Life Insurance Premiums Dwiguna Joint Life and Last Survivor with Makeham Law

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Abstract  
In this paper we determined dual life insurance premiums with the combined status of last survivor and join life involving two insurance participants who have kinship relationships such as husband and wife, brother and sister, where they work in the same agency. In determining the policy that will be made by the life insurance party does not require two policies to be made, but simply have one policy. So by having one policy it is expected that the premium paid by life insurance participants to the life insurance company will be smaller than if you have to pay in two policies. Determination of dual-use life insurance premiums that will be paid by participants to the insurance party based on the chance of death of both participants of life insurance, by stating a condition that will continue as long as there is at least one surviving member and will stop after the death of the last person of its members (all members die), and is also a state that lasts as long as all members of the combined several people can survive and will stop after one of its members first dies. To determine a single premium and an annual premium use the cash value of the initial life annuity of a dual-purpose life insurance. Meanwhile, the cash value of early life annuities was influenced by interest rates and discounted factor and also influenced the combined life chances of two insurance participants. Furthermore, before the annual premium is determined, it is first determined a single premium. In formulating the chances of death insurance participants use the Makeham Law, which is a development of the Gomperzt law. In this distribution there are constants used to determine the chances of life and the chance of death.

Keywords  
Annuity, Premium, Last Survivor, Joint Life, Makeham Law
1. Introduction

One of the efforts to anticipate the risks of old age is to include each worker in a life insurance program. The old days led to an inability to earn the maximum income and resulted in economic hardship for the worker himself and his family. Life insurance is an insurance that provides payment of sum insured a certain amount of money for the death of the insured to the heirs or people who are entitled to receive it in accordance with the provisions and agreements that have been approved by the insured in the life insurance policy. The insurance policy itself is a contract or agreement between the insurance company and the insured (Kalfin et al., 2020; Kalfin et al., 2021). A married couple working in the same agency makes it possible to take out an insurance program with the same policy to facilitate premium payments and sum insured payments (Sirait et al., 2020).

Life insurance based on the number of insured is divided into two types are single life insurance (individual) and combined life insurance (Futami, 1994). In individual life insurance is limited to one participant only, while in the combined life insurance there are two or more insurance participants, such as husband and wife, brother and sister, children and parents. In this paper discussed is a combined life insurance consists of people by taking the example of husband and wife who work in the same agency. According to (Bowers et al., 1997) Combined life insurance is distinguished into two, namely joint life insurance and last survivor life insurance. Joint life insurance is a condition that lasts as long as all members of a combination of several people can survive and will stop after one of its members first dies. While the last survivor's life insurance is a condition that will continue as long as there is at least one member who is still alive and will stop after the death of the last person of his members (all members died).

Based on the coverage period is divided into pure dual-use life insurance, life insurance for life, term life insurance and dual-use life insurance. According to (Futami, 1993) dual-use life insurance is a type of insurance that is a combination of pure dual-use life insurance and term life insurance that means in or at the end of the coverage period to policyholders, both death and survival will be paid sum insured. For the payment of sum insured on the insurance dwiguna joint life states the sum insured will be paid if one of the husband and wife there is a death first, while in the insurance dwiguna last survivor is if the insurance participant is husband and wife at the last time of death or at the end of the coverage period is still alive then the sum insured will be paid.

In dual life insurance joint life and last survivor (Jhon and Albert, 2016; Matvejevs, 2001) single premiums and annual premiums are determined. Premium determination can be calculated through the life annuity value of the insurance participant. In this paper will be used annual premiums. Annual premium is a series of payments made by the insurance participant to the insurance company once a year within the time limit of the agreed insurance contract, while a single premium is the payment of insurance premiums made by the participant only once during the time of the contract that has been approved when entering into the insurance participant. Premium payments are also affected by discount factors, life chances and chances of dying from insurance participants. (Hasriati and Nababan, 2019). In previous research has discussed determining the insurance premiums of last survivor by using pareto distribution (Rytgaard, 1990). by estimating parameters for ages x and y years. Then Hasriati also determined the premium by gompertz law. Furthermore, in this research using makeham law which is a development of the law. The determination of Makeham distribution and survival function is done by stating in the form of development of gompertz mortality acceleration. Which is in this case. This constant has a role in determining the life chances and chances of death of insurance participants, so that it will ultimately affect the amount of premiums that will be paid by the insurance participants and the sum insured that will be received by the insurance participants.

2. Methods

2.1 Makeham Survival and Distribution Functions

The survival function (Bowers et al., 1997). is related to the distribution function and the opportunity density function. The distribution function of the X continuous random variable annotated $F(x)$ is a function related to the $f(x)$ opportunity density function.

Definition 2.1 from (Bain and Engelhardt, 1992) The random variable X is said to be a continuous random variable if there is an \( f(x) \) function, so that the cumulative disribusi function is expressed as

$$F(x) = \Pr(X \leq x) = \int_{-\infty}^{x} f(u)du.$$  

There is a term survival function called survival function. The survival function is noted with \( S(x) \), expressed as

$$S(x) = \Pr(X > x).$$
So that the relationship between survival function and distribution function is as follows

\[ S(x) = 1 - F(x) \]

William Matthew Makeham an English mathematician in 1860 presented a theory of mortality hokum as follows:

\[ \mu(x) = A + Bc^x, B > 0, A \geq -B, c > 1, x \geq 0 \]

With, \(0.001 < A < 0.003, 10^{-6} < B < 10^{-3}, 1.08 < c < 1.12\). So that the life chances of an x-year-old life insurance participant can survive up to t years later with makeham distribution are:

\[ p_x = e^{\int_0^t A + Bc^x \, dt} \]
\[ p_x = e^{-A} e^{-\frac{B}{\ln c} c^x (c^t - 1)} \]

If \(-A = \ln s\) and \(-B / \ln c = \ln g\), then,

\[ p_x = s^{A} g^{c^x (c^t - 1)} \]

### 2.2 Annuities Living On Individual Status With Makeham's Law

An annuity is a payment of a certain amount made each time lapse and a certain length of time on an ongoing basis (Dickson et al., 2009). The cash value of a live annuity whose payment is made once at the beginning of the year over a certain period of time called an initial term life annuity is

\[ \ddot{a}_{x\mid n} = 1 + v^2 p_x + v^3 2p_x + \cdots + v^{n-1} n-1p_x. \]

\[ \ddot{a}_{x\mid n} = \sum_{t=0}^{n-1} v^t t^p_x \]

Furthermore, an initial life annuity is obtained for life insurance participants aged x years with a period of n years under Makeham law as follows:

\[ \ddot{a}_{x\mid n} = \sum_{t=0}^{n-1} v^t s^t g^{c^x (c^t - 1)} \]

Then in the same way can be obtained the value of an initial life annuity futures life insurance participants aged y years with a period of n years according to Makeham's law that is

\[ \ddot{a}_{y\mid n} = \sum_{t=0}^{n-1} v^t s^t g^{c^{y} (c^t - 1)} \]

### 2.3 Chance to Die Joint Life and Last Survivor With Makeham Law

Life insurance not only provides protection for one insured person, but the life insurance company also provides protection for two or more insured people. Thus, insurance participants can include other family members in a policy such as husband and wife, mother and child, sister and brother and so on in the hope that the premiums paid are less than paying two life insurance policies. The number of insured is limited to two people.

Combined life insurance is divided into two parts, namely joint life insurance and last survivor life insurance. Joint life insurance is a condition that lasts as long as all members of a combination of several people can survive and will cease after the first death of its members (Jhon and Albert, 2016). While the last survivor's life insurance is a condition that will continue as long as there is at least one member who is still alive and will stop after the last death of its members or all its members die (Hasriati and Nababan, 2019). The difference between joint life insurance and last survivor life insurance lies in the time of payment of premiums. Joint life insurance is a combined
life insurance that payments premiums until the first death of the participant, while the last survivor life insurance is a combined life insurance whose premium payment is carried out until the last death of the participant.

In combined life insurance in determining the amount of premium paid is required survival function for combined status (Bowers et al., 1997). Survival function for combined status is obtained from the relationship of survival function for individual status. Suppose $T(x)$ declares a continuous random variable for life insurance participants who are $x$ years old and $T(y)$ states a continuous random variable for life insurance participants who are $y$ years old. Continuous random variables for joint life insurance $T(x)$ and $T(y)$ are $T(xy) = \min\{T(x), T(y)\}$ with obtained cumulative distribution function for combined life insurance that is

$$F_{T(xy)}(t) = t q_{xy}, \quad 1 - S_{T(xy)}(t) = t q_{xy}.$$ 

So that obtained the survival function of joint life insurance. Life chances of joint life insurance are expressed in the form of,

$$p_{xy} = p_x q_y.$$  

(2)

By substituting the equation (1) into the equation (2) obtained the life chances of participants of joint life insurance under gompertz law as follows:

$$p_{xy} = g^{c^x e^{ct-x-1}} g^{c^y e^{ct-x-1}},$$

$$p_{xy} = g^{c^x + c^y} e^{ct-x-1}. \quad (3)$$

Bowers et al. (1997). also states the continuous random variable for last survivor life insurance with $T(x)$ and $T(y)$ being $T((xy) = \max\{T(x), T(y)\}$ with its cumulative distribution function as follows:

$$F_{T(xy)}(t) = P(\max\{T(x), T(y)\} \leq t),$$

$$F_{T(xy)}(t) = P(T(x) \leq t \text{ dan } T(y) \leq t),$$

$$F_{T(xy)}(t) = P(T(x) \leq t)P(T(y) \leq t).$$

We obtained,

$$F_{T(xy)}(t) = t q_x t q_y,$$

$$t q_{xy} = t q_x t q_y.$$ 

Then,

$$1 - t p_{xy} = t q_x t q_y,$$

$$1 - t p_{xy} = (1 - t p_x ) (1 - t p_y)$$

$$= 1 - t p_x - t p_y + t p_x t p_y$$

$$t p_{xy} = t p_x + t p_y - t p_{xy}.$$ 

Furthermore, joint life insurance relationship with last survivor life insurance is obtained as follows:

$$t p_{xy} = t p_x + t p_y - t p_{xy}.$$ 

So that the life chances of last survivor life insurance participants obtained under Makeham's law as follows:

$$p_{xy} = s^x g^{c^x (e^{ct-x}-1)} + s^y g^{c^y (e^{ct-y}-1)} - s^x s^y g^{c^x+c^y} (e^{ct-x-1}). \quad (4)$$

3. Results and Discussion

3.1 Annuity Joint Life and Last survivor With Makeham Law

Early life term annuities (Musian, 2003) for the combined status there are two, namely an initial life annuity of a joint life term and an annuity of the initial life of the last survivor. The cash value of the initial life term annuity is annotated with $a_{xy|t}$ is the cash value of an initial life annuity futures joint life. The cash value of the initial life
Annuitization of the joint life term is the cash value affected by the discount factor and the life opportunity for the combined status \( p_{xy} \) over a period of \( n \) years is
\[
\bar{a}_{xy:|n|} = 1 + v p_{xy} + v^2 2p_{xy} + v^3 3p_{xy} + \cdots + v^{n-1} n-1p_{xy},
\]
\[
\bar{a}_{xy:|n|} = \sum_{t=0}^{n-1} v^t t p_{xy}. \tag{5}
\]

By substituting the equation (3) into the equation (5) obtained the cash value of the initial life annuity of the term joint life of life insurance participants aged \( x \) and \( y \) years over a period of \( n \) years under Makeham's law that is
\[
\bar{a}_{xy:|n|} = \sum_{t=0}^{n-1} v^t s^t g(c^x+c^y)(c^{t-1}).
\]
The cash value of the initial annuity of futures is annotated with \( \bar{a}_{xy:|n|} \) is the cash value of the initial life annuity of the last survivor. The cash value of the last survivor's initial life annuity is the cash value affected by the discount factor and the life chances for combined status \( t p_{xy} \) over a period of \( n \) years are,
\[
\bar{a}_{xy:|n|} = 1 + v p_{xy} + v^2 2p_{xy} + v^3 3p_{xy} + \cdots + v^{n-1} n-1p_{xy},
\]
\[
\bar{a}_{xy:|n|} = \sum_{t=0}^{n-1} v^t t p_{xy}. \tag{6}
\]

By substituting the equation (4) into the equation (6) obtained the cash value of the initial life annuity last survivor term from life insurance participants aged \( x \) and \( y \) years over a period of \( n \) years under Makeham's law,
\[
\bar{a}_{xy:|n|} = \sum_{t=0}^{n-1} v^t \left( s^t g(c^x(c^{t-1}) + s^t g(c^y(c^{t-1}) - s^2 t g(c^x+c^y)(c^{t-1}))\right)
\]

### 3.2 Single Premium Joint Life and Last Survivor

A single dual-purpose life insurance premium (Gray and Pitts, 2012) is the sum of a single premium of pure dual-purpose life insurance and term life insurance noted with
\[
A_{xy:|n|} = A_{xy:|n|}^1 + A_{xy:|n|}^4.
\]
The single premium of dual-purpose combined life insurance for insurance participants aged \( x \) and \( y \) years with insurance coverage period for \( n \) years and sum insured of 1 unit of payment is declared as
\[
A_{xy:|n|}^1 = v^n n p_{xy},
\]
and the single premium of term life insurance for combined living status is
\[
A_{xy:|n|}^4 = \sum_{t=0}^{n-1} v^{t+1} | q_{xy}.
\]
We obtained
\[
A_{xy:|n|} = v^n n p_{xy} + \sum_{t=0}^{n-1} v^{t+1} | q_{xy}.
\]
\[
A_{xy:|n|} = v^n n p_{xy} + v \sum_{t=0}^{n-1} p_{xy} - \sum_{t=0}^{n-1} v^{t+1} t p_{xy}. \tag{8}
\]
Then the substitution of the equation (6) to the equation (8) then we obtained
\[
A_{xy:|n|} = v^n n p_{xy} + v \bar{a}_{xy:|n|} - (\bar{a}_{xy:|n|} - (1 - v^n n p_{xy})).
\]
\[ A_{x:y:n} = 1 - d \bar{a}_{x:y:n}. \]  

So obtained a single premium of dual life insurance joint life assuming Makeham is,

\[ A_{x:y:n} = 1 + d \sum_{t=0}^{n-1} v^t s_{x,y}^t g^{(c^x + c^y)(c^t - 1)} \]

The single premium of life insurance last survivor dwiguna from participants aged \( x \) and \( y \) years with insurance coverage period for \( n \) years is noted with \( A_{x:y:n} \). Suppose to state the discount factor, \( n p_{x:y} \) states the chance of life and \( t q_{x:y} \) states the chance of dying delayed for the status of last survivor, then the single premium of life insurance last survivor pure dual,

\[ A_{x:y:n}^1 = v^n \ n \ P_{x:y}, \]

and the single premium of life insurance last survivor futures is stated by

\[ A_{x:y:n}^1 = \sum_{t=0}^{n-1} v^{t+1} \ t q_{x:y}. \]

Single premium of life insurance last survivor dwiguna is the sum of the single premium of life insurance last survivor dwiguna pure and life insurance last survivor futures that is

\[ A_{x:y:n} = A_{x:y:n}^1 + A_{x:y:n}^1. \]

By substitution of equations (9) and (10) to equations (11) it is obtained

\[ A_{x:y:n} = v^n \ P_{x:y} + \sum_{t=0}^{n-1} v^{t+1} \ t q_{x:y}. \]

Furthermore, by using the Makeham Law obtained,

\[ A_{x:y:n} = 1 - d \bar{a}_{x:y:n}. \]

3.3 Annual Premiums for Joint Life Insurance and Last Survivor

The bi-life insurance annual premium for the combined life status of participants aged \( x \) and \( y \) years with insurance coverage period for \( n \) years and sum insured paid at the end of the policy year is stated in (Futami, 1994) as

\[ P_{x:y:n} = \frac{A_{x:y:n}}{\bar{a}_{x:y:n}}. \]

Substitution of equations (12) to equations (14) obtained

\[ P_{x:y:n} = \frac{1 - d \bar{a}_{x:y:n}}{\bar{a}_{x:y:n}}. \]
Then obtained the annual premium of dual life insurance joint life assuming Makeham

\[ P_{xy|n} = \frac{1}{d_{xy|n}} - d. \]

The annual premium of life insurance last survivor dwiguna with insurance coverage period for \( n \) years, noted with \( P_{xy|n} \). The amount of annual premium on the life insurance of the last survivor can be stated as

\[ P_{xy|n} = \frac{A_{xy|n}}{d_{xy|n}} \]

\[ P_{xy|n} = \frac{1}{d_{xy|n}} - d. \]

So using the Makeham law obtained,

\[ P_{xy|n} = \frac{1 - d \sum_{t=0}^{n-1} v^t \cdot s^2 t g(x + e^t)(c^t-1) + s^t g(x + e^t)(c^t-1) - s^2 t g(x + e^t)(c^t-1)}{\sum_{t=0}^{n-1} v^t \cdot s^2 t g(x + e^t)(c^t-1)} \]

Example 1 A 50-year-old private employee with his wife of 45 years wants to join the last survivor life insurance program with a coverage period of 10 years and the age of coverage given to the heirs of Rp100,000,000. If the applicable interest rate is 0.025 then determine the annual premium of life insurance last survivor dwiguna with the following case:

(i) At the time the private employee was alive and his wife died
(ii) At the time of the private employee’s death and his wife was still alive
(iii) If both participants live until the end of the policy year

Table 1. Annual premiums for life insurance last survivor with Makeham law

| Year | \( P_{x|n} \) | \( P_{y|n} \) | \( P_{xy|n} \) |
|------|----------------|----------------|----------------|
| 0    | 9.774.017,37   | 9.592.950,679  | 8.813.971,34   |
| 1    | 10.869.004,77  | 10.687.460,5   | 9.898.958,959  |
| 2    | 12.234.564,35  | 12.053.233,07  | 11.257.359,54  |
| 4    | 16.318.512,04  | 16.140.984,07  | 15.342.772,45  |
| 6    | 24.461.120,61  | 24.297.248,26  | 16.618.340,4   |
| 7    | 32.591.318,02  | 32.443.935,35  | 31.507.69      |
| 8    | 48.839.617,08  | 48.727.850,74  | 48.192.463,09  |
| 9    | 97.560.975,6   | 97.560.975,6   | 97.560.975,6   |
4. Conclusion

Sum insured Last survivor life insurance is paid to the heirs until the last death of the insurance participant. The cash value of an annuity for the last survivor's dual life insurance is influenced by the discounted factor and the life chances of the insurance participants. By using makeham mortality acceleration, premiums are obtained that increase until the last death of insurance participants in the case of last survivor. So for a married couple who work in the same agency it would be better to use last survivor insurance than to use individual life insurance.

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