to represent the problem using a scenario-based approach. The fast-forward selection algorithm is implemented to reduce the set of scenarios.

One of the challenges of stochastic programming is the size of the problems and the solution of these in reasonable computational times. For future research related to the mobile collection system of blood products modeled as a two-stage problem, it is proposed to evaluate scenarios reduction methods to obtain shorter solution times. In addition to this, approximation methods can be used for stochastic models such as the L-Shape Method or Cutting Plane Approximation. Heuristics, Markov processes or other solution methods can be developed to optimize the blood supply chain proposed in this study.

Possible extensions of the model proposed are to consider that the bloodmobile can be parked more than one day in a potential site depending on the profit of the site. It can also be considered that the shuttle makes more visits during the day to the bloodmobiles in operation to analyze how this activity affects the average age of blood products in the supply chain.

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