

# **A New Jerk Chaotic System with Three Nonlinearities and Its Circuit Implementation**

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## **Abstract**

A new jerk chaotic system with three nonlinearities is investigated in this work. By modifying a jerk chaotic system with two cubic nonlinearities obtained by Sprott (1997), we obtain a new jerk chaotic system with three nonlinearities (two cubic nonlinearities and a transcendental nonlinearity). Dynamics of the new chaotic system with three nonlinearities are analysed by means of phase portraits, Lyapunov exponents, Lyapunov dimension, bifurcation diagram and Poincaré map. Then an electronic circuit realization is shown to validate the chaotic behaviour of the new jerk chaotic system. Finally, the physical circuit experimental results of the jerk chaotic attractor show agreement with numerical simulations.

## **Keywords:**

Chaos, chaotic systems, jerk systems, Lyapunov exponents, and bifurcation diagram

## 1. Introduction

The behaviour of chaotic system was accidentally discovered by Lorenz, when he was designing a 3-D model for weather prediction in 1963 (Lorenz, 1963). Then, Rossler constructed several three-dimensional quadratic autonomous chaotic systems in 1976 (Rossler, 1976), which is algebraically simpler than the Lorenz system. In 1999, Chen and Ueta proposed a three-dimensional autonomous differential equation with only two quadratic terms (Chen and Ueta, 1999). In 2000, Malasoma presented the simplest dissipative jerk equation that is parity invariant (Malasoma, 2000). In 2010, Sprott described many chaotic systems with algebraically simple flows (Sprott, 2010). In 2009, Liu constructed a four-wing chaotic system with cubic nonlinearity (Liu, 2009). In 2016, Pham created a no-equilibrium hyperchaotic system with cubic nonlinearity (Pham et. al, 2016). In 2016, Zhang and Han constructed an autonomous chaotic system with cubic nonlinearity (Zhang and Han, 2016). In 2016, Vaidyanathan and Volos constructed a novel conservative jerk chaotic system with two cubic nonlinearities and discussed its adaptive backstepping control (Vaidyanathan and Volos, 2016).

Chaos theory can be applied in various disciplines, such as physics (Chen et. Al, 2014), economy (Idowu et. al, 2018), ecology (Wang and Jiang, 2012), random bit generators (Virte, et. al, 2014), laser (Shahverdiev and Shore, 2013; Yuan et. al, 2014), chemical reactions (Budroni et. al, 2017; Yadav et. al, 2017), robotics (Vaidyanathan et. al, 2017a), text encryption (Parvees et. al, 2017), image encryption (Liu et. al, 2018), voice encryption (Vaidyanathan et. al, 2017b), and secure communication systems (Sambas, et. al, 2013, 2016a).

Recently, there is significant interest in the chaos literature in finding jerk chaotic systems (Vaidyanathan et. al, 2014; Sambas et. al, 2017; Kom et. al, 2018). In this work, by modifying a jerk chaotic system with two cubic nonlinearities obtained by (Sprott, 1997), we obtain a new jerk chaotic system with three nonlinearities (two cubic nonlinearities and a transcendental nonlinearity).

In Section 2, the basic dynamical properties of the new jerk chaotic system have been discussed in detail. We discuss the bifurcation diagram, Lyapunov exponents, Lyapunov dimension and Poincaré map analysis. In Section 3, a circuit implementation of the new jerk chaotic system is shown to facilitate practical feasibility of the theoretical model. Section 4 concludes this work with a summary of the main results.

## 2. A New Jerk Chaotic System with Three Nonlinearities

In literature (Sprott, 1997), Sprott obtained a jerk chaotic system described by:

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -Az + xy^2 - x^3 \end{cases} \quad (1)$$

Sprott showed that the system (1) with two cubic nonlinearities displays chaotic behaviour when  $A = 3.6$ . In this work, we propose a new jerk chaotic system with three nonlinearities by adding a transcendental nonlinearity to Sprott system (1) as follows:

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -az - b|x|z + xy^2 - x^3 \end{cases} \quad (2)$$

In the new jerk system (2), we note that  $x, y, z$  are state variables and  $a, b$  are positive, constant, parameters. The new system (2) has two cubic nonlinearities ( $x^2y, x^3$ ) and a transcendental nonlinearity ( $|x|z$ ). We show that the system (2) displays chaotic behavior and strange attractor when:

$$a = 2, \quad b = 0.35 \quad (3)$$

For numerical calculations, we take the initial conditions for the new system (2) as:

$$x(0) = x_0 = 0.1, \quad y(0) = y_0 = 0.1, \quad z(0) = z_0 = 0.1 \quad (4)$$

For numerical simulation of the new chaotic system (2), we have used the classical fourth-order Runge-Kutta method in MATLAB.

Figures 1 (a)-(c) show the projections of the new chaotic system (2) on to the  $x$ - $y$  plane, the  $x$ - $z$  plane and the  $y$ - $z$  plane, respectively. Figure 1 (d) shows the 3-D phase portrait of the strange attractor of the new chaotic system (2). Lyapunov exponents of the new chaotic system (2) are determined using Wolf's algorithm (Wolf et. al, 1985) in MATLAB for the parameter values (3) and the initial conditions (4) as follows:

$$L_1 = 0.1704, \quad L_2 = 0, \quad L_3 = -2.5921 \quad (5)$$

The Lyapunov dimension of the new chaotic system (2) is obtained as:

$$D_L = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0657 \quad (6)$$

The maximal Lyapunov exponent (MLE) of the new chaotic system (2) is  $L_1 = 0.1704$ . The time-evolution of the Lyapunov exponents of the system (2) is depicted in Figure 2 (a). Since the sum of the Lyapunov exponents of the new chaotic system (2) is negative, it is evident that the system (2) is dissipative. Thus, the system orbits of the new jerk chaotic system (2) are ultimately confined into a specific limit set of zero volume and the asymptotic motion settles onto a strange chaotic attractor.

The dynamic behaviour of the new chaotic system (2) with respect to the bifurcation parameter  $b$  is investigated. The bifurcation diagram in Figure 2(d) is achieved by plotting the local maxima of the state variable  $Z_{max}$  when changing the value of  $b$ . The numerical result of Lyapunov exponents spectrum is shown in Figure 2(c). The bifurcation diagram agrees well with the Lyapunov exponent spectrum as shown in Figure 2(c). For  $0.2 \leq b \leq 0.74$  strange attractor is displayed as the new chaotic system (2), while for values of  $b > 0.74$  is a transition to periodic behavior. In addition, the Poincare map of new chaotic system (2) is shown in Figure 2 (b), which also reflects the chaotic properties of the system.

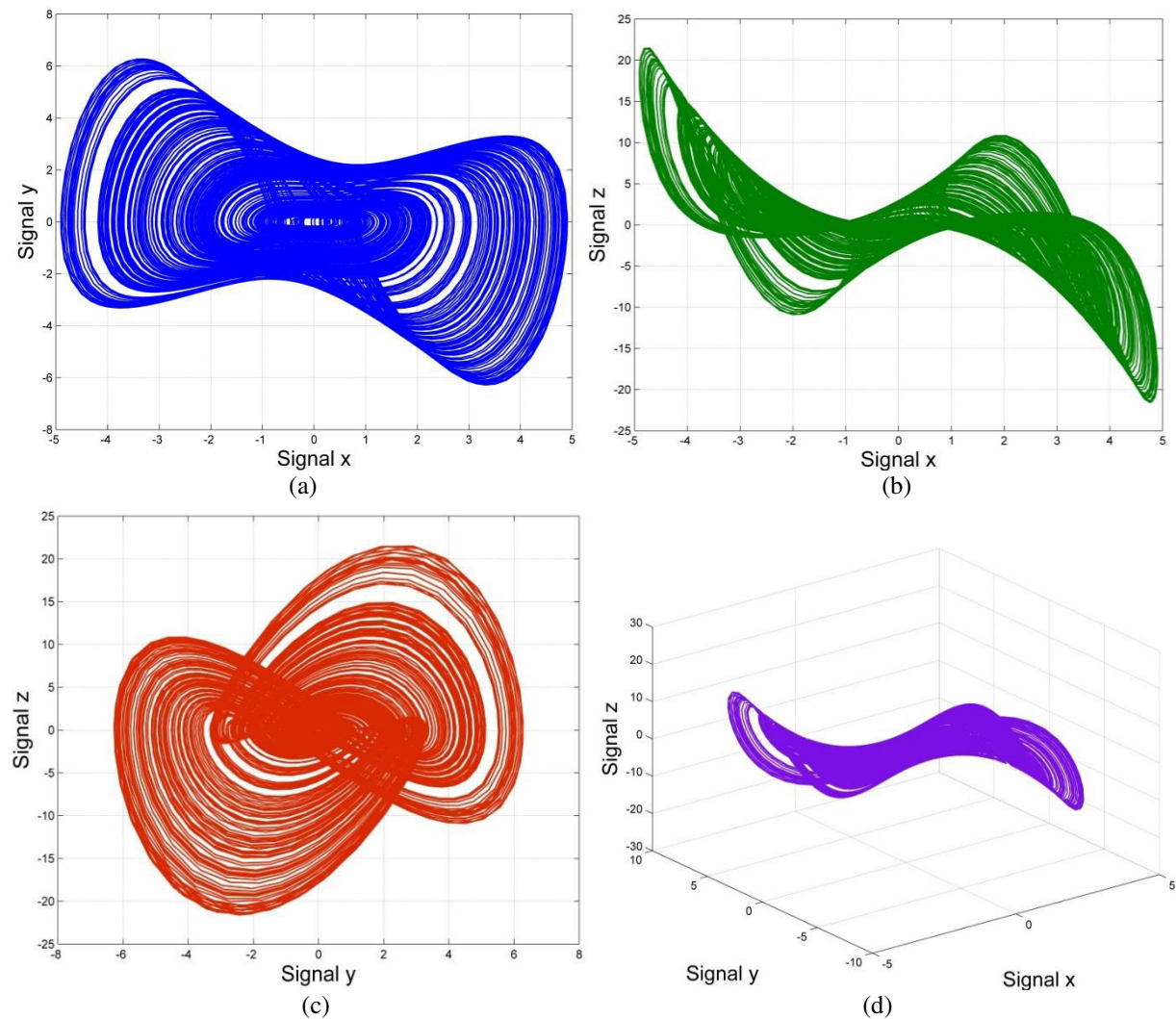


Figure 1: Numerical simulation results using MATLAB, for  $a = 2$  and  $b = 0.35$ , in (a)  $x$ - $y$  plane, (b)  $x$ - $z$  plane, (c)  $y$ - $z$  plane and (d)  $x$ - $y$ - $z$  plane.

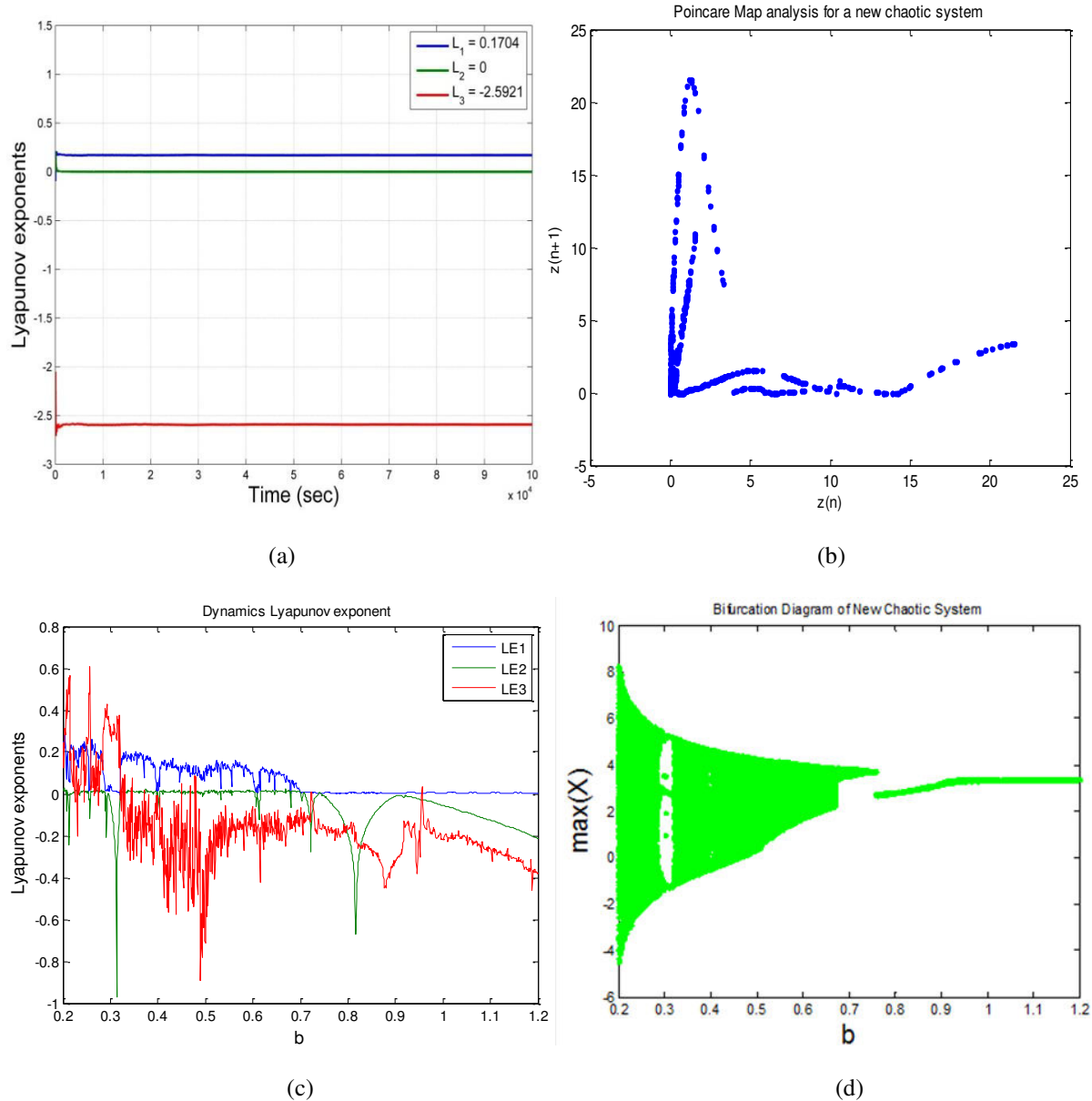


Figure 2: Dynamical analysis of new chaotic system (2) using MATLAB for  $a = 2$  and  $b = 0.35$  (a) Lyapunov exponents of the highly chaotic system (b) Poincare map of system (2) in the plane  $z(n+1)$  versus  $z(n)$  (c) Lyapunov spectrum of system (2) when varying the parameter  $b$  (d) Bifurcation diagram of system (2) versus the parameter  $b$ .

### 3. Circuit Implementation of the New Chaotic System

Chaotic behaviour in electric circuits have been studied with great interest. The chaotic dynamics of new chaotic system (2) has also been realized by an electronic circuit based on (Sambas et. al 2016b, 2018; Vaidyanathan et. al, 2018). For the rescaled on circuit design can be seen on (Li et. al, 2016, 2017). The circuit design of the new chaotic system (2) by MultiSIM is shown in Figure 3. The operational amplifiers TL082CD and associated circuitry perform the basic operations of addition, subtraction, and integration. The nonlinear terms of new chaotic system (2) are implemented with the analog multipliers AD633JN. By applying Kirchhoff's laws to the designed electronic circuit, its nonlinear equations are derived in the following form:

$$\begin{cases} \dot{x} = \frac{1}{C_1 R_1} y \\ \dot{y} = \frac{1}{C_2 R_2} z \\ \dot{z} = -\frac{1}{C_3 R_3} z - \frac{1}{10 C_3 R_4} |x| z + \frac{1}{100 C_3 R_5} x y^2 - \frac{1}{100 C_3 R_6} x^3 \end{cases} \quad (7)$$

We choose the circuit elements as  $R_1 = R_2 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = 10 \text{ k}\Omega$ ,  $R_3 = 5 \text{ k}\Omega$ ,  $R_4 = 2.857 \text{ k}\Omega$ ,  $R_5 = R_6 = 100 \Omega$ ,  $C_1 = C_2 = C_3 = 10 \text{ nF}$ . The supplies of all active devices are  $\pm 15 \text{ Volt}$ . The existence of the chaotic attractor can be clearly seen from Figures 4 (a)-(c). By comparing it with Figures 1 (a)-(c), it can be concluded that a good qualitative agreement between the numerical simulation by MATLAB and the experimental realization by MultiSIM is obtained.

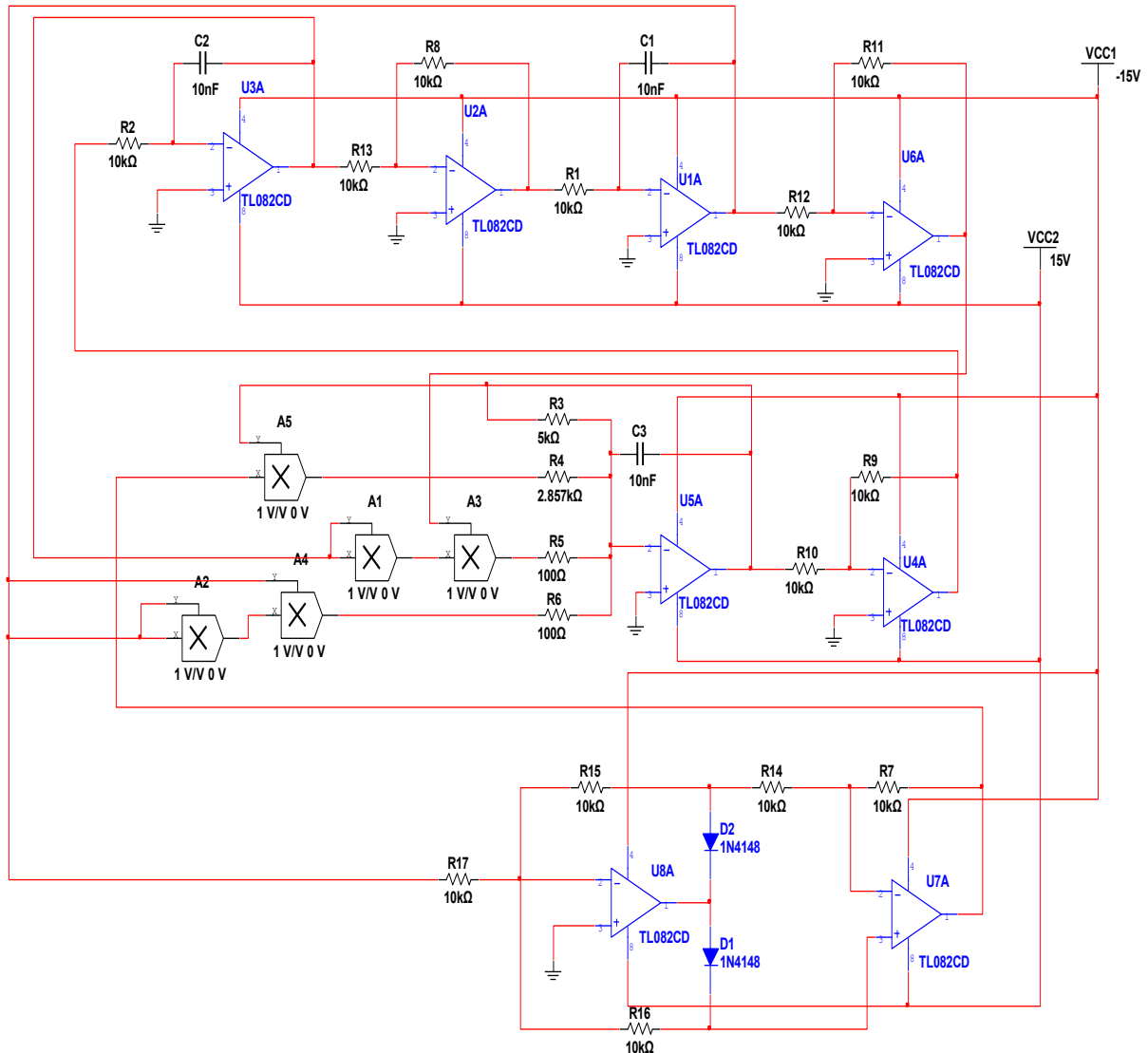


Figure 3: Circuit design for new chaotic system (2) by MultiSIM

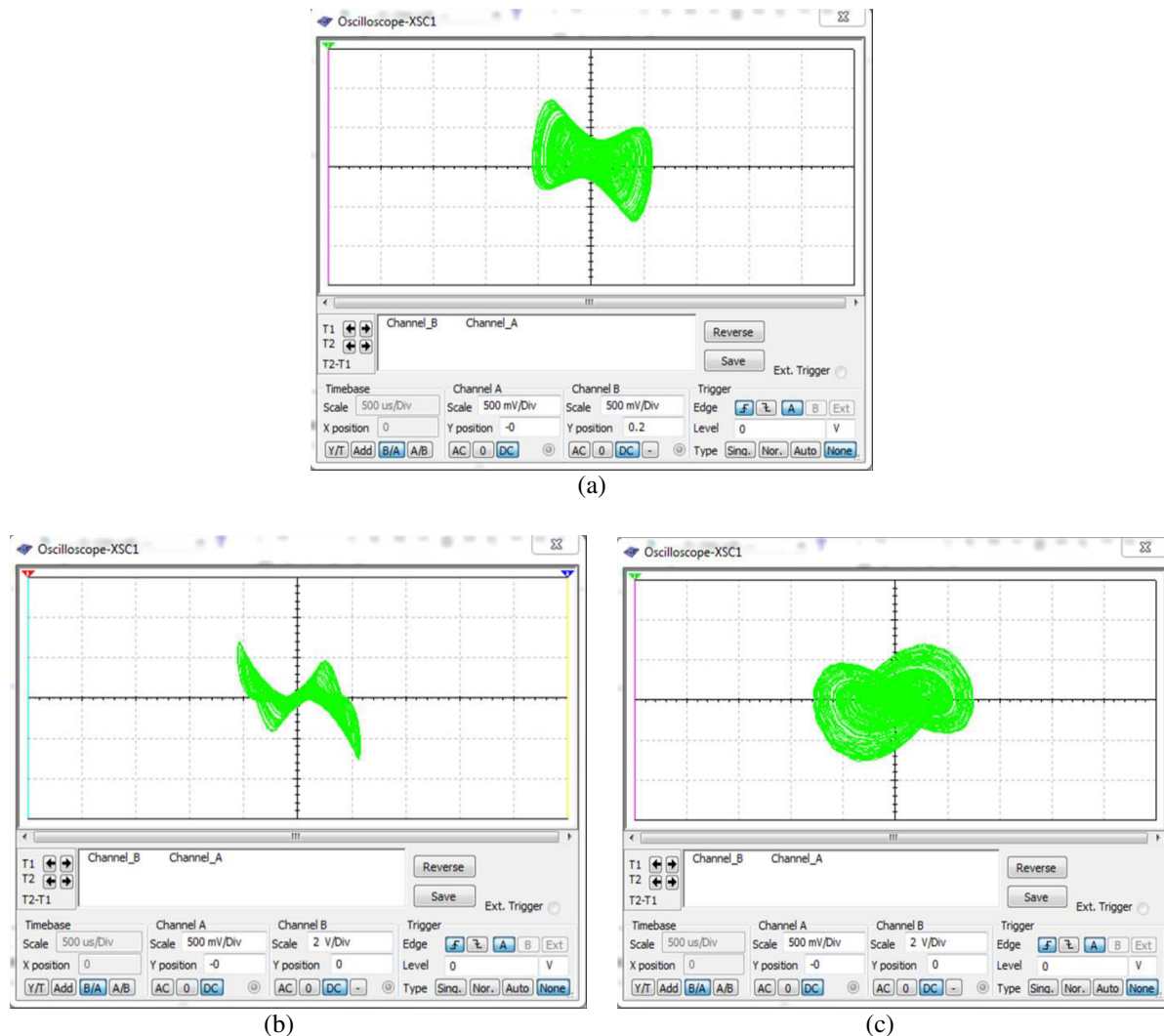


Figure 4: The phase portraits of new chaotic system (2) observed on the oscilloscope in different planes  
(a) x-y plane, (b) x-z plane, (c) y-z plane by MultiSIM

#### 4. Conclusion

In this work, a new jerk chaotic system with three nonlinearities was derived. Dynamics of the new chaotic system were analyzed in detail by means of phase portraits, Lyapunov exponents, Lyapunov dimension, bifurcation diagram and Poincaré map. Furthermore, an electronic circuit realization was shown to validate the chaotic behavior of the new jerk chaotic system. Finally, it was established that the physical circuit experimental results of the new jerk chaotic circuit show good qualitative agreement with the MATLAB simulations of the new jerk chaotic system.

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