Single Stage Single Commodity and Multi Period Warehouse Location Problem (SSCMPWLP) With Location, Distribution Inventory and Shortage Costs

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Abstract

In this paper we pose the location-allocation problem (single stage) with shortages and inventory variables in a style similar to Sharma (1991). Vimal Kumar (2012) gave a novel method of eliminating shortages and this is considered here (this leads to reduced number of variables). Next he problem SSCMPWLP is formulated in the conventional manner where no variable can be eliminated. We derive a formulation where the resulting 0-1 MILP is solved by Bender's decomposition and the associated all real problem is solved by Dantzig-Wolfe decomposition where the sub problems are posed as min-cost-network flow problems. It offers computational advantages. Thus we give three different methods to solve the problem SSCMPWLP. An empirical investigation is underway to see which of these three methods perform the best.

Keywords
Warehouse location problem, Lagrangian relaxation, Dantzing-Wolfe decomposition

1. Introduction
In this paper we directly give the novel formulation of the problem posed as above. Here goods (single commodity) are moved from plants to warehouses to markets. Plants are already located and warehouses are to be located at points that represent potential warehouse locations. We consider inventories at warehouses (as in Sharma (1991)) and consider inventories and shortages at markets. We also consider inventory and shortage costs and this means we have a multi-period problem. We wish to minimize sum total of transportation, location costs at warehouse points, the inventory costs at markets and warehouses and shortage costs at markets. We give a mix 0-1 integer linear programming formulation of the problem (in fact we give three approaches to solve this problem). We discuss its important merits in the discussion and conclusions.

2. Problem Formulation
Here, we have developed a single stage warehouse location problem. In this mathematical model we have considered shortage and inventory distribution. To reduce the complexity of the problem we formulated the model for single type of product or service.

2.1 Constants of the problem
Index i (1..I) for plants, j (1..J) for warehouses, k (1,,K) for markets and t (1..T) for time periods

cpw_{ijt}	cost of unit quantity transported from plant i to warehouse j in period t
cwm_{jkt}	cost of unit quantity transported from warehouse j to market k in period t
$supply_{it}$	max supply possible from plant I in period t
cwi_{jt}	cost of unit inventory at warehouse j in period t
cmi_{kt}	cost of unit inventory at market k in period t
csm_{kt}	cost of shortage at market k in period t
f_j	fixed cost of locating a warehouse at point j
dem_{kt}	demand at market k in time period t

2.2 Decision Variables

xpw_{ijt}	quantity transported from plant i to warehouse j in period t
xwm_{jkt}	quantity transported from warehouse j to market k in period t
xwi_{jt}	ending inventory at warehouse j in period t
xmi_{kt}	ending inventory at market k in period t
xsm_{kt}	ending shortage at market k in period t
$QTY\_RECD\_W_{jt}$	quantity received at warehouse j in period t
$QTY\_RECD\_M_{kt}$	quantity received at market k in period t
$Xm\_beg\_inv_{kt1-1}$	beginning inventory at market k in t1-1 period
$Xm\_end\_inv_{kt1}$	ending inventory at market k in t1 period
$Xm\_beg\_shortage_{kt1-1}$	beginning shortage at market k in t1-1 period
$Xm\_end\_shortage_{kt1}$	ending shortage at market k in t1 period
$\lim\_warehouse\_cap_j$	capacity for warehouse j
$\lim\_inv\_cap\_at\_m_k$	inventory capacity of market k
y_j	binary variable: equal to zero means no warehouse located at j otherwise it is one

It is noted that xmi is same as X_end_inventory

2.3 Mathematical Model

$$Min\ Z = \sum_{i,j,t} (cpw_{i,j,t} * xpw_{i,j,t}) + \sum_{j,k,t} (cwm_{j,k,t} * xwm_{j,k,t}) + \sum_{j,t} (cwi_{j,t} * xwi_{j,t}) + \sum_{k,t} \{(cmi_{k,t} * xmi_{k,t}) + (csm_{k,t} * xsm_{k,t})\} + (f_j * y_j)$$

$S.t.$

$$QTY\_\text{RECD\_W}_{j,t} = \sum_i xpw_{i,j,t} \quad for\ all\ \ j\ and\ t \qquad .... 1$$

$$\sum_j xpw_{i,j,t} \leq \text{supply}_{i,t} \qquad for\ all\ \ i\ and\ t \qquad .... 2$$

$$xwi_{j,t-1} + QTY\_RECD\_W_{j,t} = \sum_k xwm_{j,k,t} + xwi_{j,t} \quad for\ all\ j\ and\ t \qquad ....3$$

$$QTY\_RECD\_M_{k,t} = \sum_j xwm_{j,k,t} \quad for\ all\ k,t \qquad .... 4$$

$$\sum_{t=1}^{t_1} QTY\_RECD\_M_{k,t} + xshm_{k,t_1} = \sum_{t=1}^{t_1} dem_{k,t} + xmi_{k,t_1} \quad for\ all\ k\ and\ t_1 = 1,2,....,T \quad ....5$$

$$\sum_t [QTY\_RECD\_W_{j,t} + \sum_t xwi_{j,t}] + \sum_{k,t} xwm_{j,k,t} \leq M * y_j \quad for\ all\ j \qquad ....6$$

$$xwi_{j,t} \leq \lim\_\ warehouse\_cap_j \quad for\ all\ j,t \qquad ....7$$

$$xmi_{k,t} \leq \lim\_\ inv\_cap\_at\_m_k \quad for\ all\ k,t \qquad .... 8$$

$$all\ real\ \text{variable}s\ are\ greater\ than\ or\ equal\ to\ zero;\ and\ y_j = (0,1)\ for\ all\ j \qquad ....9$$

Method 1: We can eliminate $xshm_{k,t_1}$ by using (5) (and we can have computational advantages for solving a LP). Similarly, we can eliminate $QTY\_RECD\_W_{j,t}$ and $QTY\_RECD\_M_{k,t}$. Demand numbers can be suitably inflated to include safety stocks to take care of service levels. With reduced number of variables (elimination of xshm(k,t)) we can have computational advantages.

Method 2: we write equation (5) for t-1 and t and subtract (5) t1-1 from (5) t1 to get the following:

QTY_RECD_M(k,t1) + xshm(k,t1) – xshm(k,t1-1) = dem(k,t1) + xmi(k,t1) – xmi(k,t1-1) …………..10

Here none of the variables can be eliminated. Hence probably one may say that it may take more computational time as it has more number of variables than in method 1. We are in the process of carrying computational investigation to verify the claim.

Method 3: We add (11) and (12) as linking constraints as follows.

Xm_beg_inv(k,t1) = Xm_end_inv(k,t1-1) for all t1 ……………….11
Xm_beg_sht(k,t1) = Xm_end_sht(k,t1-1) for all t1 ……………….12

With this (10) becomes the following:

Xm_beg_inv(k,t1) + QTY_RECD_M(k,t1) + Xm_beg_sht(k,t1+1) =
Dem(k,t1) + Xm_end_inv(k,t1) + Xm_end_sht(k,t1-1) ………….13

In the same vein we add the following linking constraint:

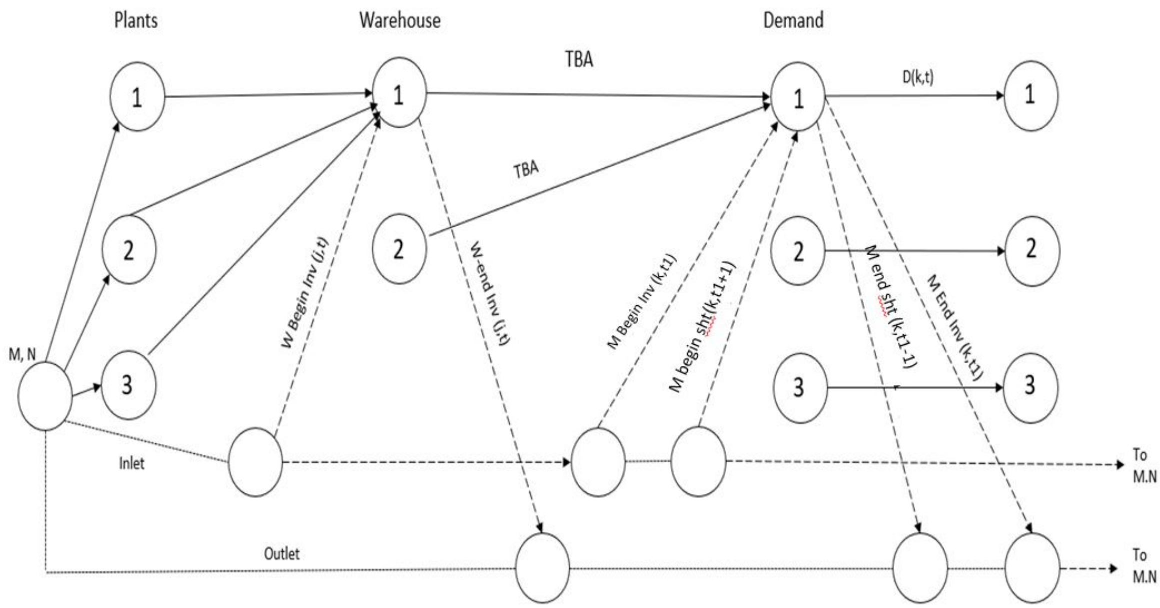
Xw_beg_inv(j,t1) = Xw_end_inv(j,t1-1) for all t1 …………..14

With this we cast the equation (3) as following:

Xw_beg_inv(j,t1) + QTY_RECD_W(j,t1) = sum(k), xwm(j,k,t1) + Xw_end_inv(j,t1) …………..15

Now the entire network flow problem (1) to (15) (see figure 1) minus (6) (and minus (11), (12) and (14)) has a block diagonal structure (See Figure 1) with each period problem being posed as a network flow problem (for a period t1) as given in figure given below. Now the constraints (11), (12) and (14) act as a reduced basis (in the context of Dantzig-Wolfe decomposition: see Sharma (1991)). The problem with constraint (6) also one can easily initiate the Benders' decomposition procedure. Advantage here is we solve min-cost-flow problem as in the given figure below which is of much smaller size and may offer computational advantages (besides encouraging use of parallel processing). This means we can initiate a Bender's decomposition procedure, and here all real problem is solved by Dantzig-Wolfe decomposition to reach optimality (where a sub problem as in figure given below (a min-cost-flow network problem that can be efficiently solved by any of the procedures such as network simplex, Busaker's code (dual simplex) or the primal-dual Out-of-Kilter algorithm).

It will be interesting to see which method does empirically better on large sized problems.



3. Conclusion

Thus in this paper we have given three methods for solving the SSCMPWLP. We have undertaken an investigation to empirically determine which method does the best. Method three can be accelerated by the inclusion of 'feasibility' constraints in the all integer sub problem.

References

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Biography

R. R. K. Sharma is BE (mechanical engineering) from VNIT Nagpur India and Ph.D. in management from I.I.M., Ahmedabad, INDIA. He has nearly 3 years of experience in automotive companies in India (Tata Motors and TVS-Suzuki). Now he has 32 years of teaching and research experience at the Department of Industrial and Management Engineering, I.I.T., Kanpur, 208016 INDIA. To date he has written 376 papers **(**peer reviewed). He has developed over 10 software products. Till date he has guided 62 M TECH and 21 Ph D theses at IIT Kanpur. He has been Sanjay Mittal Chair Professor at IIT KANPUR (15.09.2015 to 14.09.2018); and is currently HAG scale professor at IIT Kanpur. In 2015, he received "Membership Award" given by IABE USA (International Academy of Business and Economics). In 2016 he received "Distinguished Educator Award" from IEOM (Industrial Engineering and Operations Management) Society, USA. In 2019 and 2020 he was invited by Ministry of Human Resources Department, India to participate in NIRF rankings survey for management schools in India. In 2019, he was invited to participate in QS ranking exercise for ranking management schools in South Asia.

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KK Lai is an accomplished academician with several international publications. He is currently Vice-Chairman of CYUT (Chao Yang University of technology) Taiwan.