Effective X-bar&R Chart for Monitoring Aluminum Extrusion Process

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Abstract

Aluminum extrusion is considered as one of the very challenging processes to produce good quality and low cost products. This paper presents an optimization design of the $\bar{X}\&R$ chart for effective statistical monitoring of output of aluminum extrusion process. The proposed optimal $\bar{X}\&R$ chart is compared with traditional 3-σ $\bar{X}\&R$ chart, in terms of the average extra quadratic loss (*AEQL*), for detecting a wide range of shifts in the mean and variance. The results reveal that the optimal $\bar{X}\&R$ chart overtakes the 3-σ $\bar{X}\&R$ chart by 94%, in terms of *AEQL*. In addition, the former has the smallest out-of-control average time to signal (*ATS*) over almost all shifts. Consequently, the replacement of traditional $\bar{X}\&R$ chart by the optimal one is recommended to detect the mean and variance shifts more efficiently and as a result, reduce the process variation and avoid economic loss.

Keywords
Aluminum Extrusion, Control Chart, Statistical Process Control

1. Introduction

Aluminum extrusion usage has been increased in the last decades. It approximately represents one-third of the global aluminum production. Products of aluminum extrusion are usually seen in our daily life from cars and airplanes to wall elements and window frames. It is considered as a complex process since it depends on material properties and many process parameters, such as initial temperature of billet, die geometry, friction at the outlet, cooling temperature and ram speed. Products produced by the extrusion process can have complex geometrical designs achieving customized cross-sections. However, due to the nature of the process, variations in geometry always takes place, which lower the capability to meet strict tolerance limits of the final product.
Lucignano et al. (2010) used two neural networks to optimize the aluminum extrusion process. Zhang et al. (2012) developed a numerical model to study metal flow attributes during aluminum extrusion. The numerical results were validated by comparing with the nose ends of two extrudates in practice, and the comparison showed that the numerical model developed in this work could

provide the useful guidance for an efficient production. Reggiani et al. (2013) designed a multi-objective optimization of extrusion dies. The objective functions were the optimal die strength, welds quality, profile tolerances and exit flow balance. The proposed methodology to the extrusion process optimization provided an efficient automated solution for practical productions problems of the die. Garbacz et al. (2015) proposed an inspection system that can be applied for on-line monitoring of aluminum extrusion processes and the inspection of defects arising in extruded products. The proposed system is hybrid between computer analysis, simultaneous infrared, and visible image analysis for surface inspection of the profile.

Raknes et al. (2018) adopted an alternative approach to overcome dimension variations in aluminum extrusion. He used Finite Element Analysis (FEA) to simulate the extrusion of a U-shaped profile and to provide response surfaces with for FEA program, he used a two-level fractional factorial Design of Experiment (DOE) to collect the appropriate data. The factors with significant effect on the dimension accuracy of the final part were obtained. The results of the study showed to reduce cross-sectional variations effectively from traditional extrusion tolerances.

In this paper, we propose an effective Statistical Process Monitoring (SPM) through optimizing $\bar{X}$& R control chart to improve the effectiveness for detecting mean and variance shifts.

2. Control chart

Shewhart (1931) proposed statistical control charts as a method for quality monitoring. The control chart procedure asserts the improvement of quality by monitoring the process rather than correcting defects in the final product. The Shewhart control charts are now known as the X, $\bar{X}$, R and S charts for variable data and p, np, c and u charts for attribute data. These charts are common in practice as they are easy to construct and implement. When constructing control charts, samples are drawn from the process in different periods and sample statistics are then computed and plotted on control charts. If any special cause has occurred, the chart will give an out-of-control signal (the statistic will fall outside the control limits). The combination of a Shewhart chart has been employed widely for this purpose even though computing systems are available in most of today's manufacturing industries. This is attributable to the simplicity of these combinations for understanding, implementation and design compared with many new advanced charts.

The construction and implementation of the Shewhart control chart for any process requires determination of three design parameters: the sampling interval rate h, the sample size n, and the width coefficient of the control limit k. For a Standard Shewhart control chart, these parameters are always the same for the duration of operation to be monitored.

Since 1999, there has been a good attention to the joint monitoring of the process mean and variance (Costa, 1999, Rahim and Costa, 2000, Reynolds Jr and Stoumbos, 2001, Li et al., 2010, Ou et al., 2012, Tasias and Nenes, 2018). The combination of a Shewhart $\bar{X}$ chart and an R chart has been widely used for this purpose. Costa and Rahim (2004) proposed joint $\bar{X}$&R charts with two stage sampling (TTS), each with a different size. Through the first stage, every single item of the sample is inspected and, based on the result, if the process is in control, the sampling is stopped. Otherwise, it goes on to the second stage, where the remaining sample items are inspected. They compared the TTS $\bar{X}$&R charts with the Variable Sample Size (VSS) $\bar{X}$&R using Average Run Length (*ARL*) as a performance measure. It is found that the TTS $\bar{X}$&R charts provides the same protection against false alarms and has the same average number of inspected items. Wu and Chen (2006) developed a joint economic design of the Variable Sampling Interval (VSI) $\bar{X}$&R chart and studied the optimal sample size that minimizes the expected total cost associated with the test procedure using a genetic algorithm. Also, they provided an example of how to obtain the best sample size. Kasarapu and Vommi (2011) applied differential evolution (DE) in which a population-based evolutionary optimization technique has to implement a joint $\bar{X}$&R control charts. The developed model is compared with the earlier designs which are based on conventional optimization techniques. It is found that the designs obtained using the designed DE are better than most of the compared cases.

Typically, a sample size n between 4 and 6 is usually recommended for the $\bar{X}$&R chart (Montgomery, 2007). When a process shift occurs, the mean μ and standard deviation σ of x will change.

$$\mu = \mu_0 + \delta_\mu \sigma_0, \qquad \sigma = \delta_\sigma \sigma_0 \tag{1}$$

where δ_μ and δ_σ are the mean shift and standard deviation shift, respectively, in terms of σ_0. When the process is in control, $\delta_\mu = 0$ and $\delta_\sigma = 1$. The sample mean value is converted to z which follows a standard normal distribution as follows:

$$z = \frac{\bar{x} - \mu_0}{\sigma_0} \tag{2}$$

The $\bar{X}$&R chart has five parameters: the sample size n, sampling interval h, upper control limit UCL, lower control limit LCL, and the upper control limit H of the R chart element. If $LCL \leq \bar{X} \leq UCL$ and the sample range $R \leq H$, the process is deemed to be in control. Otherwise, it is considered to be out of control. The sample size n is the only independent design variable to be optimized, while the other four parameters h, UCL, LCL and H are dependent design variables, and are determined by the two constraints on inspection rate and false alarm rate.

3. Aluminum extrusion process

There are two known types of aluminum extrusion processes; direct and indirect. In the direct extrusion, the die is stationary and the ram forces the billet through the die profile which is designed to obtain the required aluminum section. While, in the indirect process, the die is contained within the ram, which moves into the stationary billet from one end, forcing the metal to flow into the ram, acquiring the shape of the die as it does so as shown in Figure 1.

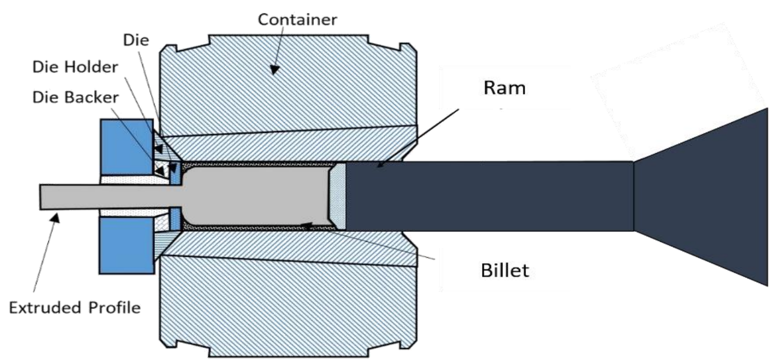

Figure 1. Direct aluminum extrusion

Figure 2 describes the sequence of direct aluminum extrusion process. In this study, we focus on step 3, i.e., the extrusion operation. Before the extrusion operation, the billet is softened in a heating furnace. The melting point of aluminum varies with the purity of the metal, but it is usually between 450° and 600°. Extrusion typically occurs after the heating process. The ram inside the press starts applying pressure to the billet forcing it through the die. About 10 percent of the billet, including its outer skin, is left behind in the container. The completed extrusion is cut off at the die and the remainder of the metal is removed to be used again. A stretcher is used to allow the extruded part cool and be stretched to straighten the product and correct any bending that may have occurred during or after extrusion. After elongation, conveyors feed the work to the saw to be cut again to the required commercial length. To obtain the desired properties of the extruded part aging or age-hardening is done. It takes place in a temperature controlled furnace. The aging process ensures the uniform precipitation of fine particles through the metal, yielding maximum strength, hardness, and elasticity for the specific extrusion alloy. Extrusion rates vary, depending on the alloy used and the shape of the die. A hard alloy, given a complex shape, may emerge from the press very slowly while a softer alloy with a simple shape may be extruded at a faster rate. Besides, the extrusion is cooled after emerging from the die, either through the air or water quenches, depending on the used alloy.

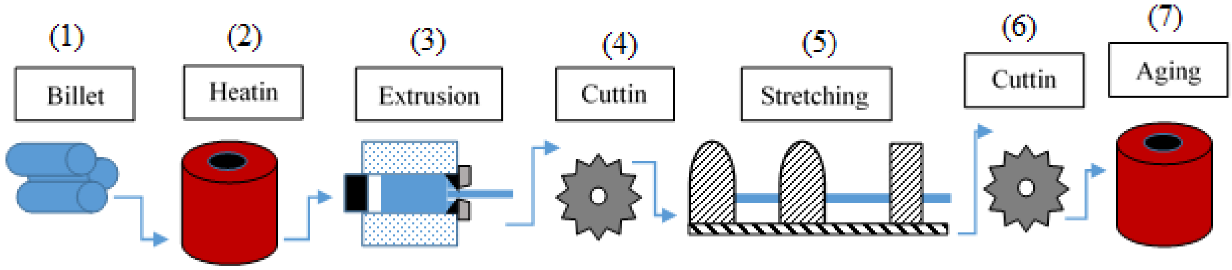

Figure 2. Sequence of aluminum extrusion process

4. Case Study

This case study focuses on the quality of aluminum profiles produced by aluminum extrusion manufactured by aluminum International Company that was established in 2005 in Egypt. The aluminum profile of interest is shown in Figure 3. The length (x) of the extruded part (BA905) produced by the illustrated profile is a critical dimension that has to be monitored by an SPC control chart for detecting a wide range of shifts in mean and variance. The length x is specified as 28.80 ± 0.16 mm.

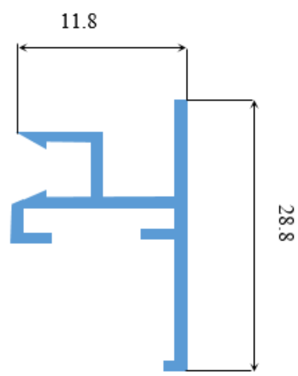

Figure 3. Part of interest (BA905)

The control chart here is used as a monitoring tool based on which we decide whether to take an action or not. In other words, if the values of length we monitor are within the control limits, no action is required. On the other hand, if the control chart signals, the assignable causes should be identified and a corrective action should be taken.

In this case study, the team is interested in detecting the two-sided mean shift and an increasing variance shift. The internal quality team currently uses a traditional 3-σ $\bar{X}$&R chart (with n = 5) for monitoring x. The team investigates designing an optimal $\bar{X}$&R chart to detect the process shifts more efficiently.

Following the framework proposed by Haridy et al. (2016) to carry out the design of the above chart, the following three specifications have to be determined beforehand: (1) The minimum allowable value τ of the in-control Average Time to Signal ATS_0, (2) the allowable inspection rate r and (3) the mean values μ_{δ_μ} and μ_{δ_σ} of the random shift in mean and standard deviation, respectively. The optimization model goes as:

Objective function: Minimize *AEQL*

Subject to:

Constraint for ATS_0: $ATS_0 = \tau$ (3)

Constraint for inspection rate: $h = n / r$ (4)

The optimal values of the independent design variable (n) are searched so that the objective function *AEQL* is minimized. The dependent design variable (h, UCL, LCL and H) are adjusted to meet the two constraints (3) and (4). It is assumed that the shift in mean and variance is following Rayleigh distribution.

The value of τ is determined based on the allowable false alarm rate. The value of r is equal to the ratio between the average sample size n and the sampling interval h and depends on the available resources such as workforce and measurement instruments. Usually, only the in-control (or long run) value of r is considered as the inspection rate in the short out-of-control period has little influence on the long run value of r and is of much less concern (Arnold and Reynolds, 2001).

When the process was in control, the Quality Control (QC) engineer collected 30 samples of size 5 at phase I to estimate the mean μ_0 and standard deviation σ_0 of the thickness x. As shown in Figure 4, it can be seen that the distribution of the data is well presented by a normal distribution with μ_0 = 28.80 and σ_0 = 0.05. The available resources allow the QC engineer to use an inspection rate of 1.

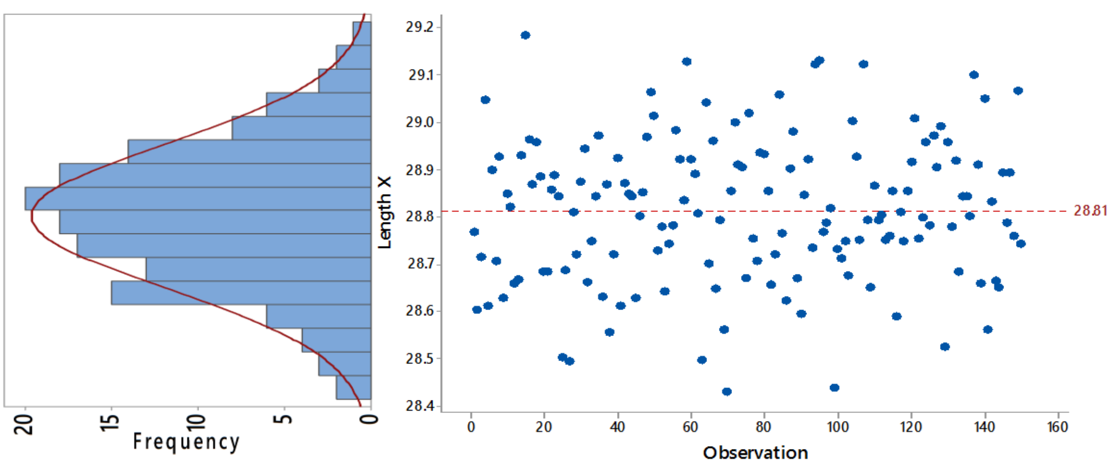

Figure 4. Sample data collected when the process is in control

The QC engineer used the historical records of out-of-control to estimate the distribution of the process shift. It was found that the random shifts δ_μ and δ_σ can be adequately represented by a Rayleigh distribution with a mean of μ_{δ_μ} = 3.513 and μ_{δ_σ} = 4.164, respectively.

The traditional 3-σ $\bar{X}$&R chart currently used in the factory gives a false alarm rate of 733. Hence, the optimal $\bar{X}$&R chart is also designed under the same τ = 733 for a fair comparison. The values of the charting parameters can be found in Table 1 where UCL and LCL are the upper and lower control limits for the $\bar{X}$ chart element, respectively while H is the upper control limit for the R chart element. The values of Average Extra Quadratic Loss values (*AEQL*) are also indicated in Table 1. The index *AEQL* is a performance measure used to evaluate the overall performance of the control charts over a wide domain of shifts rather than at a specific shift. *AEQL* directly relates the chart performance with the economic outcome based on Taguchi's loss function (Ross, 1988).

$$AEQL = \frac{1}{\delta_{\mu,max} \cdot (\delta_{\sigma,max} - 1)} \int_{0}^{\delta_{\mu,max}} \int_{1}^{\delta_{\sigma,max}} (\delta_\mu^2 + \delta_\sigma^2 - 1) \cdot ATS(\delta_\mu, \delta_\sigma) \cdot f(\delta_\mu) \cdot f(\delta_\sigma) d\delta_\mu d\delta_\sigma \qquad (5)$$

The smaller the *AEQL*, the better the chart is. It is found that the optimal $\bar{X}$ & R chart with a sample size *n* of 2 can outperform the traditional 3-σ $\bar{X}$ & R chart by 94% in terms of *AEQL*.

Table 1. Charting parameters and *AEQL* values for the 3-σ $\bar{X}$ & R and optimal $\bar{X}$ & R chart

Chart	Charting parameters					*AEQL*
	n	*h*	*UCL*	*LCL*	*H*	
3-σ $\bar{X}$ & R	5	5	28.733	28.867	0.248	88.045
Optimal $\bar{X}$ & R	2	2	28.689	28.911	0.232	45.355

Table 2 shows that the optimal $\bar{X}$ & R chart outperforms the 3-σ $\bar{X}$ & R chart over almost all process shifts (small, medium and large shifts) except for small pure mean shifts (δ_μ = 2.2, δ_σ = 1) in terms of the Average Time to Signal (*ATS*).

Table 2. *ATS* values of the 3-σ $\bar{X}$ & R and optimal $\bar{X}$ & R chart

δ_σ	Chart	δ_μ					
		0	2.2	4.4	6.6	8.8	11
1	3-σ $\bar{X}$ & R	733.0	2.641	2.500	2.500	2.500	2.500
	Optimal $\bar{X}$ & R	733.1	3.120	1.002	1.000	1.000	1.000
3	3-σ $\bar{X}$ & R	3.439	2.816	2.513	2.500	2.500	2.500
	Optimal $\bar{X}$ & R	3.099	2.091	1.248	1.029	1.001	1.000
5	3-σ $\bar{X}$ & R	2.601	2.565	2.518	2.502	2.500	2.500
	Optimal $\bar{X}$ & R	1.597	1.481	1.264	1.104	1.030	1.006
7	3-σ $\bar{X}$ & R	2.521	2.517	2.508	2.503	2.501	2.500
	Optimal $\bar{X}$ & R	1.286	1.257	1.189	1.115	1.059	1.025
9	3-σ $\bar{X}$ & R	2.506	2.506	2.504	2.502	2.501	2.500
	Optimal $\bar{X}$ & R	1.169	1.158	1.132	1.097	1.064	1.038
11	3-σ $\bar{X}$ & R	2.502	2.502	2.501	2.501	2.500	2.502
	Optimal $\bar{X}$ & R	1.107	1.095	1.077	1.058	1.041	1.107

5. Conclusion

This study aims at improving the effectiveness of detecting shifts in mean and variance in the aluminum extrusion process in a manufacturing plant, and hence lowering the number of defective items produced. The proposed case study emphasizes the supremacy of the optimal $\bar{X}$ & R chart with *n* = 2 over the traditional 3-σ $\bar{X}$ & R chart with *n* = 5 in terms of *AEQL* and *ATS*. It is found that the optimal $\bar{X}$ & R chart is more effective than the 3-σ $\bar{X}$ & R chart by 94%, in terms of *AEQL*. In addition, the former has the smallest out-of-control *ATS* for almost all shifts except for small pure mean shifts.

This study is conducted assuming that the quality characteristic x of the output of the aluminum extrusion process follows a normal distribution with a known in-control μ_0 and standard deviation σ_0, while δ_μ and δ_σ are assumed to follow Rayleigh distribution. It is interesting to carry out further studies on the performance of the $\bar{X}\&R$ chart when μ_0 and σ_0 are unknown and need to be estimated, or when x follows non-normal distributions.

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Biographies

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