A New Four-Dimensional Two-Wing Chaotic System with a Hyperbola of Equilibrium Points, its Properties and Electronic Circuit Simulation

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Abstract

In this research paper, we report the finding of a new four-dimensional, two-wing, chaotic system with two quadratic nonlinear terms. It is worthwhile to note that the new dissipative chaotic system has two-wing chaotic attractor and that it has a hyperbola of equilibrium points. Thus, it follows that the new two-wing chaotic system exhibits a hidden attractor. Phase plots of the new two-wing chaotic system are illustrated, properties are elucidated, and electronic circuit simulations are also depicted to verify the feasibility of the theoretical mechanical chaotic system developed in this work.

Keywords
Chaos, chaotic systems, curve equilibrium, electronic circuits.

1. Introduction

In the last three decades, several applications have been developed in science and engineering for chaotic systems (Azar and Vaidyanathan, 2016, Vaidyanathan and Volos, 2016). Many applications of chaotic systems have been made in secure communication systems (Cicek *et al*., 2018; Sun *et al.,* 2018; Jayawickrama *et al*., 2018; Seneviratne and Leung, 2017), image encryption (Li *et al*., 2018; Ullah *et al*., 2018; Wu *et al*., 2018), data encryption (Chen *et al*., 2017; Gan et al., 2017), Tokamak systems (Vaidyanathan, 2015), delay systems (Pham *et al.,* 2015), circuits (Volos *et al*., 2015), fuzzy control (Boulkroune *et al*., 2016), oscillators (Bao *et al*., 2018; Messias and Reinol, 2018; Mishra and Yadava, 2018), chemical reactor (Budroni, et al., 2018), etc.

In the chaos literature, considerable attention has been devoted to the construction and analysis of chaotic systems with a line equilibrium (Jafari and Sprott, 2013; Li *et al*., 2014) and chaotic systems with a curve equilibrium (Gotthans and Petrzela, 2015; Kingni *et al*., 2017).

In this work, we derive a new four-dimensional two-wing chaotic system with a hyperbola of equilibrium points. Our new chaotic system exhibits a hidden chaotic attractor (Leonov *et al*., 2011; Sambas *et al.,* 2018) as it possesses an infinite number of equilibrium points on a hyperbola. This work makes a new valuable addition to the chaotic systems with a special curve of equilibrium points. We also unveil an electronic circuit simulation of the new chaotic system with a hyperbola of equilibrium points. It is known that checking the feasibility of a chaotic system with electronic circuit realization has practical applications (Sambas *et al*., 2016; Vaidyanathan and Rajagopal, 2016; Pham *et al*., 2016; Vaidyanathan *et al*., 2017; Volos *et al*., 2017).

2. A new four-dimensional two-wing chaotic system

In this section, we describe our results of a new four-dimensional two-wing chaotic system. Our dynamics is given by

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) - x_4 \\ \dot{x}_2 = x_1 x_3 \\ \dot{x}_3 = b - x_1 x_2 \\ \dot{x}_4 = x_3 \end{cases} \tag{1}$$

In Equation (1), the state variables are designated as x_i, $(i = 1,2,3,4)$. Also, a, b are positive constants. We shall show that the system (1) displays hidden chaotic attractor for the choice $(a,b) = (5,25)$.

Indeed, for the initial state $x(0) = (0.2, 0.2, 0.2, 0.2)$ and $(a,b) = (5,25)$, the Lyapunov exponents of the system (1) are determined using MATLAB as $(LE_1, LE_2, LE_3, LE_4) = (1.0060, 0, 0, -6.0060)$. This pinpoints that the system (1) is a dissipative chaotic system with maximal Lyapunov exponent (MLE) as $LE_1 = 1.0060$, which is positive. In addition, the Kaplan-Yorke dimension of the chaotic system (1) is determined as follows:

$$D_{KY} = 3 + \frac{LE_1 + LE_2 + LE_3}{|LE_4|} = 3.1675 \tag{2}$$

The new chaotic system (1) stays unchanged by the transformation of coordinates given by

$$(x_1, x_2, x_3, x_4) \mapsto (-x_1, -x_2, x_3, -x_4) \tag{3}$$

This shows that the chaotic system (1) has a rotation symmetry about the x_3 −coordinate axis. Consequently, every non-trivial trajectory of the system (1) has a twin trajectory.

The equilibrium points of the four-dimensional chaotic system (1) are got by solving the equations:

$$a(x_2 - x_1) - x_4 = 0 \tag{4a}$$

$$x_1 x_3 = 0 \tag{4b}$$

$$b - x_1 x_2 = 0 \tag{4c}$$

$$x_3 = 0 \tag{4d}$$

Since $(a,b)=(5,25)$ for the chaotic case, the equilibrium points of the chaotic system (1) are given by the three equations:

$$x_3 = 0,\ x_1 x_2 = 25,\ \ 5(x_2 - x_1) - x_4 = 0 \tag{5}$$

From Equation (5), we see that the new chaotic system (1) has a hyperbola of equilibrium points as illustrated in Figure 1.

The system (1) exhibits a hidden chaotic attractor for $(a,b)=(5,25)$ as it possesses an infinite number of equilibrium points. Its phase portraits for $X(0)=(0.2,0.2,0.2,0.2)$ and $(a,b)=(5,25)$ are illustrated in Figures 2-5. From these figures, it is clear that the new chaotic system (1) exhibits a two-wing chaotic attractor.

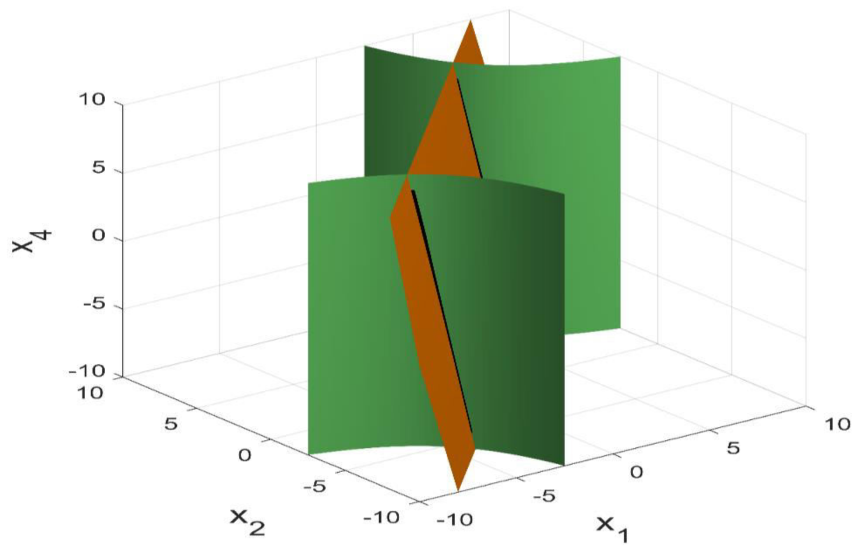


Figure 1. The hyperbola of equilibrium points of the new chaotic system (1) for $(a,b)=(5,25)$ and $x_3 = 0$

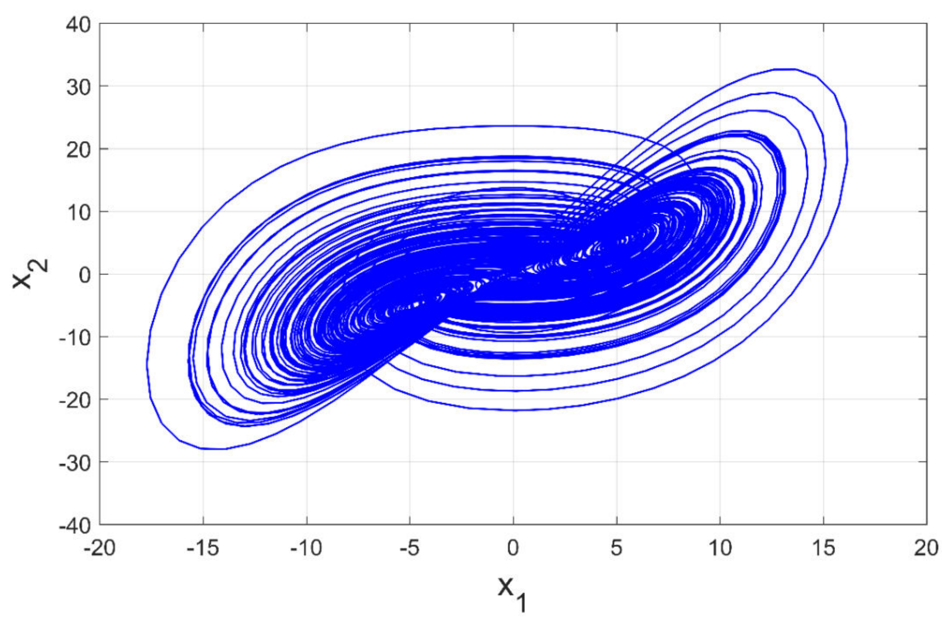


Figure 2. MATLAB simulation of the hidden attractor of the new chaotic system (1) for $(a,b)=(5,25)$ and $X(0)=(0.2,0.2,0.2,0.2)$ in the $(x_1,x_2)-$ plane

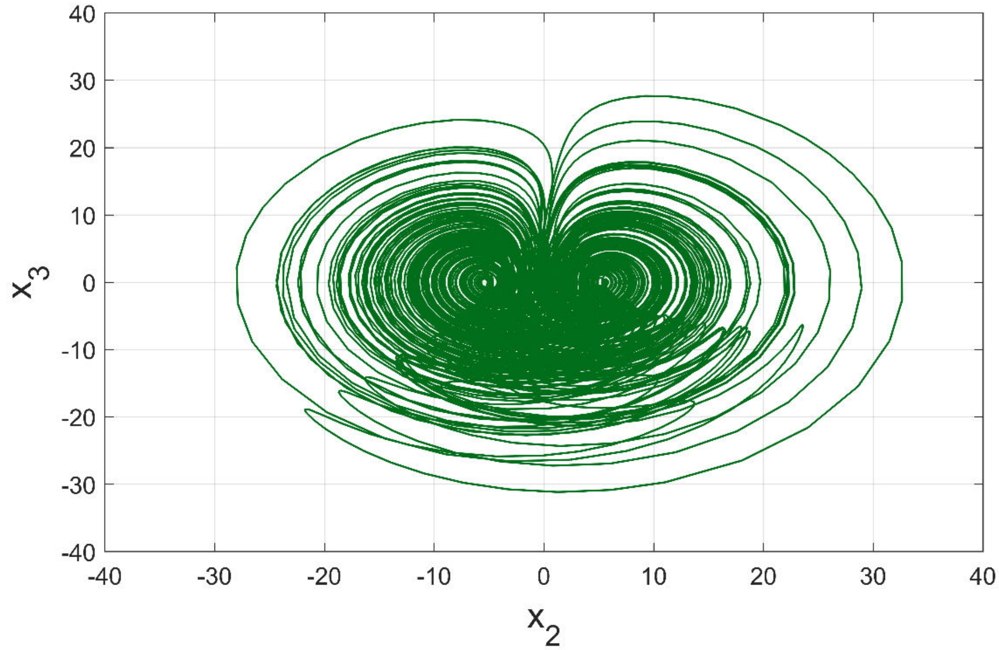


Figure 3. MATLAB simulation of the hidden attractor of the new chaotic system (1) for $(a,b)=(5,25)$ and $X(0)=(0.2,0.2,0.2,0.2)$ in the $(x_2,x_3)-$plane

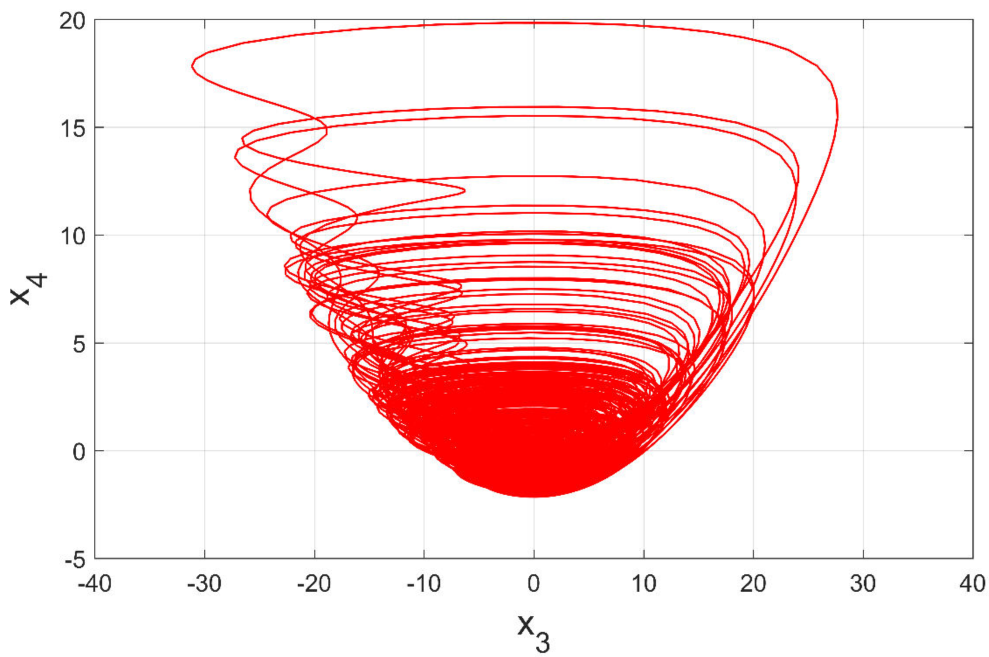


Figure 4. MATLAB simulation of the hidden attractor of the new chaotic system (1) for $(a,b)=(5,25)$ and $X(0)=(0.2,0.2,0.2,0.2)$ in the $(x_3,x_4)-$plane

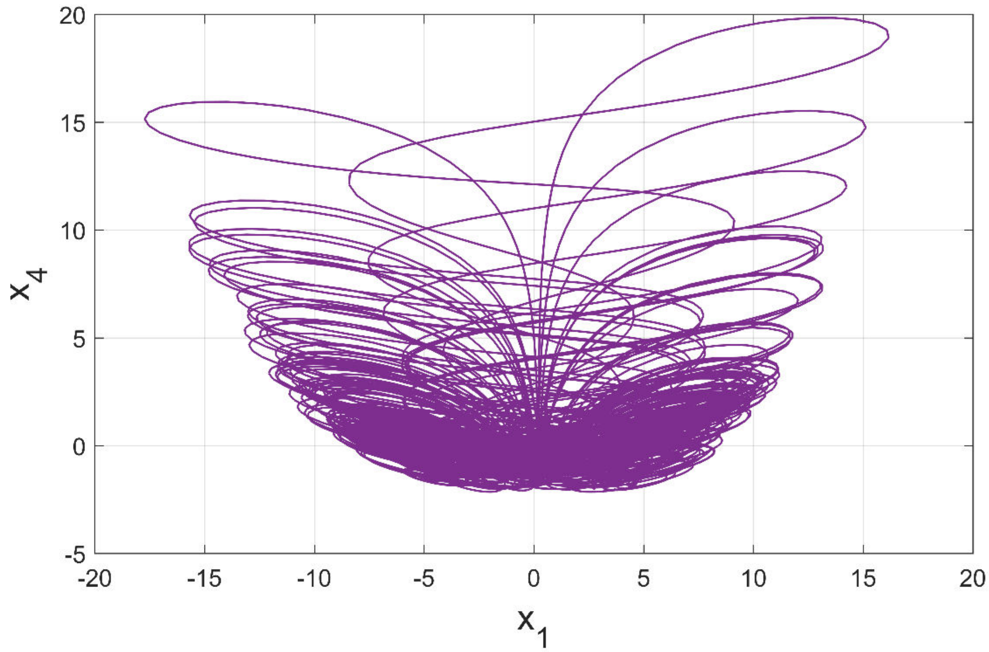


Figure 5. MATLAB simulation of the hidden attractor of the new chaotic system (1) for $(a,b)=(5,25)$ and $X(0)=(0.2,0.2,0.2,0.2)$ in the $(x_1,x_4)-$ plane

3. Electronic circuit realization new four-dimensional two-wing chaotic system

In this section, the new four-dimensional two-wing chaotic system (1) is realized by an electronic circuit shown in Fig. 6. It has four integrators (U1A, U2A, U3A, U4A), three invertings (U5A, U6A, U7A) and two signals multipliers (A1, A2) by using the AD633JN. The TL082CD operational amplifiers are used in this work and the supplies of all active devices are ±15 Volt.

In this study, a linear scaling is considered as follows:

$$\left.\begin{aligned} \dot{x}_1 &= a(x_2-x_1)-x_4 \\ \dot{x}_2 &= 2x_1x_3 \\ \dot{x}_3 &= \frac{b}{2}-2x_1x_2 \\ \dot{x}_4 &= x_3 \end{aligned}\right\} \tag{6}$$

By applying Kirchhoff's laws to this circuit, its dynamics are presented by the following circuital equations:

$$\left.\begin{aligned} \frac{dVc_1}{dt} &= \frac{1}{C_1R_1}Vc_2-\frac{1}{C_1R_2}Vc_1-\frac{1}{C_1R_3}Vc_4 \\ \frac{dVc_2}{dt} &= \frac{1}{C_2R_4}Vc_1Vc_3 \\ \frac{dVc_3}{dt} &= \frac{1}{C_3R_5}V_1-\frac{1}{C_3R_6}Vc_1Vc_2 \\ \frac{dVc_4}{dt} &= \frac{1}{C_4R_7}Vc_3 \end{aligned}\right\} \tag{7}$$

Where V_{C1}, V_{C2}, $V_{C3,}$ V_{C4} are the voltages across the capacitors C_1, C_2, C_3 and C_4, respectively. Here the design approach based on the operational amplifiers (Sambas et. al, 2017; Vaidyanathan et. al, 2018) is applied.

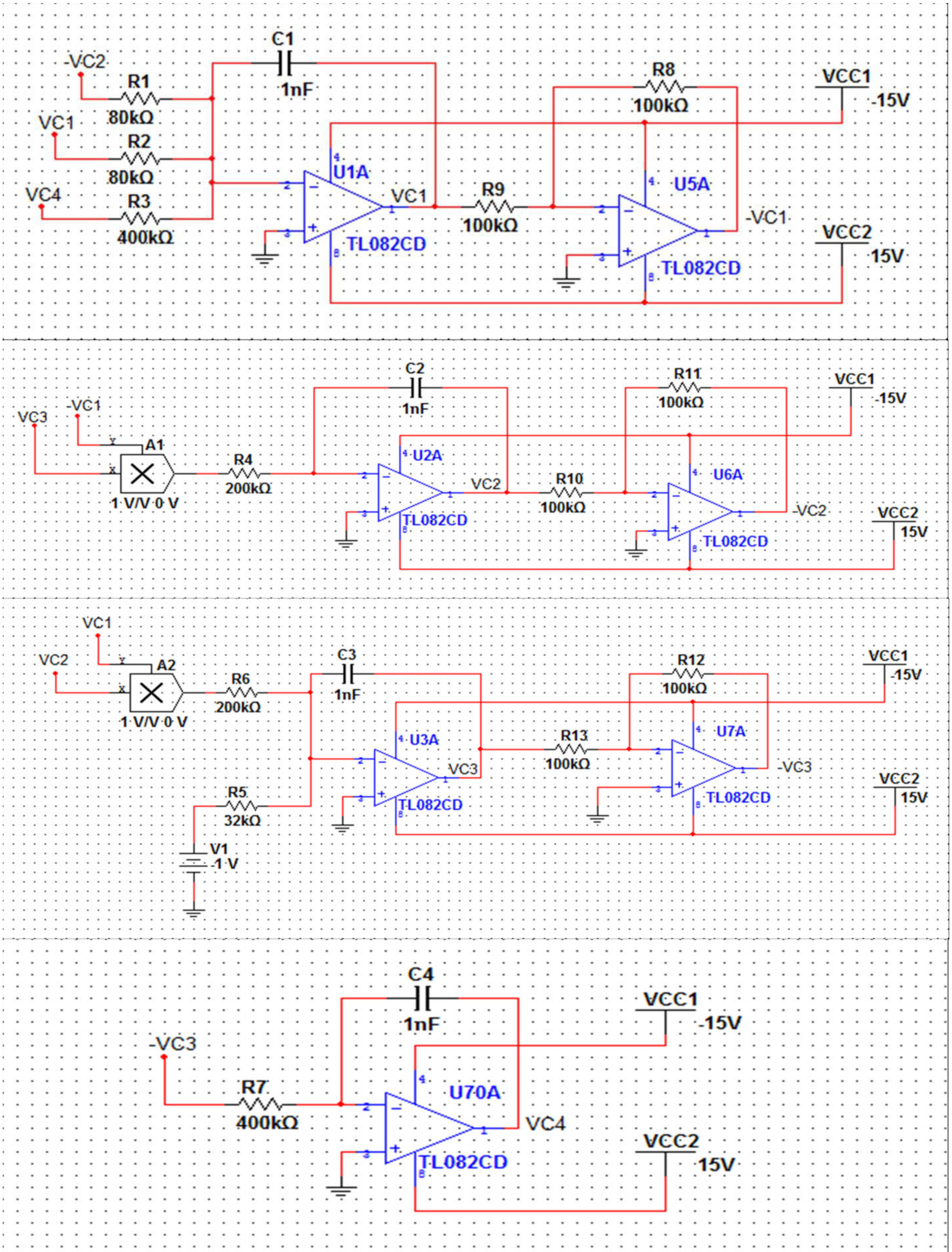
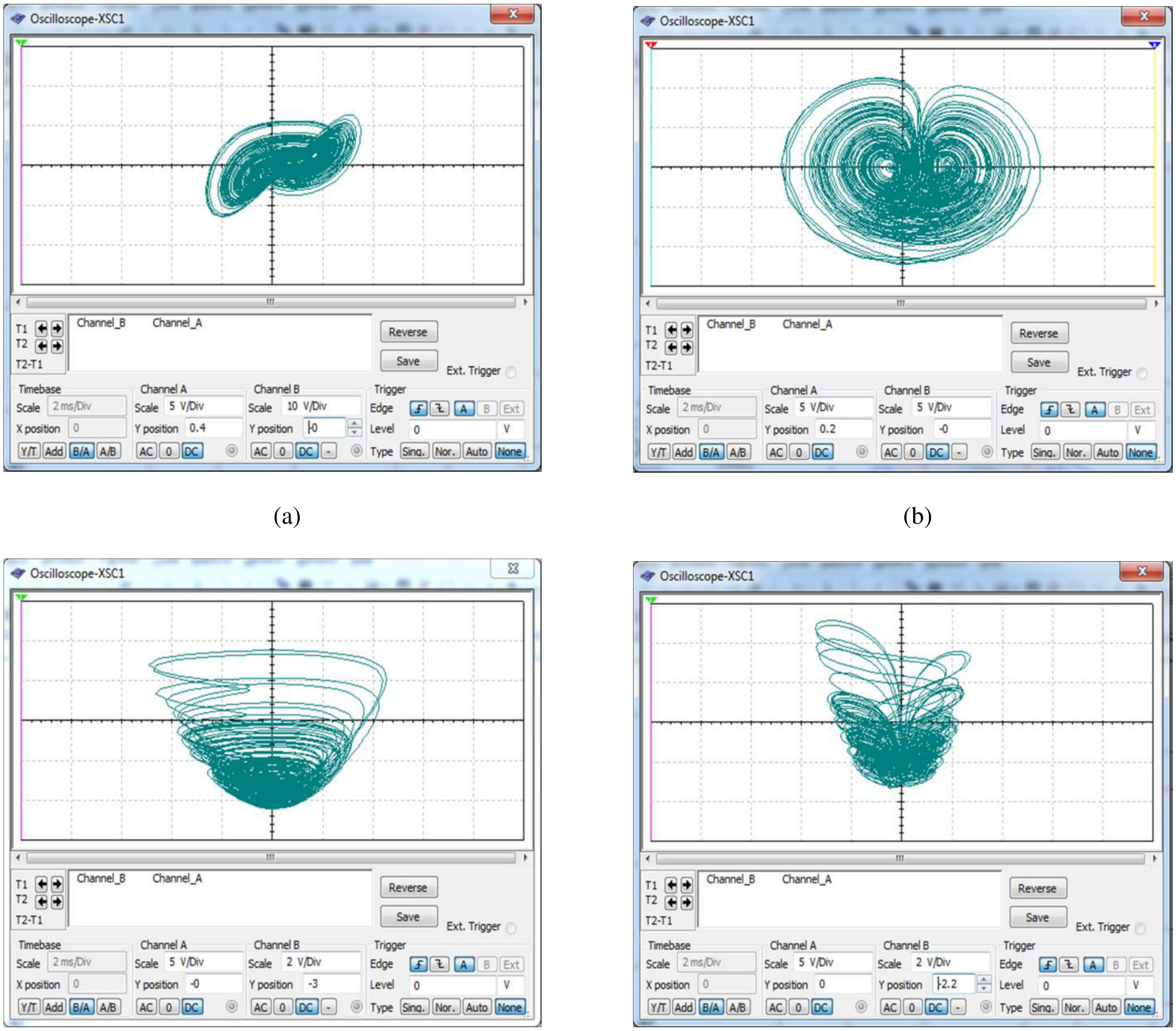


Figure 6. Circuit design for the proposed new four-dimensional two-wing chaotic system (1)

The values of components in Figure 6 are chosen to match the parameters of new four-dimensional two-wing chaotic system (1) as follows: $R_1 = R_2 = 80$ KΩ, $R_3 = R_7 = 400$ KΩ, $R_4 = R_6 = 200$ KΩ, $R_5 = 32$ KΩ, $V_1 = -1$ V_{DC}, $R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = 100$ KΩ, $C_1 = C_2 = C_3 = C_4 =$ 1nF. The circuit simulations of the phase plots are displayed in Figures 7 (a)-(d), which show the chaotic attractors in x_1- x_2 plane, x_2- x_3 plane, x_3- x_4 plane and x_1- x_4 plane, respectively. From the comparison of the phase portraits of Figs. 2-5 with that of Figures 7 (a)-(d), a good agreement between the chaotic behavior obtained form MATLAB simulation and MultiSIM results can be concluded.



(a) (b) (c) (d)

Figure 7 MultiSIM chaotic attractors of the new four-dimensional two-wing chaotic system (1) (a) x_1- x_2 plane, (b) x_2- x_3 plane, (c) x_3- x_4 plane and (d) x_1- x_4 plane.

4. Conclusions

In this work, we described our new results of a four-dimensional, two-wing, chaotic system with two quadratic nonlinear terms. We proved that the new dissipative chaotic system has a hyperbola of equilibrium points and that it exhibits a two-wing attractor. Thus, it follows that the new two-wing chaotic system exhibits a hidden attractor. Phase plots of the new two-wing chaotic system were illustrated, and electronic circuit simulations using MultiSIM are also depicted to verify the feasibility of the theoretical mechanical chaotic system developed in this work.

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