Determination the Smoothing Constant that Minimizes Mean Absolute Error and Mean Square Deviation

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Abstract

Exponential smoothing technique has become one of the quantitative techniques are very important in forecasting. The accuracy of forecasting based on this method depends on a parameter called the smoothing constant. Selection of smoothing constant value becomes very crucial because in forecasting prosecuted forecasting error is minimal. This paper discusses the selection of the optimal smoothing constant value which minimizes the mean square error (MSE) and the mean absolute deviation (MAD). Trial and error method is used to determine the optimal value of the smoothing constant based on the two criterias (MSE and MAD). Based on the analysis carried out, there was no regularity of the relationship between the amount of data and the smoothing constant value that minimized MAD and MSE.

Keywords:
smoothing constant, MAD, MSE, single exponential smoothing.

1. Introduction

Forecasting is a method to estimate the many aspects of a business or other activities. Forecasting methods can be divided into two categories: quantitative and qualitative. Quantitative forecasting methods based on the use of past data or historical data in the form of time series data. Forecasting methods that are often used are regression (Ashfahani, 2020; Prabowo *et al*., 2020b), arithmetic and geometric method (Prabowo *et al*., 2020a) and time-series method (Taylor, 2003; McKenzi and Gardner, 2010; Hyndman *et al*., 2002; Corberen-Vallet *et al*., 2011).

Time-series data is a set of observation or research data were measured continuously (successive) at a point in time or at a specific time period. Forecasting basically gives the future value of time series data for a particular variable, for example, sales volume variable.

Exponential smoothing is one of the techniques of time series which are widely used in forecasting. One of the methods for forecasting time series data is the single exponential smoothing (SES) method. Use of the term single exponential smoothing to differentiate it from the double exponential smoothing method. The single exponential smoothing method is also called the simple exponential smoothing method. Another name is the exponential weighted moving average (EWMA). This method can only be applied if the time series data does not contain trend data and seasonal data.

Exponential smoothing gives greater weighting to the latest observations and give a chance to the data of the past to play a role in the determination of the current forecast. In other words, an exponential weighting scheme will give maximum weight to the most recent observations and perform weighting of decreasing the data is getting old. Thus, the longest data will be weighted most minimal. Weight in the exponential smoothing technique is given by a parameter called the exponential smoothing constant, given the symbol α . Forecast values will vary greatly depending on the values of the smoothing constant use. Therefore, the error is the difference between actual and forecast values, thus forecast errors also depends upon the magnitude of the value (α).

The use of the word exponential in a method name refers to the smoothing constant, which decreases in weight with the length of the data. The oldest data will have the smallest weight and the latest data will be given the greatest weight. The illustration in Table 1 is made to explain the exponential decrease in weight, taking for example α = 0.30, 0.50, and 0.70.

Table 1 Exponential decline of the smoothing constant in the SES model

α	0.30	0.50	0.70
$\alpha(1-\alpha)$	0.21	0.25	0.21
$\alpha(1-\alpha)^2$	0.147	0.125	0.063
$\alpha(1-\alpha)^3$	0.1029	0.0625	0.0189
$\alpha(1-\alpha)^4$	0.07203	0.03125	0.00567
$\alpha(1-\alpha)^5$	0.050421	0.015625	0.001701
$\alpha(1-\alpha)^6$	0.0352947	0.007813	0.000510
$\alpha(1-\alpha)^7$	0.02270629	0.003906	0.000153

Equation (1), which is the equation for forecasting time series data using the single exponential smoothing method, shows the exponential weight loss that is listed in Table 1:

$$\begin{aligned} F_{t+1} &= \alpha X_t + \alpha(1-\alpha)X_{t-1} + \alpha(1-\alpha)^2 X_{t-2} + \alpha(1-\alpha)^3 X_{t-3} + \alpha(1-\alpha)^4 X_{t-4} + \ldots.. + \\ &\alpha(1-\alpha)^{n-1} X_{t-(n-1)} + (1-\alpha)^n F_{t-(n-1)} \end{aligned} \tag{1}$$

In Equation (1), it can be seen that the coefficient or weight value decreases with the length of the data. The oldest data $X_{t-(n-1)}$ has the smallest weight $\alpha(1-\alpha)^{n-1}$ and the newest data X_t has the highest weight α . Philosophically, the SES model is great at making the most recent data the most important data and conversely, the latest data the least important data.

Another thing to note is that the values of $\alpha(1-\alpha)^{n-1}$ and $\alpha(1-\alpha)^n$ are the same so that the graph is horizontal for $n \to \infty$, namely $\lim_{n\to\infty} \alpha(1-\alpha)^{n-1} = \lim_{n\to\infty} \alpha(1-\alpha)^n$. From this, the selection of the number of data n greatly affects the smoothing constant value.

In Equation (1), there are two types of data at the same point in time, namely $X_{t-(n-1)}$ and $F_{t-(n-1)}$ respectively weighted $\alpha(1-\alpha)^{n-1}$ and $(1-\alpha)^n$. For $n \to \infty$ then $\alpha \approx (1-\alpha)$ so that $\alpha(1-\alpha)^{n-1} = (1-\alpha)^n$. It means both data have the same weight.

In equation (1), weight loss occurs by following a geometric series (measuring series) with a ratio $(1-\alpha)$ so that the curve will resemble a graph of an exponential function $f(x) = exp(-x)$ with $x = \alpha, \alpha(1-\alpha), \alpha(1-\alpha)^2, \ldots..$ Since the graph moves from top left to bottom right, we can define exponential smoothing as a forecasting method for moving averages whose weight decreases exponentially from the longer observed values (Makridakis *et al.*, 1999: 79).

It is also important to pay attention to the comparison between the selected value α and the ratios that make up the geometric series, namely $(1-\alpha)$. If $\alpha = 0{,}5$, then $\frac{\alpha}{1-\alpha} = 1$. If $\alpha < 0{,}5$, then $\frac{\alpha}{1-\alpha} < 1$. If $\alpha > 0{,}5$, then $\frac{\alpha}{1-\alpha} > 1$. What are the meaning of these ratio or its reciprocal?

As seen in equation (1), the existence of the smoothing constant is very important and determines the value of the forecast. Because F_{t+1} very dependent on the smoothing constant α, the choice of α value is something that must be considered. Next, the latest research on smoothing constant α.

The accuracy of forecasting using the single exponential smoothing method really depends on the smoothing constant used (Mu'azu, 2014). The smoothing constant states the percentage of forecast errors. The selection of the smoothing constant used plays an important role in determining the response of the forecast results to historical data. For α small number, the forecast data is not very responsive to the historical data collected. The opposite is true so that the value forecast data depends on the variation α that used.

Dielman (2006) reports a smoothing constant (α) selection technique that minimizes sum of square error and sum of absolute error. Several years earlier, Paul (2001) used the same technique as Dielman, but was chosen to minimize mean squared error and mean absolute deviation.

Stevenson (2009) states that the selection of smoothing constants basically depends on policy or through trial and error and generally selects a smoothing constant value between 0.05 and 0.50. Rao (2012) provides a rule for selecting smoothing constants $\alpha < 0{,}5$ and specifically for $\alpha = 0{,}2$ and $\alpha = 0{,}3$ gives good results. Marzena and Toporowski (2012) reported that the use of smoothing constant p close to 0 gives good results for time series data that do not contain cyclic data and are free of irregular components.

The results reported by Marzena and Toporowski are in line with Chiang's conclusions several years earlier. Chiang (2005) provides guidelines for selecting α. Smoothing constant α that is close to 0 can be chosen if the time series data are slightly different, and α close to 1 can be used if the predicted value can experience significant changes in the actual data value. In education, Ravinder (2013) compares the smoothing constant α generated by the *Solver* program in Excel with the value α should have. This comparison leads to several different results, so studying single exponential smoothing method with a *Solver* requires special attention. Furthermore, Susilo *et al*. (2016) used the Winter's exponential smoothing method for forecasting volume of water. Beside, Mukarromah *et al*. (2016) examined the determination of the smoothing constants used in secondary data with MAPE and MAD. The research resulted in α values of 0.86 and 0.83 which minimized MAD and MSE.

Brown (in Mu'azu, 2014) states that for forecasting purposes a value can be chosen α between 0.7 - 0.9. Mu'azu (2014) states that there is no empirical research that supports and justifies the above conclusions. Also, there is no theory that discusses it. Furthermore, from the results of his research Mu'azu (2014) provides a mathematical model to determine the value of the alpha smoothing constant α in the single exponential smoothing method.

For smoothing $$\alpha = \left(\frac{n-1}{3n}\right) \tag{2}$$

For forecasting $$\alpha = 1 - \left(\frac{n-1}{3n}\right) \tag{3}$$

Equations (2) and (3) show that the smoothing constant is only a function of number data. For each natural number n, the numerator has a smaller value than the denominator, so the α value will be less than 1. Furthermore, from (2) for n the bigger we obtained $\lim_{n\to\infty} \frac{n-1}{3n} = \lim_{n\to\infty} \frac{\frac{n}{n} - \frac{1}{n}}{\frac{3n}{n}} = \lim_{n\to\infty} \frac{1-\frac{1}{n}}{3} = \frac{1}{3} = 0{,}333....$ where $0 < \alpha < 1$ and n states the amount of data. Thus, the α value is between 0.167 (for $n = 2$) to 0.333.

The idea of this research is based on equations (8): $F_{t+1} = \left(\frac{1}{n}\right)X_t + \left(1 - \frac{1}{n}\right)F_t$. The equation is identical to equation (9) i.e. $F_{t+1} = \alpha X_t + (1-\alpha)F_t$ so we obtained $\alpha = \frac{1}{n}$. Therefore, in this study, a mathematical model was created to determine the α smoothing constant whose behavior is similar to the function $\alpha = f(n) = \frac{1}{n}$. The results will be compared with the smoothing constant mathematical model produced by Mu'azu (2014) in equations (2) and (3).

The purpose of this study is to create a mathematical model to determine the magnitude of the smoothing constant α in the single exponential smoothing method, where $0 < \alpha < 1$. One of the techniques used in determining the α value is a mathematical approach by creating a mathematical model based on the time series data obtained. Therefore, the formulation of the problem of how a mathematical model is proposed to determine the magnitude of the smoothing constant in the single exponential smoothing method is based on inspection of a set of time series data.

2. Research Methodology

This research was done by studying the literature. Several articles discussing the smoothing constant became the foundation in finding the results obtained in this study. In this study, a mathematical equation is obtained which is used to estimate the smoothing constant. To arrive at these results, the following research steps were taken:

Stage 1: Time series data collection
At this stage, time series data are collected. These data are trend-free and seasonal-free, so that forecasting can be done with a single exponential smoothing. We also use research results with trend-free and seasonality-free time series data to obtain alpha values.

Stage 2: Determination of the smoothing constant
Based on the data collected, predictions were made to get the smoothing constant value.

Stage 3: Determination of the mathematical model for the smoothing constant
Based on the obtained smoothing constant values, a mathematical model is determined which satisfies all the smoothing constant values.

3. Results and Discussions

3.1 Moving Average Method

One of the methods used to analyze time series data is the Moving Average method. According to Makridakis *et al.* (1999), the Moving Average (MA) method with the order T can be written as

$$MA(T) = F_{T+1} = \frac{X_1 + X_2 + \dots + X_T}{T} = \frac{1}{T}\sum_{i=1}^{T} X_i \quad (4)$$

$$MA(T) = F_{T+2} = \frac{X_2 + X_3 + \dots + X_T + X_{T+1}}{T} = \frac{1}{T}\sum_{i=2}^{T+1} X_i \quad (5)$$

Formulas (4) and (5) can be continued so that they are obtained

$$MA(T) = F_{T+N} = \frac{X_N + X_{N+1} + \dots + X_{N+(T-1)}}{T} = \frac{1}{T}\sum_{i=N}^{T+(N-1)} X_i \quad (6)$$

where

F_{T+N} : forecast value for the period $T+N$ for $N = 1,2,\dots\dots$

X_i : *i*-th period data

Other writings for moving average of order T is

$$MA(T) = F_{t+1} = \frac{X_{t-T+1} + \dots + X_{t-1} + X_t}{T} = \frac{1}{T}\sum_{i=t-(T-1)}^{t} X_i \quad (7)$$

where

F_{t+1} : forecast value for the period *t+1*

X_t : *t*-th period data

From equation (5) that in the MA method by order T or $MA(T)$, the earliest data is omitted in the next step. Furthermore, equation (5) can be written as

$$F_{T+2} = F_{T+1} + \frac{1}{T}\left(X_{T+1} - X_1\right) \quad (8)$$

From equation (8) it can be seen that each new forecast (F_{T+2}) is only an adjustment compared to the forecast for the previous period (F_{T+1}). The adjustment is $\frac{1}{T}$ of the difference between the newest data X_{T+1} and the oldest data X_1. If a larger number is chosen for T, the adjustment value becomes smaller. This means that moving averages with a large order T will produce the forecast that does not differ too much.

3.2 Single Exponential Smoothing Method

Equation (4) can also be represented in several analogous ways, for example by replacing $n = t-1$ amd $1 = t-n$ with $T = n$ and $T = t-1$.

$$F_{t+1} = F_t + \frac{1}{n}\left(X_t - X_{t-n}\right) \tag{9}$$

If the old observation X_{t-n} is not available then X_{t-n} can be replaced by the approach, for example the predictive value that was previously F_t, so that (9) can be represented as

$$F_{t+1} = \left(\frac{1}{n}\right)X_t + \left(1 - \frac{1}{n}\right)F_t \tag{10}$$

Formula (10) is able to explain if the time series data is stationary then the formula is good enough as a prediction for the actual data. However, if the data contains a trend, then the use of formula (10) must be avoided. Formula (10) explains that the latest actual data weight X_t is $\left(\frac{1}{n}\right)$ and the weight is $\left(1 - \frac{1}{n}\right)$ for the last prediction of F_t. Depending on the selected positive n value, the last actual data or last estimate may be given more weight.

Furthermore, the equation model (10) is called the single exponential smoothing equation. In double smoothing, the results obtained are smoothed again so that the model (10) is called double smoothing.

Let $\alpha = \frac{1}{n}$ and $1 - \frac{1}{n} = 1 - \alpha$. Look at the SES equation model (10) which is presented in another form in equation (11):

$$F_{t+1} = \alpha X_t + (1-\alpha)F_t \tag{11}$$

The results of the translation of the SES equation model (11) is

$$F_{t+1} = \alpha X_t + (1-\alpha)\left[\alpha X_{t-1} + (1-\alpha)F_{t-1}\right]$$

$$F_{t+1} = \alpha X_t + \alpha(1-\alpha)X_{t-1} + (1-\alpha)^2 F_{t-1}$$

where α is smoothing constant.

If the translation of this function is repeated by substituting F_{t-1} with its components, F_{t-2} with its components, and so on, we will get equation (12):

$$\begin{aligned} F_{t+1} = {} & \alpha X_t + \alpha(1-\alpha)X_{t-1} + \alpha(1-\alpha)^2 X_{t-2} + \alpha(1-\alpha)^3 X_{t-3} + \alpha(1-\alpha)^4 X_{t-4} + \ldots + \\ & \alpha(1-\alpha)^{N-1} X_{t-(N-1)} + (1-\alpha)^N F_{t-(N-1)} \end{aligned} \tag{12}$$

As in equation (9), in equation (10) the initial value of $F_{t-(N-1)}$ can be selected for the last actual data of $X_{t-(N-1)}$. Furthermore, the SES equation model (8) can also be expressed by equation (13):

$$F_{t+1} = F_t + \alpha\left(X_t - F_t\right) \text{ atau } F_{t+1} = F_t + \alpha\left(e_t\right) \tag{13}$$

where e_t declare a forecast error. If the initial predicted value of F_t is the actual last data for X_t, then the estimation error of e_t is 0, for any α. It means $F_{t+1} = F_t = X_t$

Mathematically, formula (11) explains that for a greater smoothing constant α, it means that the error value will increase and consequently the forecast results will be no better. However, to obtain the best SES model equation for the set of data we have is to minimize forecast errors, so the alpha value is not a reference in selecting the best model.

3.3 Mean Square Error and Mean Absolute Deviation

The difference between forecasting techniques with time series analysis has been described by Montgomery and Johnson (Guardner, 2006; Synder *et al.*, 2002). In the period t, forecasting technique based on exponential smoothing is given by

$$F_t = A_0 \qquad \text{for } t = 1$$

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + ... + \alpha(1-\alpha)^{t-1} A_0 \qquad \text{for } t > 1 \tag{14}$$

with

F_t : forecasting in the period t

A_{t-1} : actual value in the period $(t-1)$

α : exponential smoothing constant

From equation (14), for different periods, can be written

$$F_1 = A_0$$
$$F_2 = \alpha A_1 + \alpha(1-\alpha)A_0$$
$$F_3 = \alpha A_2 + \alpha(1-\alpha)A_1 + \alpha(1-\alpha)^2 A_0$$
$$F_4 = \alpha A_3 + \alpha(1-\alpha)A_2 + \alpha(1-\alpha)^2 A_1 + \alpha(1-\alpha)^3 A_0$$
$$F_5 = \alpha A_4 + \alpha(1-\alpha)A_3 + \alpha(1-\alpha)^2 A_2 + \alpha(1-\alpha)^3 A_1 + \alpha(1-\alpha)^4 A_0$$

and so on.

The use of various sizes (criteria) to determine the accuracy of forecasting has been discussed by Mentzer and Kahn (1995). In this article, mean square error (MSE) and mean absolute deviation (MAD) is used to measure the accuracy of forecasting.

Mean Square Error (MSE) definied as $MSE = \sum_{i=1}^{t} \frac{(F_t - A_t)^2}{t}$. This formula can express with $MSE = \frac{(F_1 - A_1)^2 + (F_2 - A_2)^2 + ... + (F_t - A_t)^2}{t}$. Substitute the values of F_i in MSE, we get

$$MSE = \frac{1}{t}\left(\begin{array}{l} [A_0 - A_1]^2 + [\alpha A_1 + \alpha(1-\alpha)A_0 - A_2]^2 + .[\alpha A_2 + \alpha(1-\alpha)A_1 + \alpha(1-\alpha)^2 A_0 - A_3]^2 + ... + \\ [\alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + ... + \alpha(1-\alpha)^{t-1} A_{t0} - A_t]^2 \end{array} \right) \tag{15}$$

Furthermore, *mean absolute deviation* (MAD) defined as $MAD = \sum_{i=1}^{t} \frac{|F_t - A_t|^2}{t}$ or $MAD = \frac{|F_1 - A_1|^2 + |F_2 - A_2|^2 + ... + |F_t - A_t|^2}{t}$. Substitute the values F_i in MAD, we

$$MAD = \frac{1}{t}\left(\begin{array}{l} |A_0 - A_1|^2 + |\alpha A_1 + \alpha(1-\alpha)A_0 - A_2|^2 + .|\alpha A_2 + \alpha(1-\alpha)A_1 + \alpha(1-\alpha)^2 A_0 - A_3|^2 + ... + \\ |\alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + ... + \alpha(1-\alpha)^{t-1} A_{t0} - A_t|^2 \end{array} \right) \tag{16}$$

Furthermore, the trial and error method will be used to determine the value of smoothing constant that minimizes error. By choosing the values of the varying smoothing constant α $(0 < \alpha < 1)$, can be calculated MSE and MAD for each selected value.

3.4 Application with Data

Exponential smoothing method will be used to take a simple example. Data on past sales of a company for a period is given in Table 2. Thus, the data appear in Table 1 is actual data. Six numbers data for the period 0, 1, 2, 3, 4, and 5 are given in the Table 2.

Table 2 Actual data sales

Period	Actual value A_t

t	(in thousand)
0	10
1	8
2	14
3	13
4	12
5	12.5

For smoothing constant values α are different, can be calculated value corresponding MSE and MAD. MSE is calculated using equation (15) and the results are given in Table 3. Furthermore, MAD is calculated using equation (16) and the results are given in Table 4.

MSE value is different for each α that selected. MSE value decreases as the value α . After reaching value α = 0.88 MSE value is increasing. Variations the MSE value associated with α is written in Table 3. MAD value is different for each α that selected. MAD value decreases as the value α . After reaching value α = 0.83 MAD value is increasing. Variation MAD value associated with the use tabulated in Table 4.

Table 3 MSE Value for each of the Smoothing Constant Tested

Smoothing constant α	*Mean Square Error* (MSE)
0	133.85
0.10	76.374
0.20	45.241
0.30	28.300
0.40	18.950
0.50	13.685
0.60	10.687
0.70	9.020
0.80	8.211
0.82	8.130
0.84	8.074
0.85	8.054
0.86	8.040
0.87	8.033
0.88	8.031
0.89	8.034
0.90	8.044
0.95	8.267
1	8.450

Table 4 MAD Value for each of the Smoothing Constant Tested

Smoothing constant α	*Mean Absolute Deviation* (MAD)
0	10.700
0.10	8.038
0.20	6.028
0.30	4.535
0.40	3.443
0.50	2.663
0.60	2.241
0.70	2.041
0.75	1.961
0.78	1.919
0.80	1.894
0.82	1.869
0.83	1.858
0.84	1.867
0.88	1.921
0.90	1.949
0.95	2.022
0.98	2.068
1	2.100

To determine the optimum value of the smoothing constant α , selected MSE and MAD smallest. The value of the smallest MAD and MSE corresponding to the optimum. Thus, for this case, α is selected as the smoothing constant α = 0.88 for the criteria MSE and α = 0.83 for MAD criteria (see Table 5).

Table 5 The Optimum Value that Minimizes the MSE and MAD

Criteria	**MinimumValue**	**Smoothing constant** α

MSE	8.031	0.88
MAD	1.858	0.83

By using the value that has been selected, the values forecast for the period to 1, 2, 3, 4, 5 and 6 can be calculated. The use of two criteria which it produces two different options α (not always, different criteria give the same value α) produced two singles exponential smoothing equation calculated by equation (14). Table 6 summarizes the results obtained for α = 0.88 and α = 0.83

$$F_t = A_0 \qquad \text{for } t = 1$$

$$F_t = \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + ... + \alpha(1-\alpha)^{t-1} A_0 \qquad \text{for } t > 1$$

Table 6 The Forecast Value Based on MSE and MAD

Period t	Actual Value A_t (in thousand)	Forecasting Value that minimized MSE α = 0.88	Forecasting Value that minimized MAD α = 0.83
0	10	-	-
1	8	10	10
2	14	8.096	8.051
3	13	13.291	12.750
4	12	13.034	12.765
5	12.5	12.124	11.794
6	-	12.455	12.068

Forecasting results based on Table 6 are very satisfying. Using α = 0.88 gives more accurate results than α = 0.83. If we use Equation (3), we get $\alpha = 1 - \left(\frac{6-1}{18}\right) = 0.72$. Relatively speaking, these results are not too different from the results obtained in Table 5.

In their research, Zulfatul et al. (2016) obtained alpha = 0.8 for import duty data that minimizes MAD. However, for customs and excise data, the α values obtained were very different, namely $\alpha = 0.1$ and alpha $\alpha = 0.38$. Even though the same amount of data is used, the alpha values are very different and do not converge to one particular value. Zulfatul *et al*. (2016) uses 18 data, so that with Equation (3) is obtained $\alpha = 0.69$. This result cannot be said to be the same as $\alpha = 0.80$

Ravinder (2013a) uses 21 problems with the amount of data each between 4 and 44. The results obtained are given in Table 7.

Table 7 Alpha values that minimize MSE and MAD from 21 problems studied by Ravinder (2013a).

Problem #	**Number of data**	**Alpha value that minimizes MSE**	**Alpha value that minimizes MAD**
1	6	1.00	0.30
2	11	0.31	0.13
3	5	0.54	0.49
4	12	1.00	1.00
5	12	1.00	0.77
6	11	1.00	0.69
7	4	1.00	1.00
8	5	1.00	1.00
9	5	1.00	1.00
10	4	0.00	0.00
11	24	0.34	0.21
12	44	1.00	1.00
13	12	0.34	0.30
14	10	0.54	0.32
15	12	0.31	0.58

16	16	0.71	0.63
17	9	0.45	0.69
18	10	0.00	0.00
19	12	0.14	0.26
20	5	1.00	1.00
21	12	0.43	0.41

Based on Table 7, for the same amount of data, an alpha value that converges to a certain number is not obtained. Likewise, it was not found that an increase in the amount of data would cause an increase or decrease in the value of the smoothing constant.

Several studies have been added to determine the relationship between the amounts of data used in forecasting using the single exponential smoothing method with smoothing constants. Ravinder (2013b) concluded that when there is no trend in the data, simple exponential smoothing produces a small alpha value, namely for 12 data, an alpha between 0 and 0.30 is generated. If there is more data, which is more than or equal to 36, then the alpha value lies between 0 and 0.15. These results provide a pattern that the greater the amount of data used, the smaller the smoothing constant.

Referring to Equation (3), for as many as 12 and 36 data an alpha is obtained $\alpha = 0{,}69$ dan $\alpha = 0{,}68$. The results obtained by Ravinder (2013b) do not show compatibility with the model built by Mu'azu (2014). Gorgess and Zahra (2018) used 120 data and obtained an alpha value that minimized MSE of 0.94. Velumani *et al*. (2019), produced an alpha of 0.3 for research with 12 data. Amalia, *et al*. (2020) use 3,225 data and produce an alpha that minimizes MSE and MAD is 0.90. Meanwhile, Hasan and Dhali (2017) used 15 data and produced an alpha of 0.68 for MAD and MSE. Olaniyi *et al*. (2018) used 8 data and obtained an alpha of 0.9. Meanwhile, Septiyana and Bahtiar (2020) used 12 data and produced an alpha of 0.20. Gustriansyah (2017) obtained an alpha value of 0.5 for forecasting with 9 pieces of data. Karmaker (2017) uses 7 pieces of data and produces alpha 0.31 (MAD) and alpha 0.14 (MSE). Finally, Adeniran and Stephens (2018) used 17 pieces of data and obtained an alpha of 0.90. The tabulations of these data are provided in Table 8.

Analysis of the data in Table 8 does not produce a certain pattern as stated by Ravinder (2013b). These data also do not support the formula generated by Mu'azu (2014) in Equations (2) and (3).

Table 8 Alpha values that minimize MSE and MAD from various research results

Researchers	**Number of data**	**Alpha value that minimizes MSE / MAD**
Zulfatul *et al*. (2016)	18	0.8 ; 0,1 ; 0.38
Gorgess and Zahra (2018)	120	0.94
Velumani *et al*. (2019),	12	0.30
Amalia, *et al*. (2020)	3,225	0.90
Hasan and Dhali (2017)	15	0.68
Olaniyi *et al*. (2018)	8	0.90
Septiyana and Bahtiar (2020)	12	0.20
Gustriansyah (2017)	9	0.50
Karmaker (2017)	7	0.31
Adeniran and Stephens (2018	17	0.90

4. Conclusion

Exponential smoothing is a method/technique in handling time series data that are widely used in various fields to predict future events. However, the problem that arises is the selection of smoothing constant value α associated with data analysis. In this paper some examples of advanced and proffered two criteria. Based on the analysis carried out, there was no regularity of the relationship between the amount of data and the smoothing constant value that minimized MAD and MSE.

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