Value-at-Risk Contribution under Asset Liability Models by Using Expenential Weighted Moving Average Approaches

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Abstract

This paper discusses the Value-at-Risk Contribution under the asset liability model using the EWMA approach. It is assumed that asset returns and liabilities are time series data following the Exponential Weighted Moving Average (EWMA) model. Return of surplus which is the difference between asset return and liability is analyzed using asset liability model. In this case the risk of surplus return is measured using the Value-at-Risk model. When investments are made on multiple assets, each asset will contribute to the establishment of Value-at-Risk from the investment portfolio, which can be measured using the Value-at-Risk Contribution model. Using Value-at-Risk Contribution, it can be seen how much Value-at-Risk surplus investment portfolio, and what is the proportion of Value-at-Risk contribution of each surplus of investment asset. Based on the calculation of Value-at-Risk Contribution, can be considered for investors in investing in some assets analyzed.

Keywords:
Asset return, EWMA model, asset liability model, Value-at-Risk, Value-at-Risk Contribution.

1. Introduction

Financial risk management aims to balance between risk and profit in accordance with established policies. These policies range from the limits of dynamic tactical decisions to strategic decisions in allocating capital to invest (Sukono et al., 2016). In investing, investors are generally well aware that there is a potential risk of loss that needs to be measured and carefully considered (Dowd, 2002). Usually risk is measured by using variance or standard deviation, but variance or standard deviation is an average risk measure. So it can not accommodate all the risk events that occur (Sukono et al., 2017.b). Then comes the idea of risk measurement using quantitative, better known as Value-at-Risk (VaR) (Khindanova & Rachev, 2005). VaR has become popular and has been widely used for risk measurement, by financial institutions for internal importance in decision making (Bohdalova, 2013).

In making investment decisions, to minimize the risk of loss is usually done by portfolio setting. Portfolio formation is done by diversifying investments that is spreading investment in several assets with the aim to reduce risk (Sukono et al., 2017.a). Then there is the problem of how to measure the risk contribution of each investment asset to the overall risk of its investment. Haaf & Tasche (2002) and Huang et al. (2007), has conducted a study on how to measure marginal risk contribution to total portfolio risk. The study shows that Value-at-Risk Contribution (VaRC) risk measures can be used to measure the total risk contribution of individual assets to Value-at-Risk. Thus

VaRC can be used to show VaR contribution for different risk factors, as well as to calculate risk on some investment sub-portfolios (Sukono et al., 2018.a).

A financial institution that collects funds from the public, usually collected funds are partly used to invest in an asset in order to obtain a surplus. Because funds are collected from the community, the financial institution has obligations to be paid back to the community. So the surplus obtained by financial institutions must be paid partly to the public. Gersner et al. (2007), analyzed the investment assets with the obligations of financial institutions using asset-liability models. In the asset-liability model that the surplus return is the difference between the return of the investment asset and the return factor liability. The return data of investment assets and liability returns are often time series data. To estimate VaR time series data, Gabrielsen et al. (2012) do so using the RiskMetric model. In assessing VaR by using the RiskMetric model, what needs to be known is volatility? Volatility can be determined by using the normal distribution and exponential weighted moving average (EWMA).

Referring to Haaf & Tasche (2002), Huang et al. (2007), Gersner et al. (2007), and Gabrielsen et al. (2012), in this paper the VaRC analysis is undertaken under the asset-liability model using the EWMA approach. This analysis is conducted with the aim to find alternative method of VaRC measurement, for data return following model asset liability, where return of investment asset and return of liability is time series data.

2. Mathematical Models

2.1 Calculation of Stock Return

Let's say P_t price or value of an asset-liability at a time t ($t=1,\dots,T$ and T number of observation data), and r_t return asset-liability at time t. The amount of asset-liability return can be determined by the equation (Sukono et al., 2018.b):

$$r_t = \ln P_t - \ln P_{t-1} \ . \quad (1)$$

Return data r_t hereinafter used in RiskMetric modeling as follows.

2.2 Normality Test of Return Data

Normality test here is done by Kolmogorov-Smirnov (KS) approach, which is the most basic and most used statistical test. The first KS test was introduced by Andrey Nikolaevich Kolmogorov in 1933 and then tabulated by Nikolai Vasilyevich Smirnov in 1948. According to Arnold & Emerson (2011), the KS test is used for one-sample test allows the comparison of a frequency distribution with some standard distribution, such as the normal Gaussian distribution.

The KS test measures the proximity of the intermediate distance $F(x)$ with $Fn(x)$ when n assumed to be of enormous value, Arnold & Emerson (2011) defines its cumulative distribution function or cdf (cumulative distribution function) as follows:

$$D = \sup_x |Fn(x) - F(x)| \quad (2)$$

where $\sup_x$ is the supremum of some distance D.

Statistic value of D (*Most Extreme Differences*) in the KS test consists of:

1) *D Positive* ($D^+ = \sup_x [F_n(x) - F(x)]$) , is a reduction that produces the greatest positive number.
2) *D Negative* ($D^- = \sup_x [F(x) - F_n(x)]$) , is a reduction that produces the largest negative number.
3) *D Absolute* ($D = \max\{D^+, D^-\}$), is the largest number between absolute values D^+ and D^-.

Kolmogorov-Smirnov Z is the result of the square root of the number of N samples and the largest absolute difference between the empirical cdf and the theoretical CDF of Arnold & Emerson (2011), is almost equal to the square root of the N number of samples multiplied by D Absolute:

$$Z = \sqrt{N.D\ Absolute} \quad (3)$$

According to Arnold & Emerson (2011), "Kolmogorov-Smirnov Z" is the Absolute D turned into a standardized score (Z score); the standardized score is the Z value in the standard normal distribution. That is, the way the test is almost the same as testing the D value, only this time under the normal distribution using the help of standard normal distribution table, where: H_0 rejected if Z -count (Kolmogorov-Semirnov) greater than Z -table at the level of significance α .

The basic concept of the Kolmogorov-Smirnov normality test (KS) is by comparing the distribution of data (to be tested for normality) to the normal standard distribution. The standard normal distribution is data that has been transformed into Z-Score and assumed to be normal. So in fact the KS test is a different test between the tested data normality with normal raw data (Arnold & Emerson, 2011).

The hypothesis in the Kolmogorov-Smirnov One Sample test is as follows:

H_0 : There is no difference between the data tested and the normal distribution.

H_1 : There is a difference between the data tested and the normal distribution.

If $P-Value > \alpha$, then H_0 received, and if $P-Value < \alpha$, then H_0 rejected. $P-Value$ is the probability of statistic KS and α level of significance.

2.3 Model RiskMetric

Model *RiskMetric* dikenalkan oleh J.P Morgan pada tahun 1994, model *RiskMetric* dipergunakan untuk menghitung *VaR,* dan telah menjadi tolak ukur dalam menghitung risiko pasar. *RiskMetric* mengasumsikan bahwa log *return* (*continously compounded*) harian mengikuti distribusi normal dengan *mean* nol dan variansi diestimasi menggunakan *Exponential Weighted Moving Average* (EWMA) (Qian, 2005; Haaf & Tasche, 2002).

RiskMetric mengasumsikan bahwa *return* mengikuti:

$$r_t = r_t \,|\, \Omega_t \sim N(0, \sigma_t^2), \tag{4}$$

And the variance follows recursive EWMA or non-recursive EWMA. Using recursive EWMA when data is used very much, the equations used are:

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2 . \tag{5}$$

Using non-recursive EWMA when the amount of data used is small, the equations used are:

$$\sigma_t^2 = (1-\lambda)\sum_{i=1}^{\infty} \lambda^{i-1} (R_x - r_{t-i})^2 , \tag{6}$$

with R_x is the average value of asset-liability returns (Tsay, 2005).

Where the values can be used $\lambda = 0.94$ for daily data, and $\lambda = 0.97$ for monthly data. Using the RiskMetric model, the 1-day VaR on t day is calculated by the equation:

$$VaR = -z_\alpha \sigma_t P_t . \tag{7}$$

Where σ_t = standard deviation, sign (-) is intended as a loss, and P_t = asset prices (assets-liabilities). When α assumed to be 1%, then the percentile of the normal distribution is -2.33, so the daily VaR formula is:

$$VaR = -(-2.33\sigma_t P_t) = 2.33\sigma_t P_t .$$

For daily portfolio *VaR* calculations use the same formula with a single asset return. To calculate *k*-day *VaR* horison is:

$$VaR(k) = \sqrt{k} \text{ x} VaR. \tag{8}$$

Where *k* is used to denote the time horizon (Dowd, 2002).

2.4 Model EWMA

The Exponential Weighted Moving Average (EWMA) model is used to estimate the volatility in RiskMetric. Volatility is a measure of dispersion which in statistics is measured by variance σ^2 or standard deviation σ . Estimating volatility until the unit of daily time is calculated using $\lambda = 0.94$ and using the return squared r^2 . If estimating variance with RiskMetric using time series r^2 for n-periods, then the recursive EWMA variance is defined as (Dowd, 2002; Tsay, 2005):

$$\sigma_t^2 = \frac{r_{t-1}^2 + \lambda r_{t-2}^2 + \lambda^2 r_{t-3}^2 + \ldots + \lambda^{n-1} r_{t-n}^2}{1 + \lambda + \lambda^2 + \ldots + \lambda^{n-1}} . \tag{9}$$

Because (9) converges to $\frac{1}{1-\lambda}$ for $n \to \infty$, then $\sigma_t^2 = (1-\lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2$.

This can be proven as follows:

$$\sigma_t^2 = \frac{r_{t-1}^2 + \lambda r_{t-2}^2 + \lambda^2 r_{t-3}^2 + \ldots + \lambda^{n-1} r_{t-n}^2}{1 + \lambda + \lambda^2 + \ldots + \lambda^{n-1}}$$

$$\sigma_t^2 = r_{t-1}^2 + \lambda r_{t-2}^2 + \lambda^2 r_{t-3}^2 + \ldots + \lambda^{n-1} r_{t-n}^2 . \left(\frac{1}{1 + \lambda + \lambda^2 + \ldots + \lambda^{n-1}}\right)$$

It is an infinite geometry series, so the number of series can be calculated using the formula:

$$S_n = \frac{1}{1-r} \text{, for } n \to \infty . \tag{10}$$

Series $(\frac{1}{1+\lambda+\lambda^2+\ldots+\lambda^{n-1}})$, for $n \to \infty$ can be calculated by the formula (10), where $r = \lambda$

So the number of series is $(\frac{1}{1+\lambda+\lambda^2+\ldots+\lambda^{n-1}}) = \frac{1}{\frac{1}{1-\lambda}}$. The variances in EWMA can be written as:

$$\sigma_t^2 = \frac{(r_{t-1}^2 + \lambda r_{t-2}^2 + \lambda^2 r_{t-3}^2 + \ldots)}{\frac{1}{(1-\lambda)}}$$

$$\sigma_t^2 = (1-\lambda)(r_{t-1}^2 + \lambda r_{t-2}^2 + \lambda^2 r_{t-3}^2 + \ldots) . \tag{11}$$

From equation (11) it can be simplified to be:

$$\sigma_t^2 = (1-\lambda)\sum_{i=1}^{\infty}\lambda^{i-1} r_{t-i}^2 = (1-\lambda)[\lambda^0 r_{t-1}^2 + \lambda^1 r_{t-2}^2 + \lambda^2 r_{t-3}^2 + \ldots]$$

$$= (1-\lambda)[r_{t-1}^2 + \lambda r_{t-2}^2 + \lambda^2 r_{t-3}^2 + \ldots] = (1-\lambda)[r_{t-1}^2 + \sum_{i=1}^{\infty} \lambda^i r_{t-1-i}^2]$$

$$= (1-\lambda)[r_{t-1}^2 + \lambda \sum_{i=1}^{\infty} \lambda^{i-1} r_{t-1-i}^2]$$

$$= (1-\lambda) r_{t-1}^2 + \lambda[(1-\lambda)\sum_{i=1}^{\infty} \lambda^{i-1} r_{t-1-i}^2]$$

$$= (1-\lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2$$

Thus the recursive EWMA variance formula is:

$$\sigma_t^2 = (1-\lambda) r_{t-1}^2 + \lambda \sigma_{t-1}^2 , \tag{12}$$

and nonrekursive EWMA, the equation is:

$$\sigma_t^2 = (1-\lambda)\sum_{i=1}^{\infty}\lambda^{i-1}(R_x - r_{t-i})^2 \tag{13}$$

Where R_x is the average value of return on assets, with $\lambda = 0.94$ for daily data and $\lambda = 0.97$ for monthly data (Dowd, 2002; Tsay, 2005).

2.5 Asset-Liability Model

The model of surplus return on assets is described briefly as follows. Let's say $A_{i.t}$ asset i ($i,\ldots,N$) at time t, $L_{i.t}$ liability asset i at the time t, and $S_{i.t}$ surplus asset i at the time t. At the beginning t =0, the initial surplus is given by:

$$S_{i.0} = A_{i.0} - L_{i.0} .$$

The surplus obtained after one period is (Gersner et al., 2007):

$$S_{i.1} = A_{i.1} - L_{i.1} = A_{i.0}[1 + r_{A_{i.1}}] - L_{i.0}[1 + r_{L_{i.1}}] ,$$

Let's say r_{S_i} return surplus expressed as:

$$r_{S_i} = \frac{S_{i.1} - S_{i.0}}{A_{i.0}} = \frac{A_{i.0} r_{A_i}}{A_{i.0}} - \frac{L_{i.0} r_{L_i}}{A_{i.0}} = r_A - \frac{1}{f_{i.0}} r_{L_i} , \tag{14}$$

with $f_{i.0} = \frac{L_{i.0}}{A_{i.0}}$ (Gersner et al., 2007).

Based on equation (14) the average of the surplus return can be determined by the formula:

$$\mu_{S_i} = E[r_{S_i}] = \mu_{A_i} - \frac{1}{f_{i.0}} \mu_{L_i} . \tag{15}$$

Where μ_{S_i}, μ_{A_i} and μ_{L_i} respectively is the average of surplus returns, assets, and liabilities. Also, according to (14), the surplus variance can be determined by the formula:

$$\sigma_{S_i}^2 = \sigma_{A_i}^2 - \frac{2}{f_{i.0}}\sigma_{A_i L_i} + \frac{1}{f_{i.0}^2}\sigma_{L_i}^2 . \tag{16}$$

Where $\sigma_{S_i}^2$, $\sigma_{A_i}^2$ and $\sigma_{L_i}^2$ respectively variance of surplus returns, assets, and liabilities. While $\sigma_{A_i L_i}$ covariance between asset return and liability return (Gersner et al., 2007). Similarly, by equation (14), the covariance between surpluses can be determined by the formula:

$$\sigma_{S_i S_j} = \sigma_{A_i A_j} - (\frac{1}{f_{i.0}} + \frac{1}{f_{j.0}})\sigma_{A_i L_j} + \frac{1}{f_{i.0} f_{j.0}}\sigma_{L_i L_j} . \tag{17}$$

Where $\sigma_{A_i A_j}$ covariance between assets i with assets j, $\sigma_{A_i L_j}$ covariance between assets i with liability j, and $\sigma_{L_i L_j}$ covariance between liabilities i with liability j. Using covariance between surplus return and its variance, correlation between surplus return can be determined as:

$$\rho_{S_i S_j} = \frac{\sigma_{S_i S_j}}{\sigma_{S_i}\sigma_{S_j}} . \tag{18}$$

2.6 VaR Contribution Model

To make it easy to understand, let's say a portfolio *p* consists of two assets that is *a* and *b*, with each proportion d_1 and d_2. The variance of the portfolio value will be equivalent to the amount of variance of the two assets and the covariance between them:

$$\sigma_p^2 = d_1^2\sigma_a^2 + 2d_1 d_2 \rho_{ab}\sigma_a\sigma_b + d_2^2\sigma_b^2 . \tag{19}$$

This equation can be rewritten into a number of multiplication factors with σ_a and σ_b as follows:

$$\sigma_p^2 = \sigma_a(d_1^2\sigma_a + d_1 d_2 \rho_{ab}\sigma_b) + \sigma_b(d_1^2\sigma_b + d_1 d_2 \rho_{ab}\sigma_a) , \tag{20}$$

with ρ_{ab} the correlation coefficient between the two assets. If both sides are divided by the standard deviation of the portfolio, a standard deviation sum equation is obtained:

$$\sigma_p = \sigma_a\left(\frac{d_1^2\sigma_a + d_1 d_2 \rho_{ab}\sigma_b}{\sigma_p}\right) + \sigma_b\left(\frac{d_2^2\sigma_b + d_1 d_2 \rho_{ab}\sigma_a}{\sigma_p}\right) . \tag{21}$$

The tribes in large brackets represent the correlation between the risks of the portfolio. In the form of a normal distribution approach, $VaR(\alpha)$ is z_α times standard deviation:

$$VaR_p = z_\alpha\sigma_p = z_\alpha\sigma_a\left(\frac{d_1^2\sigma_a + d_1 d_2 \rho_{ab}\sigma_b}{\sigma_p}\right) + z_\alpha\sigma_b\left(\frac{d_2^2\sigma_b + d_1 d_2 \rho_{ab}\sigma_a}{\sigma_p}\right) . \tag{22}$$

Where $z_\alpha = \Phi^{-1}(\alpha)$ tail left from the standard normal distribution. So as to be defined the *VaR* contribution for two risks *a* and *b*, can be summed to total *VaR* (Marrison, 2002):

$$VaRC_a = z_\alpha\sigma_a\left(\frac{d_1^2\sigma_a + d_1 d_2 \rho_{ab}\sigma_b}{\sigma_p}\right),$$

$$VaRC_b = z_\alpha\sigma_b\left(\frac{d_2^2\sigma_b + d_1 d_2 \rho_{ab}\sigma_a}{\sigma_p}\right),$$

So that

$$VaR_p = VaRC_a + VaRC_b . \tag{23}$$

Equation (19) can also be expressed in the following form:

$$\sigma_p^2 = (d_1\sigma_1)^2 + 2\rho_{1.2}(d_1\sigma_1)(d_2\sigma_2) + (d_2\sigma_2)^2 . \qquad (24)$$

So the equation is obtained $VaRC_1$ and $VaRC_2$ such as the following:

$$VaRC_1 = z_\alpha \times d_1\sigma_1 \frac{[d_1\sigma_1 + \rho_{1.2}d_2\sigma_2]}{\sigma_p} \text{ and } VaRC_2 = z_\alpha \times d_2\sigma_2 \frac{[d_2\sigma_2 + \rho_{1.2}d_1\sigma_1]}{\sigma_p} . \qquad (25)$$

If it consists of more than two assets, can use the same process of grouping tribes to obtain *VaRC* as follows (Marrison, 2002):

$$VaRC_1 = z_\alpha d_1\sigma_1 \left(\frac{d_1\sigma_1 + \rho_{1.2}d_2\sigma_2 + \ldots + \rho_{1.\times}d_\times\sigma_\times}{\sigma_p} \right) . \qquad (26)$$

$$VaRC_N = z_\alpha d_N\sigma_N \left(\frac{\rho_{N.1}d_1\sigma_1 + \rho_{N.2}d_2\sigma_2 + \ldots + d_N\sigma_N}{\sigma_p} \right) . \qquad (27)$$

If there are many risk factors (assets), then *VaR* can be explained as follows (Marrison, 2002):

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} \rho_{i.j}(d_j\sigma_j)(d_i\sigma_i) \qquad (28)$$

Equation (28) can be grouped again and then divided by σ_p, and obtained the following equation:

$$\sigma_p = \sum_{i=1}^{N} d_i\sigma_i \times \frac{\sum_{j=1}^{N} \rho_{i.j}d_j\sigma_j}{\sigma_p} \qquad (29)$$

For each *i*, a tribe describing *VaR*'s contribution to factor *i*:

$$VaRC_i \equiv z_\alpha \times d_i\sigma_i \times \frac{\sum_{j=1}^{N} \rho_{i.j}d_j\sigma_j}{\sigma_p} \qquad (30)$$

The mathematical models described above will be used for the analysis of the following illustrative data.

3. Numerical Illustration

3.1 Illustration Data

The illustration data used is generated through the simulation of four asset values, say for example A_1, A_2, A_3 and A_4. Similarly, liability value data are also obtained by generated through simulations, and named for example L_1, L_2, L_3 and L_4. Asset value and liabilities data are generated respectively 500. The asset value and liability data are then determined by using each equation (1). Furthermore, asset and liability return data are tested for normality as follows.

3.2 Normality Testing of Return Data

Normality tests are conducted with the intention of ensuring that the return on assets and liabilities is normally distributed with 0 and variance σ_t^2, as required by RiskMetric and EWMA analysis. The normality test here is done by the Kolmogorov-Smirnov (KS) method, referring to equations (2) and (3) in section 2.2. Test results are given in Table-1 below.

Table-1. Results of the Normality Test of Return Data

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Asset	Distribution	Mean	*P-Value*	Liability	Distribution	Mean	*P-Value*
A_1	Normal	0.0000	0.09713	L_1	Normal	0.0004	0.05985
A_2	Normal	0.0002	0.06621	L_2	Normal	0.0001	0.07445
A_3	Normal	0.0001	0.07045	L_3	Normal	0.0000	0.09463
A_4	Normal	0.0001	0.09234	L_4	Normal	0.0003	0.07856

If determined level of significance α = 0.05, then the *P-Value* in Table-1 appears, all greater than 0.05.

Therefore, the hypothesis that the return of normal distributed for assets and liabilities are correct. Next, we estimate the volatility of asset return data and liabilities using the EWMA method as follows.

3.3 Volatility Estimates Using EWMA

In this section volatility estimates are made for each asset return and liability. Estimates were performed using the EWMA method with the help of MS Excel 2007 software. Since the number of data returns of each asset and liability is relatively small ie 500, the estimation is done by using non recursive EWMA method referring to equation (13). The estimation results are given in Table-2 below.

Table-2. Results of Volatility Estimates

Asset	Volatility	Liability	Volatility
A_1	0.00311	L_1	0.00232
A_2	0.00247	L_2	0.00423
A_3	0.00256	L_3	0.00321
A_4	0.00412	L_4	0.00332

The estimated value of the asset and liability volatility in Table-2 will then be used to estimate the volatility of surplus returns as follows.

3.4 Estimation of Variance and Correlation of Surplus Return

For estimated volatility the surplus return requires the asset return volatility and liability return values of Table-2, as well as the covariance value between the asset return and the corresponding return on the liabilities. Therefore, it is necessary to estimate the covariance between asset returns and return liabilities, and the results are given in Table-3 column $\sigma_{A_i L_i}$. Further, for the estimation of volatility the surplus return is made by referring to equation (16). If assumed $f_{i.0} = 1$ ($i = 1,\ldots,4$), the result of the estimated volatility of surplus return is given in Table-3 as follows.

Table-3. Estimated Results of Volatility the Surplus Return

Surplus	$\sigma_{A_i L_i}$	$\sigma^2_{s_i}$	σ_{s_i}
S_1	0.00112	0.00431	0.06565
S_2	0.00223	0.00447	0.06686
S_3	0.00121	0.00456	0.06753
S_4	0.00231	0.00513	0.07162

While for estimation of correlation between surpluses return is done by using equation (18). Also assuming that $f_{i.0} = 1$ ($i = 1,\ldots,4$), the result of the correlation estimation of surplus return is given in Table-4 as follows.

Table-4. Correlation Between Surplus Return

Surplus	S_1	S_2	S_3	S_4
S_1	1	0.61424	0.65933	0.78452
S_2		1	0.59871	0.63411
S_3			1	0.58583
S_4				1

The values of the volatility estimator in Table-3 and the correlation estimator between surplus returns in Table-4 will then be used to estimate *VaR* and *VaRC* as follows.

3.5 Estimates of *VaR* and *VaRC*

The estimated magnitude of VaRC involves the volatility value in Table-3 and the correlation value between the surplus returns in Table-4. But before the first need to estimate portfolio variance σ^2_p. Estimation of portfolio variance is done by referring to equation (28). If you have a portfolio that produces four kinds of surplus returns, each with a proportion $d_1 = 0.20$; $d_2 = 0.20$; $d_3 = 0.25$ and $d_4 = 0.35$, then the result of estimation of variance and standard deviation of portfolio are:

$$\sigma_p^2 = 0.027920 \text{ and } \sigma_p = 0.167094.$$

Furthermore, the estimation is magnitude $VaRC_i$ ($i = 1,\dots,4$) is performed by referring to (30). If set the level of significance α =0.05, then the standard normal distribution percentile is obtained $z_{0.05} = -1.645$. So the result of estimation $VaRC_i$ ($i = 1,\dots,4$) respectively are:

$$VaRC_1 = 0.006740;\ VaRC_2 = 0.006245;\ VaRC_3 = 0.008016 \text{ and } VaRC_4 = 0.013262.$$

So obtained $VaR_p = 0.034263$.

Based on values of $VaRC_i$ ($i = 1,\dots,4$) and VaR_p, it can be explained, that the first asset return and liability contribute a risk of 0.006740; second by 0.006245; third by 0.008016; and fourth by 0.013262 against total risk VaR_p amount 0.034263. Thus, the values of risk contribution can be used as consideration for investors in investing in assets and liabilities analyzed.

4. Conclusion

This paper has analyzed the Value-at-Risk Contribution under the asset liability model using the EWMA approach. Asset price data and liabilities analyzed are generated by simulation techniques, each of 500. The results obtained that the analysis $VaRC_1$ = 0.006740; $VaRC_2$ = 0.006245; $VaRC_3$ = 0.008016 and $VaRC_4$ = 0.013262, VaR_p = 0.034263. These values are certainly very useful for investors in investing in assets and liabilities that are analyzed.

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