Developing a Mathematical Model for the Effect of Charcoal on Odor Intensity using Differential Equations

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Abstract

Charcoal is known not only for grilling but also for its ability to absorb odor and moisture. The presence of charcoal on the container results in a change in its odor intensity with respect to the change in time. This study developed a first order, first degree differential equation that can be used to predict the odor intensity of the container with respect to the time after putting charcoal or vice-versa. The variables used were the perceived odor intensity of 20 respondents as the dependent variable and time as the independent variable. Parameters such as the container's volume and temperature were held as constants for the simplicity of the model. The data used were the mean odor intensity rating for every 8 hours. The computed odor intensity per time (t) has varying absolute percentage errors because the data used deviated. This model is effective as long as the odor intensity rating is more accurate. Therefore, it is recommended to gather data on odor intensity with the use of equipment and instruments to make the predictions more accurate. It is also recommended to modify or improve the model and consider temperature and the volume of the container as variables instead of constants.

Keywords
Charcoal, Odor Intensity, Deodorizer, Exponential Model, Differential Equations.

1. Introduction
1.1 Background of the Study
Aside from being used in grilling, charcoal is also known for its ability to absorb moisture and odor. It is commonly used as a substitute for air fresheners to keep the closed areas like cabinets, boxes, and fridge clean and smell-free. In an article written by Han (2013), he stated that by placing a small stack on a room, charcoal has done a noticeable job of reducing the humidity levels and absorbing any light odors. The room smelled neutral and unscented, much better than any product masking smells with perfume. Moreover, he takes the pieces in the sunlight for a few hours to recharge, and the pieces are ready to work their purpose again.

In the Philippines, charcoal is present as one of the ingredients of natural air fresheners. In some provinces, people are either using activated charcoal or some are using the usual charcoal being used in grilling. Charcoal, a carbon residue which is made from partially burned wood and other organic materials is most well-known used in metallurgy and cooking. It is also used for the filtration of water and air because of its porous nature. On the other hand, activated charcoal is created by heating ordinary charcoal to a very high temperature. Because of this, the elements and compounds bound with the carbon atoms are removed and all the binding sites for carbon are free for binding with incoming molecules and atoms, making the activated charcoal more porous than ordinary charcoal (Strom, 2019).

In this day and age, the use of charcoal as a natural deodorizer is very common. People prefer to use it instead of harmful air fresheners even though its effect takes longer to be observed. The potential benefits of using charcoal as natural deodorizer drives the researcher to further study this topic. The researchers became interested on how long it would take to observe the effect of charcoal on odor intensity. Knowing how long it would take to maximize the effect of charcoal would greatly benefit the people who prefer to use it instead of harmful air fresheners. With this, the researchers would like to determine the relationship between time and the effectivity of charcoal on the odor intensity by making a mathematical model using differential equations.

1.2 Objectives
The main purpose of this study is to create a mathematical model that would show the relationship between time and the effect of charcoal on the odor intensity of a closed container. Specifically, this study also aims to determine the effect of charcoal on the reduction of odor intensity of a container at a certain time or the time it will take for a container to reach a certain odor intensity after putting charcoal.

1.3 Significance of the Study
The findings and result of this study would be beneficial to the household. The determination of the optimal time to fully maximize the effect of charcoal as fridge deodorizer would definitely benefit the household. This would give them an idea about the time it will take to eliminate the odor, so they would know when to start putting charcoal before storing on the containers. It will also help them in terms of odor monitoring and management. Moreover, the mathematical model that will be constructed would definitely benefit the students. It will give them an idea about the use of Differential Equations as it is applied on real-life situations like this, and they might also use the created mathematical model for further studies and application. Lastly, for the future researchers, the findings and result of this study would be an encouragement to investigate further on the same topic.

1.4 Scope and Limitations
The mathematical model to be developed in this study is made using first order, first-degree Differential Equation. This study would focus on developing a mathematical model that would help to determine the optimal time for charcoal to fully eliminate the odor of a body. The variables to be used in this study are time as the independent variable and odor intensity as the dependent variable. The data were gathered from 20 participants who will describe the odor on a given scale for every 8 hours until it reaches the level where the odor is described as not perceptible. On the other hand, factors such as the fridge's volume and temperature will be held constant.

2. Literature Review
This chapter will discuss the reference literature and related studies. The listed references will serve as a guide for the readers to have a background and be able to fully understand the study, and for the researchers as the following will serve as their foundation to furthermore strengthen their claims.

A study entitled "Optimization of Biofiltration for Odor Control: Model Development and Parameter Sensitivity" developed a dynamic model that describes the mass transport and attenuation of odor-causing air emissions like hydrogen sulfide and other reduced sulfur compounds in a biofiltration unit. This study included mechanisms including advective flow, mass transfer from the bulk phase to the biofilm, biofilm internal diffusion, and biological reaction in the biofilm. The model formed was an ordinary differential equation, which was converted and solved using DGEAR algorithm. The model simulation found out that the existence of a water layer surrounding the biofilm in a biofiltration unit lowers the removal efficiency of hydrogen sulfide. The data gathered from two biofilters showed that biofilm internal diffusion and biofilm kinetics have a significant effect on hydrogen sulfide removal, while external mass transfer has little effect (Li, Crittenden, Mihelcic, & Hautakangas, 2002).

Likewise, Yan et al. (2014) proposed a novel odor interaction model for binary mixtures of benzene and substituted benzene using Partial Differential Equation method. Basing on the tangent-intercept method of partial molar volume, perceptual measures displaced the original parameters of corresponding formulas. Because of this, it became possible to relate a mixture's odor intensity to the individual odorant's relative odor activity value (OAV). Data gathered from testing several binary mixtures of benzene and substituted benzene were used to establish the said PDE model. Results showed that the PDE model provided easily interpretable method to relate individual components and joint odor intensity. However, disadvantages such as the expense on the fixed number of odor assessors will be avoided if the olfactory evaluation of odor intensity is achieved with the use of instruments.

Moreover, a calculation model that is able to (1) calculate maximum Perfumery Raw Materials (PRM) coverage, (2) calculating values implying odor intensity from only arbitrary gas concentration, and (3) estimating odor intensity from the calculated values directly and easily, was developed on a study entitled "Method for Predicting Odor Intensity of Perfumery Raw Materials Using Dose–Response Curve Database." The prediction accuracy of the model was tested by comparing the predicted odor intensity from the model and the evaluated value for both the single component and the mixture, which resulted to the same degree of root mean square error (RMSE) (Wakayama , Sakasai, Yoshikawa, & Inoue, 2019). This confirms that this model is able to compute and predict the odor intensity of a PRM or mixture with respect to the arbitrary gas concentrations.

On the other hand, a verified vector model which calculates the Odor Intensity of a mixture as the vector sum of its unmixed components' odor intensities was modified to avoid restriction on the assessor participation of OI evaluation. The odor intensity of the unmixed component was replaced with the corresponding logarithm of the odor activity value (OAV) because on the detected linear relationship between the two. The degree of interaction (cos a) or interaction coefficient was measured in a simplified way. With this, the model showed an effective way to relate the odor intensity of a mixture with the ln (OAV) values of its constituents. Moreover, it was considered that the modified model was applicable for odor mixtures consisting of odorants having the same chemical functional groups and similar molecular structures. (Yan, Liu, & Fang, 2015)

The aforementioned literature and studies are closely related to the study that the researchers will conduct, for it proves that the change in odor intensity is caused by numerous factors. Moreover, this proves that the change in odor intensity concerning some factors can be modeled mathematically, showing predictions that are close to the evaluated or observed results. These shed light on the researchers in their attempt to develop a mathematical model that will show the relationship between the effects of charcoal on the odor intensity of a container. Unlike the aforementioned models, this study will attempt to develop a model considering the time and the perceived odor intensity of the respondents or participants. It will not focus on the volume of the container, as well as its temperature to ensure the simplicity and the effectiveness of the mathematical model.

3. Methods

3.1 Conceptual Framework

This chapter outlines the procedures and steps employed to complete this study. The odor intensity of a container changes with respect to the change in time after putting charcoal. With this, both of these factors were considered in modeling the effect of charcoal on the odor intensity using Differential Equations. The time is considered as the independent variable, the odor rating being the dependent variable, and parameter such as the container's volume was assumed to be constant. Moreover, the container's temperature would also be considered constant due to the limited source of data.

3.2 Data Collection

The data gathered for this study consists of the ratings from ten respondents who described the odor intensity of a storage box with the length, width and height of 19 inches, 13 inches, and 9.5 inches, respectively (volume = 2,346 cubic inches) after putting two pieces of regular charcoal (130g) as follows: 5 as strong, 4 as distinct, 3 as weak, 2 as very weak, and 1 as not perceptible. The data gathering started when all of the respondents have rated the odor intensity of the storage box with the constant temperature of 33ºC as 5 or descriptively interpreted as strong. The odor intensity was checked every 8 hours due to the availability of the respondents. The researchers conducted the experiment continuously to keep the accuracy of data. The whole data gathering lasted until the overall mean observed odor intensity became 1 meaning the odor is not perceptible. Below are the odor intensity ratings of the respondents for every 8 hours.

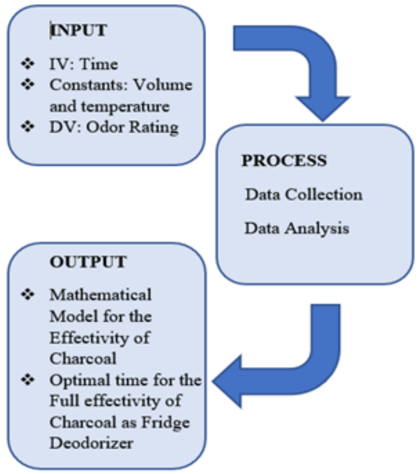

Figure 1. Conceptual Framework of the Study

3.3 Data Analysis

With the data gathered, descriptive statistics was made using Microsoft Excel. The mean response per time of observation is computed. The data showed that during the first eight hours upon putting charcoal on the box, the mean response is 3.65 which is verbally interpreted as distinct. Based on the rubric, this intensity is described by a smell that is noticeable and annoying for the respondents. On the 16th hour of observation, the respondents' mean response is 3.2 which is verbally interpreted as weak. This is described by a noticeable but not that annoying smell. After another 8 hours, there is an improvement on the odor intensity of the box based on the mean response of the respondents which is 2.8, verbally interpreted as weak. On the 32nd hour of the experiment, the respondents rated the odor intensity 2.2, which is verbally interpreted as very weak. This odor intensity is described as faint to slightly no odor. On the 40th hour, the mean response dropped to 1.8 verbally interpreted as very weak. Further improvements were observed on the 48th hour or second day of the observation, as the respondents rated the odor intensity as 1.6 which is still very weak. It took 56 hours to fully achieve a mean rating of 1, verbally interpreted as not perceptible**.** This rating is where the odor is not perceptible, or the respondents do not notice any odor at all. This rating is the standard perceptible odor intensity. This time is when the odor is fully-removed by the charcoal placed on the container. Based on the mini-experiment, it would take 56 hours to achieve the odor intensity of 1, or odor is not perceptible.

Table 1. Mean Odor Intensity per 8-hour Interval

Time (hour)	Mean Response	Verbal Interpretation
0	5	Strong
8	3.65	Distinct
16	3.2	Weak
24	2.8	Weak
32	2.2	Very Weak
40	1.8	Very Weak
48	1.6	Very Weak
56	1	Not Perceptible

Table 2. Rubric Used to Measure Odor Intensity

Odor Attribute	RATING				
	5	4	3	2	1
Intensity	Strong	Distinct	Weak	Very Weak	Not Perceptible
Quality	The smell is bad.	The smell is slightly bad.	The smell is noticeable.	The smell is faint.	There is no smell.
Acceptability	The smell is annoying.	The smell is slightly annoying.	The smell is not annoying.	The odor sensation is slightly likeable.	The odor sensation is likeable.
Observed Odor Intensity ____________					

The aforementioned ratings are based on a rubric in Table 2. The said rubric is constructed by the researchers based on the article entitled "Appraisal of Odor-Measuring Techniques" which states that odor can be measured objectively based on the odor attributes amenable to measurement by the olfactory sense (Duffee, 1968).

4. Results and Discussion

4.1 Numerical Results

Upon looking at the collected data on the mini experiment, it can be observed that the odor intensity of the container decreases after putting charcoal for hours. This just proves that charcoal is an effective natural deodorizer for it helps with odor removal. Using the mathematical model created wherein initial rating and time will be considered, it is possible to compute the time it will take to reach a certain odor intensity and vice-versa.

Table 3. The Actual Odor Intensity and the Computed Odor Intensity

Time (hour)	Actual Observed Odor Intensity	Computed Odor Intensity at time t
0	5	5
8	3.65	3.65
16	3.2	2.75
24	2.8	2.16
32	2.2	1.77
40	1.8	1.51
48	1.6	1.33
56	1	1.22

Using Microsoft Excel, the researchers tested the fit of the mathematical model on the gathered data. Based on their calculation, the computed R^2 is 0.9737. This means that the model accounts 97.37% of the variance. Theoretically, if a model could explain this percentage of the variance, the fitted values would most of the time equal the observed values and, therefore, 97.37% of the data points would fall on the fitted mathematical model.

4.2 Graphical Results

Figures 2-4 shows the graphical results.

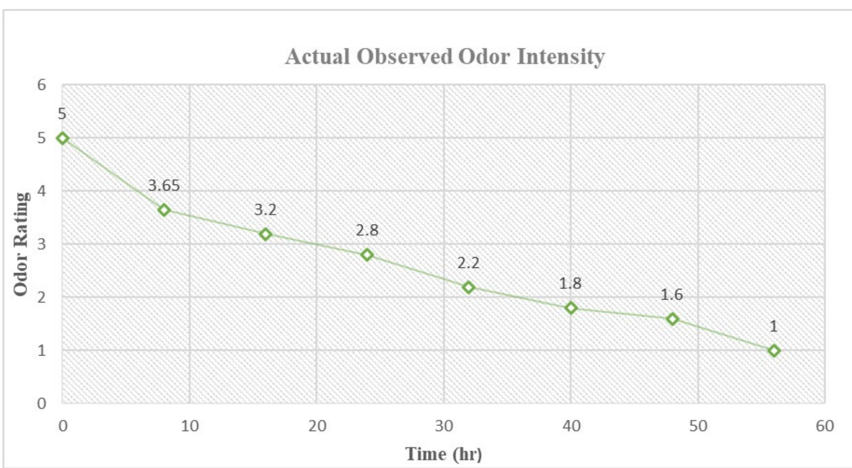

Figure 2. Change of odor intensity over time

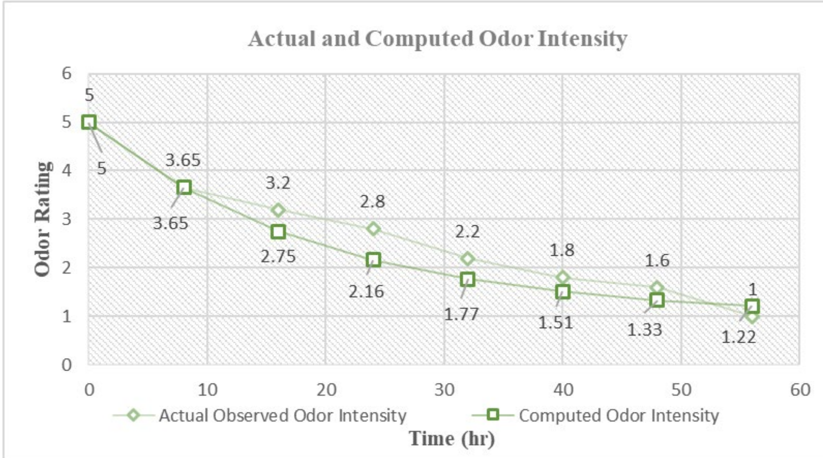

Figure 3. Fit test of the mathematical model on the gathered data

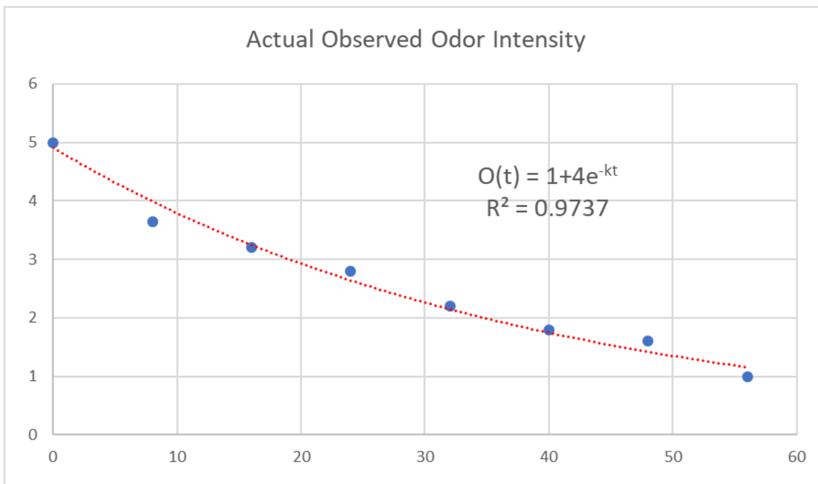

Figure 4. Comparison between observed odor intensity and computed odor intensity

4.3 Validation

To validate the results, it is needed to compute the Absolute Percentage Error (APE) with the formula:

$$APE = \frac{|Computed\ rating - actual\ rating|}{Actual\ rating}\ x\ 100$$

Table 4. Absolute Percentage Error of Computed Odor Rating

Time (hour)	Actual Rating	Computed Rating	APE
0	5	5	0%
8	3.65	3.65	0%
16	3.2	2.76	14%
24	2.8	2.16	23%
32	2.2	1.77	20%
40	1.8	1.51	16%
48	1.6	1.34	17%
56	1	1.22	22%

As seen from the table, it can be observed that the absolute percentage error per time of observation ranges from reasonable to high accuracy. The initial time where t=0 and the first observation where t=8 hours got 0% absolute percentage error because referring to the model, there is a need to compute for the proportionality constant k using the observed data from the experiment. However, the succeeding computations showed that the mathematical model can predict the actual time it will take for the odor rating to reach a certain level or the odor rating at a given time t. On the other hand, the absolute percentage errors were due to other factors such as the volume of the container, the temperature, and the fact that everyone has their preference when it comes to smell. These are the factors that are held as constants upon developing this mathematical model.

5. Conclusion

The researchers were able to develop a mathematical model that shows the relationship between the effect of charcoal on the odor rating of a container. This proves that the odor intensity of the container-when treated with the help of charcoal decreases with respect to time until it reaches the level where the odor is not perceptible anymore. When the volume and the temperature of the container are held as constant and based on the parameters and factors used in the study, the observed optimal time for the odor to be fully-removed on the container will take about 56 hours or more than 2 days.

On the other hand, the accuracy of the model is highly dependent on the data gathered from the experiment that the researchers conducted. The percentage errors of the computed odor intensity are caused by the factors being held as constants. It can be concluded that this mathematical model can be used with the odor ratings/intensity and when the temperature and the volume of the container will be held as constants.

6. Area for Future Work

Based on these findings, the researchers highly recommend using a greater scope of data on odor intensity. The use of accurate data such as the actual measurement of odor intensity with the help of equipment and instruments would greatly help for the accuracy of results. Moreover, a longer period for the data gathering and larger-scale data prompts greater modification of the model. The researchers also recommend developing a model that will consider the temperature and volume of the container being observed. Moreover, the use of objective measurements when it comes to odor intensity would also be an essential tool to achieve more accurate results. For future related studies, it is also recommended to use a device to validate the results.

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Biographies

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MC Gensen V. Gorospe is currently a student taking up Bachelor of Science in Industrial Engineering at Bulacan State University, City of Malolos, Bulacan. He is 20 years old, living in City of San Jose Del Monte Bulacan. He enjoys sports, particularly volleyball and has competed in various competitions, earning awards and medals. When he is in high school, he participated in research competition (Capstone Project). He aspires to be an advocate who educates children about mental health and LGBTQ+ awareness, so he attends various meeting to learn more.

Eldrin B. Gulapa is a twenty-year-old second-year college student, currently taking up Bachelor of Science in Industrial Engineering at Bulacan State University, City of Malolos, Bulacan, Philippines. He is an active member of his department's student quizzers and a member of the editorial board of his department's official publication. He is also a writer, competing in different levels since elementary. He also experienced joining math, science, and public speaking competitions. He authored several research papers and investigatory projects during his high school years up to the present.