

# An Application of EWMA Control Chart for Monitoring Packaging Defects in Food Industry

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## Abstract

This paper puts forward the concept of optimizing the Exponentially Weighted Moving Average (EWMA) control chart for effective monitoring of the increasing shifts in defective rate in food packaging. Optimizing the EWMA charts mainly aims at enhancing the product quality and the customers' contentment, along with reducing both production costs and waste. The Average Number of Defectives (AND) is used as an overall performance metric for guaranteeing the highest detection power of the optimal EWMA chart with respect to various sizes of shifts. Records of the packaging defects were collected from a food company in order to illustrate the implementation of the proposed optimal EWMA chart. The optimal EWMA chart is compared with the optimal NP chart and the results of this comparative study reveal that the former exceeds the latter by 69% in terms of AND.

## Keywords

Control chart, Packaging, Average number of defectives, EWMA chart, NP chart.

## 1. Introduction

Quality Control (QC) commenced with the arrival of the manufacturing era in the industrial revolution, seeking to detect and resolve issues that occur during the manufacturing phase through inspection of units and setting the corresponding specifications. QC helps in deciding whether a corrective plan needs to be implemented in a certain manufacturing stage in order to enhance the quality of products and, consequently increasing the customer satisfaction. As QC depends on statistical theories, Dr. W. Shewhart expanded the application of such theories and tools in order to apply them in quality management. The word QC was first adopted by the food industry (Herschdoerfer, 1967), as packaged food undergoes multiple phases until it reaches the consumer, therefore calling for the need of utilizing such tools to improve the process (Pereira & Aspinwall, 1991). The crucial sections of food quality control emerged first back in 2500 BC, since food contamination was a huge problem for the law in Egypt (Edith & Ochubiojo, 2012). Loader & Hobbs (1999) & Strugnell (1992) underlined that as the rejection rate of products increase, the reputation and public confidence in the company would decrease along with the direct financial losses. Therefore, Statistical Process Control (SPC) is being utilized in food industry in order to reduce the variability by detecting defects in the early stages of production.

Optimization models evolved throughout the years to reach the highest detection rate for control charts. For instance, Wu et al. (2002) suggested an algorithm for X-bar&S-bar control charts for reaching to the most effective detection power and to link the capability index of processes with the in-control and out-of-control situations. Haridy et al. (2007) put forward the Exponentially Weighted Moving Average (EWMA) chart with curtailment, which was compared to the normal EWMA control chart in terms of average numbers of defects (AND) and it was concluded that curtailed EWMA chart performs better than the EWMA by 41%, under different situations. Shamsuzzam et al. (2018) concluded that the multiattribute EWMA chart is more effective than the multiattribute NP chart by 65% in respect of AND. EWMA chart is among the most effective control charts in minimizing the variation and enhancing the quality of manufacturing systems. It is the substitute of the traditional Shewhart control chart (Roberts 2000) as it can be used to monitor small and medium shifts effectively. EWMA has a weighing factor,  $\lambda$ , ranging from 0 to 1. As  $\lambda$  gets closer to 1, it shows that more weight of importance is given to recent data, while  $\lambda$  that is closer to 0 shows that more weight is given to historical data and therefore, it is suitable for detecting smaller shifts (Montgomery, 2013). The EWMA test statistic is:

$$z_i = \lambda x_i + (1 - \lambda)z_{i-1} \quad (1)$$

The first value of the monitoring statistic  $z$  ( $i=1$ ) is set to be the same as the process mean, hence the chart is constructed by plotting  $z_i$  against  $i$ , the sample number. On the contrary, the NP chart is powerful when monitoring the number for the defective items taken from a process during a specific duration. The only parameter for the NP chart is the Upper Control Limit (UCL), which is set to monitor the position of points and it will signal once a sample point exceeds that specified limit, indicating that the behavior is out-of-control. The NP parameter is shown below, where  $p$  is the percentage of non-conforming and  $n$  is the sample size.

$$UCL = np + 3\sqrt{np(1-p)} \quad (2)$$

In order to know how fast the out-of-control is being detected, the Average Time to Signal (ATS) is utilized, which is defined as the average time needed to detect an out-of-control situation once it takes place. The smaller the ATS, the more effective a chart is in terms of detecting a problem, therefore making it easier to decide when  $p$  shift takes place, which makes the control chart more powerful in identifying the source of the problem.

This article focuses on assessing the company's data to monitor the packaging defects using a control chart. Moreover, the optimized parameters for the control charts are allocated to ensure reaching the highest overall detection effectiveness while maintaining the false alarm rate at a specific level.

The sequence of the article is as follows. Section 2 illustrates the optimal design of EWMA control chart. In section 3, the case study is shown and the optimal EWMA chart is compared against the optimal NP chart for detecting the increasing defective rate. Lastly, the remarks are set in the end section of this paper.

## 2. Optimal design of EWMA control chart

The optimal model is utilized to enhance the power of the control chart when detecting a range of different sizes of shifts. This is actually done by determining the optimal parameters of the EWMA chart through C program. The specifications needed to construct the optimal design of the EWMA control chart are:

- (1) The least allowed value  $\tau$  of the average time to signal ( $ATS_0$ ) that is set as in-control. The value is determined depending on the tolerated rate of false alarm.
- (2) The in-control fraction of the fraction nonconforming ( $p_0$ ).
- (3) The highest value of shift ( $\delta_{max}$ ) which the user desires to detect during the process.
- (4) The sample size ( $n$ ) that is set based on the considered accessible resources of the user.

The optimal model of both the np and EWMA charts can be formulated as follows:

Objective function: Minimizing AND  
 Constraints:  $ATS_0 \approx \tau$   
 Variables:  $H, \lambda$  for the EWMA chart  
                   UCL for the NP chart

where UCL and  $H$  are the upper control limits for the NP and EWMA control charts, respectively.

The AND is a beneficial tool in calculating the overall performance of a control chart, because it measures the average number of defectives once a shift happens up to the point where a control chart sends a signal across a broad

range of shifts. Hence, smaller values of AND shows less average number of defective products; and consequently better. AND can be expressed as:

$$AND = N \int_1^{\delta_{max}} \delta \times p_0 \times ATS(\delta) \times f_{\delta}(\delta) d\delta \quad (3)$$

where:

$N$ : the number of items manufactured during a specific duration. It can be removed from Equation (3) as it will not affect the comparative study.

$ATS(\delta)$ : the out-of-control  $ATS$  generated from control chart at  $\delta$ .

The stochastic shift  $\delta$  in  $p$  is said to be uniformly distribution (Castagliola et al. 2011, Haridy et al. 2013) Therefore, the probability density function of the shift  $f_{\delta}(\delta)$  is equivalent to:

$$f_{\delta}(\delta) = \frac{1}{\delta_{max} - 1} \quad (4)$$

### 3. Case Study

In this paper, we assess the company's data that was collected, in order to find the utmost type of defects in the food packaging. We collected 25 samples each of 100 food packaging over an interval of 1 hour. The case study will be conducted by utilizing pareto chart and control charts. Additionally, the optimal design of control charts are used to ensure getting the maximum detection effectiveness, while reducing the AND. The main purpose of this paper is to compare the optimal EWMA chart with the traditional EWMA chart and the NP chart, while monitoring the increasing shifts (as they declare that process is worsening and producing more defective rate). Figure 1 demonstrates the methodology of the case study:

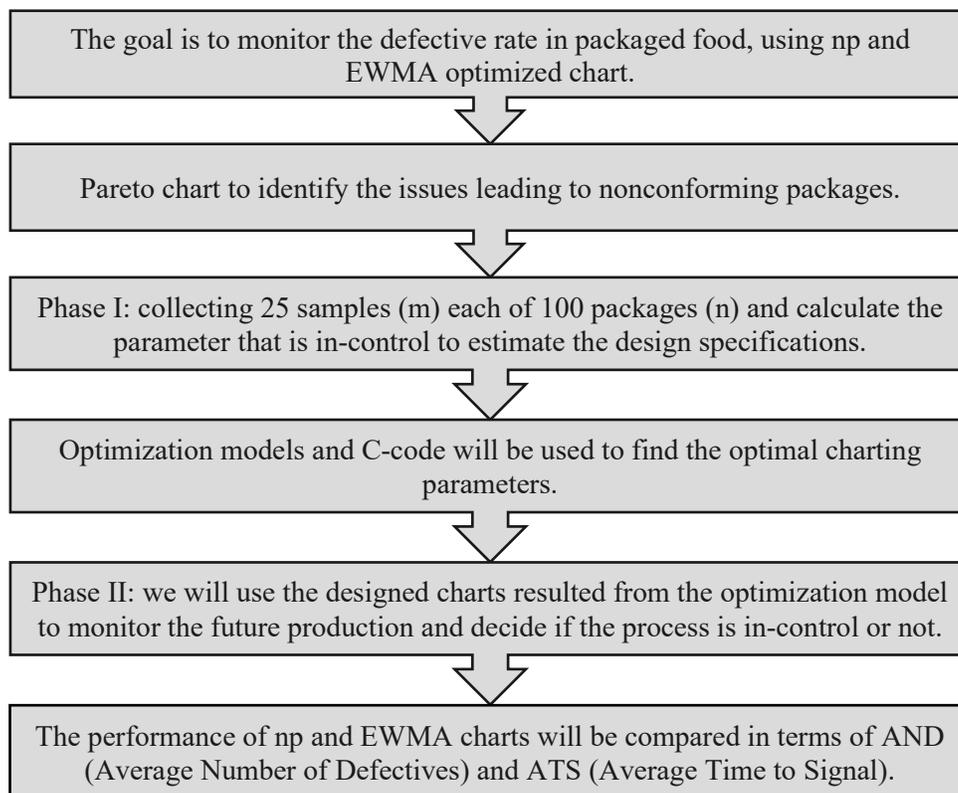


Figure 1: Methodology of the case study

We have formulated the Pareto chart based on the data collected from the food company, this was done in order to find the ruling packaging defect, as shown in Figure 2, which states that the date printing error was the major contributing element of the quality issues for the food packaging.

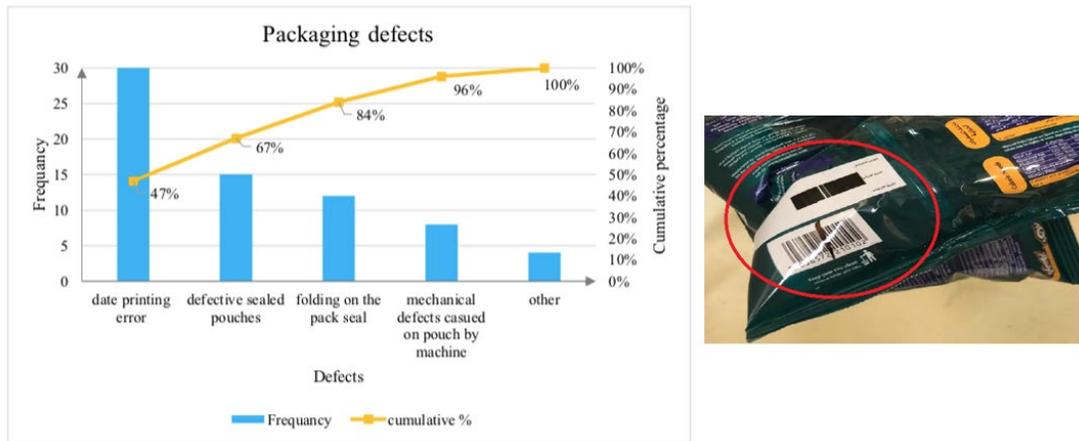


Figure 2: Pareto chart of food packaging defect

The data were based on the collection of 25 samples each of 100 packages, where the number of nonconforming items was counted in every sample, for an interval of one hour. Therefore, based on the available data we estimated the  $p_i$ , which was found to be 0.0272, for the 25 samples using the corresponding equation:

$$p_i = \frac{d_i}{n} \quad (5)$$

In phase I, the NP chart is constructed through the use of Minitab to analyze if the process is in-control, as shown in Figure 3, and the corresponding UCL and Central Line (CL) were calculated as follows:

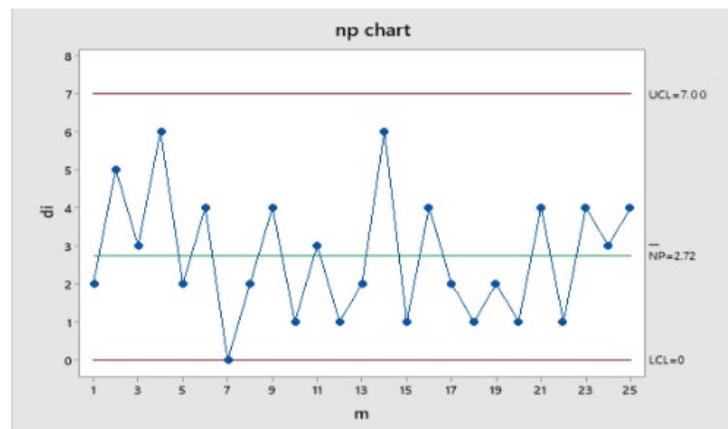


Figure 3: NP chart for initial phase

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1 - \bar{p})} = 7 \quad (6)$$

$$CL = n\bar{p} = 2.72 \quad (7)$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1 - \bar{p})} = 0 \quad (8)$$

It is concluded that all the points are falling within the control limits, therefore the NP chart is in-control. As a consequence, we can estimate the in-control  $p_0$  for phase II, as shown in the following equation:

$$p_0 = \frac{\sum_{i=1}^m p_i}{m} = \frac{\sum_{i=1}^m d_i}{m \times n} = \frac{68}{25 \times 100} = 0.0272 \quad (9)$$

During phase II, the optimal NP chart and EWMA chart ( $\lambda=0.1$ ) are constructed using an optimization algorithm. The input parameters used for the model are shown below:

- The fraction nonconforming ( $p_0=0.027$ )
- The allowable value of  $\tau$  based on false alarm that the company can handle ( $\tau = 500$ )
- The sample size used based on the available resources of company ( $n = 100$ )
- The maximum shift the user wishes to detect ( $\delta_{\max} = 5$ )

For our comparative study, we utilize the previously motioned specifications ( $\tau = 500$ ,  $n = 100$ ,  $p_0 = 0.027$  and  $\delta_{\max} = 5$ ) to design NP, EWMA chart with  $\lambda=0.1$ , and optimal EWMA charts. Table 1 summarizes the performance and the charting parameters of the three control charts, showing a clear advantage for the optimal EWMA as it has the lowest value (AND = 0.130), followed by traditional EWMA chart with  $\lambda=0.1$  (AND = 0.146), which is 11.67% higher than the optimal chart. Finally, the NP chart has the least performance, as it showed a 69.53% increase in the AND compared to the optimal EWMA.

Table 1: Charting parameters with AND for EWMA ( $\lambda=0.1$ ), optimal EWMA and NP charts

<b>The charting parameters and the AND of the EWMA (<math>\lambda=0.1</math>), optimal EWMA and np charts</b>				
	H	UCL	$\lambda$	AND
<b>EWMA (<math>\lambda=0.1</math>) chart</b>	1.092	—	0.1	0.14559
<b>Optimal EWMA chart</b>	2.584	—	0.365	0.13037
<b>np chart</b>	—	8	—	0.21971

We have monitored the performance of the control charts over a shift range ( $1 < \delta \leq 5$ ) as shown in Table 2. The three charts were also compared in terms of the normalized ATS ( $ATS/ATS_{\text{optimal EWMA}}$ ) values as shown in Figure 4. It is clear how the optimal EWMA outperforms the traditional EWMA and NP charts over most of the shift points.

Table 2: *ATS* values of EWMA ( $\lambda=0.1$ ), optimal EWMA and NP charts

Shift ( $\delta$ )	ATS		
	np Chart	EWMA ( $\lambda=0.1$ ) Chart	Optimal EWMA Chart
1	628.3503	499.29661	499.74725
1.5	48.53212	11.65251	18.18984
2	10.43086	4.37790	4.57755
2.5	3.78563	2.63409	2.28028
3	1.87007	1.87099	1.45989
3.5	1.13665	1.44699	1.05373
4	0.80761	1.17513	0.81981
4.5	0.64819	0.5	0.5
5	0.56933	0.5	0.5

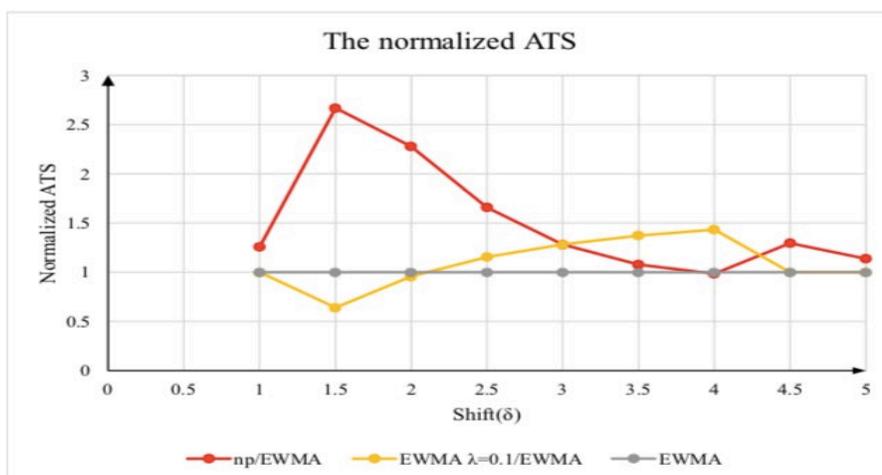


Figure 4: Normalized curves of *ATS*

Consequently, the above observations indicate that the NP with UCL (=8) is not effective, since  $ATS_0$  is 628.35, which is greater than the defined  $\tau$  (=500). On the other hand, the optimal EWMA chart showed  $ATS$  (=499.75), which is less than NP and much closer to  $\tau$  (=500), therefore it shows to be more effective over a wider range of shifts. Additionally, the optimal EWMA chart outperforms the traditional EWMA ( $\lambda=0.1$ ) for detecting most of the shift range. The optimal EWMA clearly exceeds the NP chart when detecting small to moderate  $\delta$  in p, and it also performs better than the traditional EWMA chart ( $\lambda = 0.1$ ) chart over most of the entire domain of shifts. To further illustrate the application of the NP and EWMA charts, 20 out-of-control data were used where the shift of 2 took place in p (i.e.,  $p = 2p_0 = 2 \times 0.027 \approx 0.03$ ). The out-of-control data were monitored using NP, EWMA ( $\lambda=0.1$ ) and optimal EWMA as demonstrated in Figures 5, 6 and 7, respectively. The optimal EWMA chart is found to be most effectiveness, as it exceeded the H (=2.584) preceding only 2 samples from when the shift took place (28<sup>th</sup> sample). This concludes that by adopting the optimal EWMA chart, the company will be able to detect shifts in p earlier and therefore reduce the waste and number of defectives produced.

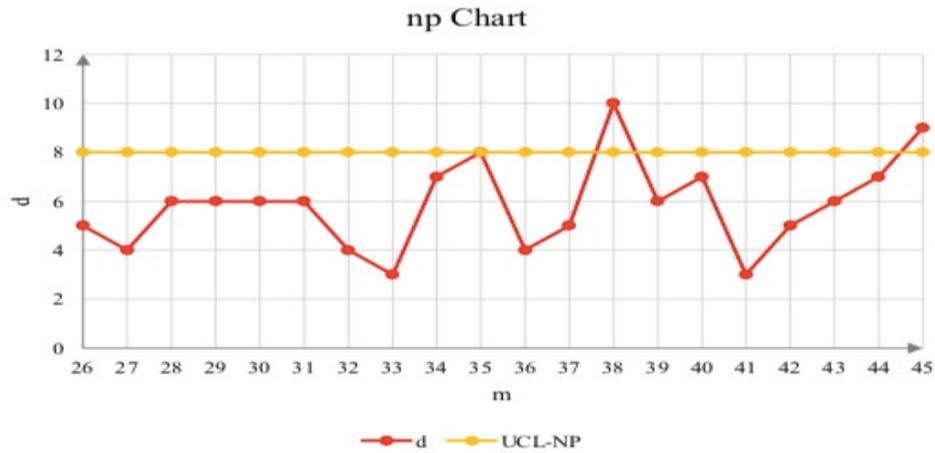


Figure 5: NP chart of phase II

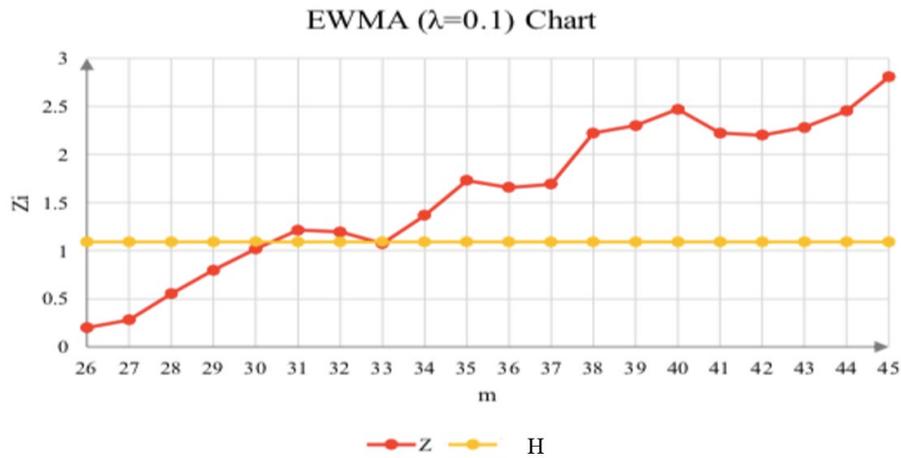


Figure 6: EWMA (λ=0.1) of phase II

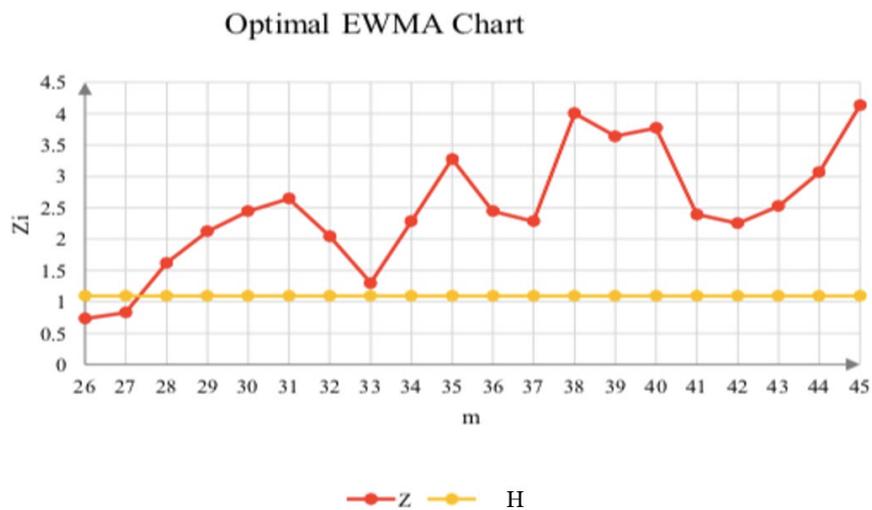


Figure 7: Optimal EWMA phase II

#### 4. Conclusion

To conclude, this paper evaluates the effectiveness of the overall performance of the optimal EWMA chart for monitoring the packaging defects in food industry. It demonstrates its high overall performance in comparison to the optimal NP and traditional EWMA charts. The optimal EWMA chart shows the highest overall performance in terms of AND. Using out-of-control data, the EWMA chart gives a signal and detects the increasing shift in packaging defects faster than the NP and traditional EWMA charts. This paper underlines the advantage of utilizing the optimization in control charts, which results in better overall effectiveness of the control charts and helps companies reduce waste and enhance the quality of the end products.

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