Application of Economic Mathematical Model in Production of Gold Sub-Sector in Indonesia

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Abstract

Gold is a metal that has a high purchasing power and Indonesia is one of the world's gold producing countries. Gold is a non-renewable natural resource, the formation process takes a long time and its availability can be used up. Thus the need for managing the availability of gold in Indonesia to prevent scarcity and can be one of the considerations in making decisions to control the price of gold in the market. The application of economic mathematical models that can be applied in the estimation of gold production in Indonesia is a sigmoid function. The sigmoid function that can be used to estimate the cumulative production of gold in Indonesia is the logistical function and the Gompertz function. Based on the research logistic model is the most appropriate model used in the prediction of cumulative production in Indonesia. The logistic model was chosen because it fulfills the two best model requirements namely, based on the highest R^2 value and the lowest $RMSE$. Logistic model shows that the cumulative production of gold in Indonesia can increase to 107895,148 Oz after the year 2040.

Keywords: Gold metal, gold price, economic mathematical models, sigmoid function, Gompertz function, logistic model.

1. Introduction

One of the chemical elements in the periodic table, namely gold, is symbolized by Au which in Latin aurum (Kryazhov, 2014). Gold is a metal that has high purchasing power and Indonesia is one of the gold producing countries in the world. The largest gold mine in the world is located in Indonesia, namely the Grasberg gold mine in Papua. Quoted from the Freeport-McMoRan third quarter financial report, gold production from Papua produced 1,232,000 Oz. The Grasberg gold mine is one of the major tax contributors in Indonesia.

In Indonesia, gold is widely used in the form of jewelry, although there is a tendency for changes in consumption patterns to investment in the form of gold bars and gold coins as purchasing power has been eroded due to the soaring price of gold in recent years. The price which increases day by day does not make people's interest decrease; on the contrary, the producers are getting more active in fulfilling market demand. As is well known, gold is a non-renewable natural resource, its formation process takes a long time and its availability can run out. Thus, it is necessary to manage the availability of gold in Indonesia in order to prevent scarcity and can be one of the considerations in making decisions to control the price of gold in the market.

Mathematics takes part in predicting future gold production. The application of economic mathematical models that can be applied in estimating gold production in Indonesia is the sigmoid function. The sigmoid functions include: logistics, Gompertz, Gaussian, Probit, and Hill. The use of functions on a data must give different values. However, it can be concluded which function model is appropriate to be applied to data (Lima et al., 2002; Achdou et al., 2016). Research by Appiah et al. (2018), conducted a comparative study of mathematical models to estimate gold production in Ghana. According to Appiah, gold is the largest export commodity in Ghana and contributes 48 percent of the country's income. The study estimated cumulative gold production in Ghana using the logistic, Gompertz, Gaussian, Probit and Hill functions. The conclusion from this study is that the Gompertz function is the best function in estimating gold production in Ghana.

Rodríguez (2020), conducted a study on the complex pattern of gold production in Canada, which is modeled and short-term forecasting is carried out using the Basic Mineral Production Equation (FEMP). For a time span with several variations in the ratio of production to reserves, the equation updates the reserves and the Ratio of Production to Reserves (PRR) based on the unit of time. The PRR values are linear, gradually adjusted according to the increasing and decreasing sub-cycle of historical data from the overall production. The Hubbert and FEMP models are introduced as case studies of the Fundamental Equations, which are presented as a function of the square product of the cumulative production and the rational function of PRR with time. Canada's current forecast for gold production shows that it is reaching its all-time high in the near term, unless there are new discoveries or influential global economic factors, this allows for an increase in the slope of the linear function provided by the PRR of the FEMP-based model.

Based on the description of the problem and the literature study above, in this paper, an economic mathematical model is applied to the gold production sub-sector in Indonesia. The aim is to find a suitable model for estimating gold production in Indonesia. In this study, the functions used are the logistical function and the Gompertz function to estimate the cumulative gold production in Indonesia. These two functions are used because the research data will form a growth curve to see how gold production is going to be in the future.

2. Research Methods

2.1 Linear Model

The term linear can be interpreted in two different ways, namely as follows (Gujarati, 2004):

a. Linearity in Variables

The first meaning of linearity is the conditional expectation of y is a linear function of X_i, for example:

$$E(Y|X_i) = \beta_1 + \beta_2 X_i \text{ , with, } i = 1, 2, 3, \dots, n$$

A function $y = f(X)$ is said to be linear in the variable X, if X to the power of one.

b. Linearity in Parameters

The second meaning of linearity is the conditional expectation of y, $E(y|X_i)$ is a linear function of its parameters, parameter of β it could be linear or it could be nonlinear for a variable X. In this case, the linear examples in parameters are:

$$E(y|X_i) = \beta_1 + \beta_2 X_i^2,$$

while for the following equation:

$$E(Y|X_i) = \beta_1 + \beta_2^{\beta_3} X_i^2,$$

is not a linear function in the due parameter β_2 not rank one (Sukono et al., 2019.a; 2019.b; 2019.c).

The nonlinear model is a function that connects the dependent variable Y with independent variables X which is not constant for every change in value of X (Suharyadi & Purwanto, 2009). Nonlinear regression is a method for obtaining a nonlinear model which states the dependent and independent variables (Yanti et al., 2014). Least square is applied to the nonlinear model by performing a procedure or algorithm that can ensure that the estimate actually meets the criteria of the objective function, namely giving the sum of square error at the minimum point.

Determining the optimum point which is believed to be the solution in determining the estimation of the nonlinear model can use the first and second derivative test operations. The first derivative test is used in several iteration procedures, namely the Gauss-Newton and Marquardt-Levenberg iterations, while the second derivative test is the Newton-Raphson and Quadratic-Hill-Climbing iterations. From several iteration procedures, this study only uses one of several forms of iteration procedures, which is applied in determining the Marquardt-Levenberg iteration method (Sukono et al., 2019.c).

2.2 Sigmoid function

A sigmoid function is a mathematical function that has an "S" curve or sigmoid curve. The Sigmoid function has the domain of all real numbers. This study uses the logistic function and the Gompertz function. The logistic function, also known as the Verhults model, was introduced by Pierre Francois Verhults in 1838. Generally, the logistic function is used in modeling population growth. The form of the logistic function equation is as follows (Appiah et al., 2018):

$$p(t_i) = \frac{k}{1+e^{-a(t_i-r)}} \,. \tag{1}$$

The Gompertz function was introduced by Benjamin Gompertz. The Gompertz model is a type of mathematical model for a time series. Initially the Gompertz function was used to model human mortality. The following is the equation for the Gompertz function (Appiah et al., 2018):

$$p(t_i) = k e^{-e^{-a(t_i-r)}}, \tag{2}$$

with

$p(t_i)$: production in years-t_i,
k : maximum cumulative production,
a : growth rate,
r : inflection point.

2.3 Nonlinier Least Square

The parameter estimation in the least square in the nonlinear model is determined by performing a procedure or algorithm that can ensure that the estimator actually meets the criteria of the objective function, namely giving the squared number of errors at the minimum value or giving the maximum value to the likelihood function. In some nonlinear problems, the method that has been successful is to write down the normal equation in detail and develop an iterative technique to solve it. Among them are: 1) Gauss Newton method (linearization method), 2) Stepest Descent method (steepest derivative), 3) Marquadrt-Levenberg.

2.4 Marquardt-Levenberg Method

Marquadrt-Levenberg is one of the methods in nonlinear estimation (Ranganatha, 2004). Marquadrt-Levenberg was developed by Levenberg (1944) and Donald Marquardt (1963). The Marquadrt-Levenberg method applies the iteration method as well as the Gauss Newton method, which is to minimize the number of squared errors, the only difference lies in the addition of scalar multiplication. λ and the identity matrix I. Marquardt-Levenberg using the Taylor series order 1, iterative step β determined by modifying (Saleh, 2010; Sukono et al., 2019.d):

$$\beta^{(n+1)} = \beta^{(n)} + \left(P^{(n)T} P^{(n)}\right)^{-1} P^{(n)T}\left(y - f^{(n)}\right),$$

become:

$$\beta^{(n+1)} = \beta^{(n)} + \left(P^{(n)T} P^{(n)} + \lambda I_k\right)^{-1} P^{(n)T}\left(y - f^{(n)}\right), \tag{3}$$

with:

$$y = f(X, \beta)$$

$$P^{(0)} = \frac{\partial f(X,\beta)}{\partial \beta^T} = \begin{bmatrix} p_{11}^0 & p_{21}^0 & \cdots & p_{p1}^0 \\ p_{12}^0 & p_{22}^0 & \cdots & p_{p2}^0 \\ \vdots & \vdots & \ddots & \vdots \\ p_{1u}^0 & p_{2u}^0 & \cdots & p_{pu}^0 \\ \vdots & \vdots & \ddots & \vdots \\ p_{1n}^0 & p_{2n}^0 & \cdots & p_{pn}^0 \end{bmatrix} = \{p_{iu}^0, k \times m\},$$

k : many parameters are estimated,
m : the number of data,
I : ordered identity matrix k x k, and
λ : scalar multiplication $0 < \lambda < 1$.

Iteration will stop when value β convergent, namely:

$$\beta^{(n+1)} = \beta^{(n)}.$$

Then calculate how much the sum of the squares of the residuals between the actual data and the predicted data using the formula:

$$RSS = \sum_{i=1}^{n} (y_i - f_i)^2, \tag{4}$$

y_i : value of gold production in years- i, and
f_i : predictive value of gold production in years –i.

2.5 Akaike Information Criterion (AIC)

Akaike (1973) introduced an information criterion called the Akaike Information Criterion. The AIC estimates the quality of each model, relative to each of the other models. Thus, AIC provides a means for model selection (Snipes et al., 2014). The AIC estimates the relative amount of information lost by a given model. As it is known that the less information is lost, the higher the quality of the model. The following is the AIC equation (Gujarati, 2004):

$$\ln AIC = \left(\frac{2k}{n}\right) - \ln\left(\frac{RSS}{n}\right), \tag{5}$$

with
k : many parameters are estimated in the statistical model,
RSS : Residual Sum of Squares or the sum of squares of the residual of the estimation model, and
n : number of data.

2.6 Root Mean Square Error (RMSE)

Root Mean Square Error, abbreviated as RMSE, is used to compare the estimation methods used, namely to determine the most accurate estimation method (Widyayati, 2009). RMSE is the average value of the squared error.

The accuracy of the measurement error estimation method is indicated by the presence of a small RMSE. RMSE is formulated as follows (Gaynor & Kirkpatrick, 1994):

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\hat{Y}_i - Y_i\right)^2} = \sqrt{\frac{1}{n}RSS}\ , \tag{6}$$

with
$\hat{Y}_i$: prediction result data,
Y_i: actual data, and
n : number of data.

2.7 Coefficient of Determination (R^2)

The coefficient of determination is symbolized by (R^2) is a tool to measure how far the model's ability to explain the variance of the dependent variable (Ghozali, 2012). The value of the coefficient of determination is between 0 and 1. If the coefficient of determination is close to 1, it means that the influence of the independent variable on the dependent is getting stronger, and vice versa, if the coefficient of determination is close to 0, the effect of the independent variable on the dependent variable is getting weaker (Saunders et al., 2012; Sukono et al., 2014). Referring to Gujarati (2004) R^2 formulated as follows:

$$R^2 = 1 - \frac{RSS}{\sum_{i=1}^{n}(Y_i - \bar{Y}_i)^2}\ , \tag{7}$$

with
Y_i : the number of actual production,
$\bar{Y}_i$: total average production, and
n : number of data.

3. Results and Discussion

3.1 Research data

The data used in this study is secondary data in the form of data on the amount of gold production in Indonesia from 1996 to 2017. The data used are obtained from the Central Bureau of Statistics website.

3.2 Determining the Logistic Model

It is known that the logistic function based on equation (1) is $p(t_i) = \frac{k}{1+e^{-a(t_i-r)}}$. The function can also be written as: $f(X_i,\beta) = \frac{\beta_0}{1+e^{-\beta_1(X_i-\beta_2)}}$. Least square is used to minimize residuals by first estimating the parameters in the Marquadrt-Levenberg iteration model. The partial derivative of the function with respect to its parameters is:

$$\frac{\partial f(X_i,\beta)}{\partial \beta_0} = \frac{1}{1+e^{-\beta_1(X_i-\beta_2)}},\ \frac{\partial f(X_i,\beta)}{\partial \beta_1} = -\frac{\beta_0(-X_i+\beta_2)e^{-\beta_1(X_i-\beta_2)}}{\left(1+e^{-\beta_1(X_i-\beta_2)}\right)^2},\ \frac{\partial f(X_i,\beta)}{\partial \beta_2} = -\frac{\beta_0\beta_1 e^{-\beta_1(X_i-\beta_2)}}{\left(1+e^{-\beta_1(X_i-\beta_2)}\right)^2},$$

then estimate the parameters using the Marquadrt-Levenberg method. For example:

$$\frac{\partial f(X_i,\beta)}{\partial \beta_0} = u_i,\ \frac{\partial f(X_i,\beta)}{\partial \beta_1} = v_i,\ \frac{\partial f(X_i,\beta)}{\partial \beta_2} = w_i.$$

Then enter the initial guess value $\beta_0 = 20000$ is selected as the minimum value, $\beta_1 = 0$ selected as value without giving effect, and $\beta_2 = 1990$ selected as the year under the data, then it is obtained:

$$P^{(0)} = \begin{bmatrix} u_1 & v_1 & w_1 \\ u_2 & v_2 & w_2 \\ u_3 & v_3 & w_3 \\ \vdots & \vdots & \vdots \\ u_{21} & v_{21} & w_{21} \end{bmatrix} = \begin{bmatrix} 0.5 & 30000 & 0 \\ 0.5 & 35000 & 0 \\ 0.5 & 40000 & 0 \\ \vdots & \vdots & \vdots \\ 0.5 & 130000 & 0 \end{bmatrix}.$$

The next step is to predict gold production in Indonesia using the Logistics function using equation (1). Enter the initial guess value $\beta_0 = 20000$, $\beta_1 = 0$, and $\beta_2 = 1990$, obtained:

$$f\left(X_i,\beta^{(0)}\right)=f_i^{(0)}=\begin{bmatrix}10000\\10000\\10000\\\vdots\\10000\end{bmatrix} \text{ and residual value } (Y_i-f_i^{(0)}),\ \text{ obtained: } Y_i-f_i^{(0)}=\begin{bmatrix}20000\\25000\\30000\\\vdots\\120000\end{bmatrix},$$

with λ=0.00001, obtained:

$$\left(P^{(0)T}P^{(0)}+\lambda I_3\right)^{-1}P^{(0)T}\left(y-f^{(0)}\right)=\begin{bmatrix}252760.668\\-0.376435056\\0\end{bmatrix},$$

so the parameter estimate β the logistic model in the 0th iteration using equation (3), namely:

$$\beta^{(1)}=\begin{bmatrix}\beta_0^{(1)}\\\beta_1^{(1)}\\\beta_2^{(1)}\end{bmatrix}=\begin{bmatrix}20000\\0\\1990\end{bmatrix}+\begin{bmatrix}252760.668\\-0.376435056\\0\end{bmatrix}=\begin{bmatrix}272760.668\\-0.376435056\\0\end{bmatrix}.$$

From the above calculations, the value of the parameter is obtained $\beta_0^{(1)}=272760.668$, $\beta_1^{(1)}=-0.376435056$, and $\beta_2^{(1)}=0$. Because $\begin{bmatrix}\beta_0^{(0)}\\\beta_1^{(0)}\\\beta_2^{(0)}\end{bmatrix}\neq\begin{bmatrix}\beta_0^{(1)}\\\beta_1^{(1)}\\\beta_2^{(1)}\end{bmatrix}$ where $\begin{bmatrix}20000\\0\\1990\end{bmatrix}\neq\begin{bmatrix}272760.668\\-0.376435056\\0\end{bmatrix}$ then the estimated value of parameter of β not yet converging. So it has to be iterated back up to the value β convergent. The iteration calculation uses Statistical Product and Service Solutions 20 (SPSS) software. The estimation results are shown in Table 1.

Table 1. Estimation Results of Logistic Model Using Parameters
Statistical Product and Service Solutions 20 (SPSS).

Parameter Estimates

Parameter	Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
β_0	107929.513	7408.039	92365.801	123493.224
β_1	1.486	4.289	-7.524	10.496
β_2	1995.261	2.616	1989.765	2000.758

From Table 1, it is obtained $\beta_0=107929.382$, $\beta_1=1.486$ and $\beta_2=1995.261\approx1995$ and sequential errors i.e. $\beta_0=740.969$, $\beta_1=4.290$, and $\beta_2=2.616$.
Then the Logistics model of gold production in Indonesia is obtained, namely:

$$p(t_i)=\frac{107929.513}{1+e^{-1.486(t_i-1995.261)}}\ . \tag{8}$$

Logistics Model Using Statistical Product and Service Solutions 20 (SPSS).

Table 2. Result of Calculation of Residual Sum of Squares (RSS) and Coefficient of Determination (R^2) Logistics Model Using Statistical Product and Service Solutions 20 (SPSS).

ANOVA[a]

Source	Sum of Squares	df	Mean Squares
Regression	237443689179.485	3	79147896393.162
Residual	17540471813.515	18	974470656.306
Uncorrected Total	254984160993.000	21	
Corrected Total	18243321440.571	20	

Dependent variable: y
a. R squared = 1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = .039.

From Table 2, RSS based on equation (4) is obtained at 17540471813.515.

Furthermore, calculating the AIC Logistics model using equation (5), obtained: $\ln AIC = -20.2575$, then

$$AIC = 1.59323\text{E-}09.$$

Using equation (5), the AIC value of the logistic model is 1.59323E-09. This means that the Logistics model can be said to not lose much information. Next calculate the RMSE Logistics Model. It is known that the RSS of the logistic model is 17540471813.515. So with equation (6), the RMSE is 28900.87. then calculate $\boldsymbol{R^2}$ Using equation (7) with the help of Statistical Product and Service Solutions 20 (SPSS) software, the coefficient of determination is obtained as shown in Table 2.Based on Table 2, it is obtained R^2 the logistics model of 3.9%.

Using Microsoft Excel 2020, with equation (8) a prediction of Gold Production in Indonesia is obtained using the Logistics Model given in Figure 1.

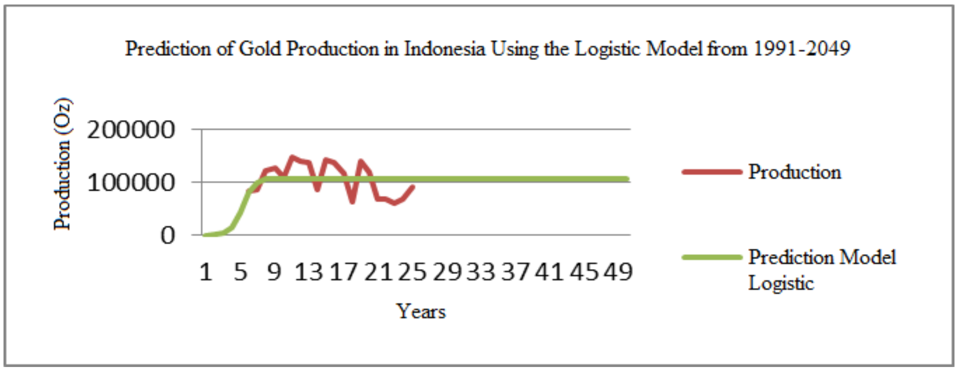

Figure 1. Graph of Prediction of Gold Production in Indonesia Logistic Model

Figure 1, shows that with the Logistics model it is estimated that the cumulative gold production is 107929.513 Oz, with a growth rate of 148.6%, with an inflection or turning point in 1995 with a production value of 43628.988 Oz. The level of information loss in the logistics model is 1.59323E-09, which means that the quality of this model is good. The RMSE is 28900.87, and the coefficient of determination of the logical model is 0.39.

3.3 Determining the Gompertz Model

It is known that the Gompertz function is $p(t_i) = ke^{-e^{-a(t_i-r)}}$ based on equation (2). The function can also be written as $f(X_i,\beta) = \beta_0 e^{-e^{-\beta_1(X_i-\beta_2)}}$. Least square is used to minimize residuals by first estimating the parameters in the Marquadrt-Levenberg iteration model. The partial derivative of the function with respect to its parameters is:

$$\frac{\partial f(X_i,\beta)}{\partial \beta_0} = e^{-e^{-\beta_1(X_i-\beta_2)}}, \frac{\partial f(X_i,\beta)}{\partial \beta_1} = -\beta_0(-X_i+\beta_2)e^{-\beta_1(X_i-\beta_2)}e^{-e^{-\beta_1(X_i-\beta)}}, \frac{\partial f(X_i,\beta)}{\partial \beta_2} = -\beta_0\beta_1 e^{-\beta_1(X_i-\beta_2)}e^{-e^{-\beta_1(X_i-\beta_2)}}.$$

Then estimate the parameters with the Marquadrt-Levenberg method. For example:

$$\frac{\partial f(X_i,\beta)}{\partial \beta_0} = g_i, \frac{\partial f(X_i,\beta)}{\partial \beta_1} = h_i, \frac{\partial f(X_i,\beta)}{\partial \beta_2} = j_i.$$

Then enter the initial guess value $\beta_0 = 20000$ is selected as the minimum value, $\beta_1 = 0$ selected as value without giving effect, and $\beta_2 = 1990$ selected as the year under the data, then it is obtained:

$$P^{(0)} = \begin{bmatrix} g_1 & h_1 & j_1 \\ g_2 & h_2 & j_2 \\ g_3 & h_3 & j_3 \\ \vdots & \vdots & \vdots \\ g_{21} & h_{21} & j_{21} \end{bmatrix} = \begin{bmatrix} 0.367894 & 44145.5 & 0 \\ 0.367894 & 51503.1 & 0 \\ 0.367894 & 58860.7 & 0 \\ \vdots & \vdots & \vdots \\ 0.367894 & 198655 & 0 \end{bmatrix}.$$

The next step is to predict gold production in Indonesia using the Gompertz function using equation (5). when the initial guess value was entered $\beta_0 = 20000$ is selected as the minimum value, $\beta_1 = 0$ selected as value without giving effect, and $\beta_2 = 1990$ selected as the year under the data obtained:

$$f(X_i, \beta^{(0)}) = f_i^{(0)} = \begin{bmatrix} 7357.59 \\ 7357.59 \\ 7357.59 \\ \vdots \\ 7357.59 \end{bmatrix} \text{ and residual value } (Y_i - f_i^{(0)}) \text{, obtained: } Y_i - f_i^{(0)} = \begin{bmatrix} 76206.4 \\ 79570.4 \\ 116504 \\ \vdots \\ 93156.4 \end{bmatrix},$$

whit λ=0.00001, obtained:

$$\left(P^{(0)T}P^{(0)} + \lambda I_3\right)^{-1} P^{(0)T}\left(y - f^{(0)}\right) = \begin{bmatrix} 350715.641 \\ -0.255801794 \\ 1990 \end{bmatrix},$$

so the parameter estimate β Gompertz model in the 0th iteration using equation (3), namely:

$$\beta^{(1)} = \begin{bmatrix} \beta_0^{(1)} \\ \beta_1^{(1)} \\ \beta_2^{(1)} \end{bmatrix} = \begin{bmatrix} 20000 \\ 0 \\ 1990 \end{bmatrix} + \begin{bmatrix} 350715.641 \\ -0.255801794 \\ 1990 \end{bmatrix} = \begin{bmatrix} 370715.641 \\ -0.255801794 \\ 1990 \end{bmatrix}.$$

From the above calculations, the value of the parameter is obtained $\beta_0^{(1)} = 272760.668$, $\beta_1^{(1)} = -0.376435056$, and $\beta_2^{(1)} = 0$. Because $\begin{bmatrix} \beta_0^{(0)} \\ \beta_1^{(0)} \\ \beta_2^{(0)} \end{bmatrix} \neq \begin{bmatrix} \beta_0^{(1)} \\ \beta_1^{(1)} \\ \beta_2^{(1)} \end{bmatrix}$ where $\begin{bmatrix} 20000 \\ 0 \\ 1990 \end{bmatrix} \neq \begin{bmatrix} 370715.641 \\ -0.255801794 \\ 1990 \end{bmatrix}$ then the parameter estimate value of β not yet converging. So it has to be iterated back up to the value of β convergent. The estimation results are shown in Table 3.

Table 3. Estimation Results of the Gompertz Model Parameters
Using Statistical Product and Service Solutions 20 (SPSS).

Parameter Estimates

Parameter	Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
β_0	107895.148	7423.140	92299.711	123490.586
β_1	1.423	4.327	-7.667	10.513
β_2	1995.116	3.053	1988.702	2001.529

From Table 3, it is obtained $\beta_0 = 107895.148$, $\beta_1 = 1.423$, and $\beta_2 = 1995.116 \approx 1995$ and sequential errors i.e. $\beta_0 = 7423.140$, $\beta_1 = 4.327$, and $\beta_2 = 3.053$.
Then the Gompertz Model obtained for gold production in Indonesia is:

$$p(t_i) = 107895.148e^{-e^{-1.423(t_i-1995.116)}}. \tag{9}$$

The value is obtained (RSS) using equation (4) which is shown in Table 4.

Table 4. Calculation Results of Residual Sum of Squares (RSS) and the Coefficient of Determination (R^2)
Gompertz Model Using Statistical Product and Service Solutions 20 (SPSS).

ANOVA[a]

Source	Sum of Squares	df	Mean Squares
Regression	237434123506.477	3	79144707835.492
Residual	17550037486.523	18	975002082.585
Uncorrected Total	254984160993.000	21	
Corrected Total	18243321440.571	20	

Dependent variable: y

a. R squared = 1 - (Residual Sum of Squares) / (Corrected Sum of Squares) = .038.

From Table 4, RSS with equation (4) it is obtained 237434123506.477.

Furthermore, calculating the AIC Gompertz Model using equation (5), obtained:

$$\ln AIC = 20.258, \text{ then } AIC = 1.59244\text{E-}09,$$

by using equation (5), the AIC value of the Gompertz model is 1.59244E-09. This means that the Gompertz model does not lose much information.

Next calculate the RMSE model of the Gompertz. It is known that the RSS of the Gompertz model is 17540471813.515. So with equation (6), the RMSE is 28908.754.

Then calculate $\boldsymbol{R^2}$using equation (7) using Statistical Product and Service Solutions 20 (SPSS) software, the coefficient of determination is obtained as shown in Table 2.Based on Table 2, it is obtained R^2 logistics model of 3.8%.

Using Microsoft Excel 2020, with equation (9) a prediction of Gold Production in Indonesia is obtained using the Logistic Model given in Figure 2.

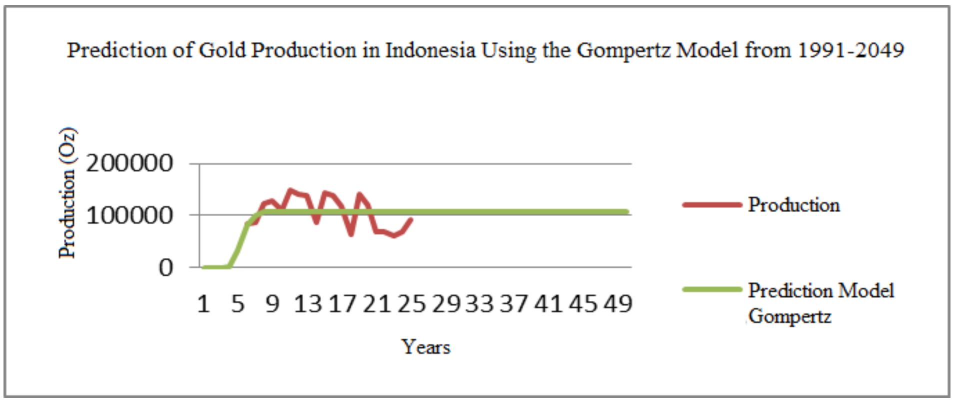

Figure 2. Graph of Gold Production Prediction in Indonesia, Gompertz Model

Figure 2 shows that with the Gompertz model it is estimated that the cumulative gold production is 107895.148 Oz, with a growth rate of 142.3%, with an inflection or turn point in 1995 with a production value of 33171.351 Oz. The level of information loss in the Gompertz model is 1.59244E-09, it can be said that this model is good. The RMSE is 28908.75, and the coefficient of determination of the Gompertz model is 0.038.

Calculation results from cumulative production, AIC, RMSE, and values R^2 each model is presented in Table 5 to be compared and then determine which model is better between the two models.

Table 5. Calculation Results of the Logistic Model and the Gompertz Model

Model	***k***	***AIC***	***RMSE***	**R^2**
Logistik	107929.513	1.59323E-09	28900.87	0.039
Gompertz	107895.148	1.59244E-09	28908.75	0.038

Based on Table 5, the identified logistic model gives a better estimate than the Gompertz model seen from the higher RMSE R^2 which is greater than the Logistics model, although the AIC value of the logistic model is greater than the Gompertz model. Because the Logistics model meets the two best model requirements, the Logistics model is a better model for predicting gold production in Indonesia than the Gompertz model.

4. Conclusion

The Logistics Model predicts a cumulative gold production of 107929.513 Oz, with a growth rate of 148.6%, with an inflection or turning point in 1995 with a production value of 43628.988 Oz. The level of information loss in this model is 1.59323E-09, the RMSE is 28900.87, and the coefficient of determination of this model is 0.039. The Gompertz model predicts a cumulative gold production of 107895.148 Oz, with a growth rate of 142.3%, with an inflection or turning point in 1995 with a production value of 33171.351 Oz. The level of information loss in this model is 1.59244E-09, the RMSE is 28908.75, and the coefficient of determination of this model is 0.038. The identified logistic model provides a better estimate than the Gompertz model seen from the smaller RMSE than the Gompertz model and a higher R^2 value. large compared to the Gompertz function. The Logistics Model predicts gold production could rise to a level of 107895.148 Oz after 2040.

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