Stress Strength Reliability of Time Dependent Strength using Jaya Algorithm: An Application to Automotive Circuits

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Abstract

In this paper, a simple methodology to obtain a reliability model for time-dependent strength is discussed. A conditional probability density function was evaluated considering the strength follows normal distribution with changing mean and variance over time. The methodology was developed taking into account the nonlinear degradation of strength over time. The maximum likelihood estimation method was used along with Jaya algorithm to estimate the model parameters. The life prediction is based on the probability that the minimum strength is less than the predetermined failure threshold. The proposed methodology was applied to data of solder strengths in automotive circuits. It was found that the proposed model gives promising results with faster compilation time.

Keywords
Time dependent reliability, maximum likelihood estimation, Jaya algorithm, solder strength data

1. Introduction
Reliability is defined as the probability that a component or a product will perform its functions under given conditions for a specific period. In today's world, a customer is seeking products that are reliable and safe. Also, with increasing developments in the sectors of transportation, infrastructure, energy generation, etc there is a need for robust design in these areas taking into account the uncertainties present in nature. Reliability studies consider these uncertainties of nature giving a rigid and dependable analysis. Thus studies on reliability modeling are given much importance in recent times(Kapur & Pecht, 2014; Prabhu Gaonkar et al., 2011; S. S. Rao, 2016). It is a known fact that reliability does not remain constant and decreases over a period of time as a result of degradation(Eryilmaz, 2013; Wu et al., 2011; Zhang & Zhao, 2017). Alternatively, it can be said that reliability decreases over a number of cycles in case of fatigue stress(D'Anna et al., 2017; Menan et al., 2015; Min et al., 1995; Sudret et al., 2003). Fatigue stress is a type of loading in which there are repetitive or cyclic loads acting on a material for a period of time. Over a large number of loading cycles, the materials tend to fail at a stress level much lesser than the strength of that material. This is known as failure due to material fatigue. Several major accidents that can be remembered in history are as a result of fatigue failure and natural uncertainties(Lewis, 2010; Salam et al., 1998). These accidents could have been prevented by robust and reliable design. Thus, failure prediction and modeling is an important aspect in material design. A lot of studies have been carried out in assessing the dynamic reliability of components under various load conditions. Also, we are aware that automobile systems are exposed to severe environmental conditions during their lifetime. The automotive electronic circuits face the problem of shearing of solder joints over a period of time or over the distance traveled. In this paper, a simple method is exhibited for evaluating time-dependent reliability by degradation modeling using Jaya algorithm which is a recent metaheuristic technique of optimization. The above methodology has been applied to a data of solder strength of automotive circuits to show its effectiveness. A plot for reliability vs time has been obtained in order to see the failure behavior of the components over a period of time and a plot for the

convergence behavior of Jaya algorithm has been obtained to show the performance of the algorithm over the number of iterations.

1.1 Objectives

The main objective of this study is to develop a simple yet effective methodology to evaluate the reliability of a component subjected to stress over a period of time taking strength degradation with time into consideration. The proposed method will seek to give close results with less compilation time. The methodology will be applied to the shear strength of solders data in automotive circuits and a plot for reliability prediction over time will be presented. The following assumptions are made:

1. The strength of solder joints follow normal distribution with non-linearly varying mean and linearly varying standard deviation.
2. The stress applied is deterministic in nature
3. The failure of circuit board is because of the failure of solder connections

2. Literature Review

In mechanical components, failure occurs when the stress applied exceeds the strength of the material. In actual practice, the strength is not constant because of uncertainties and can be assumed to follow a particular distribution(Patrick D.T. O'Connor, 2013). So, utmost care should be taken while designing the components so as to avoid the failure because of uncertainties in strength of the component. Bhuyan & Dewanji (2015) developed a model for estimating the reliability of a system undergoing strength degradation when subjected to cumulative damage due to shocks. A single model has been developed which considers strength degradation as well as damage accumulation. Park & Tang (2006) used the cycle counting approach to represent the time-dependent stress as several time-independent stress levels and developed a method to solve the problem by accumulated damage analysis with the first-order reliability analysis. Eryilmaz (2013) carried out studies in stress strength reliability for the strength following Weibull distribution and stress remaining constant over time. Analysis was also carried out for the Multicomponent form. Wu et al. (2011) introduced a dynamic reliability model for strength deterioration based on gamma distribution and stress based on Poisson process. Jose & Drisya (2020) introduced random cycle time following particular distribution and studied time-dependent stress strength reliability for stress and strength following continuous phase-type distribution. Lu & Meeker (1993) developed statistical methods using nonlinear mixed-effects model to obtain time to failure distribution for various degradation models. Reliability assessment was also carried out by finding point estimates and confidence intervals using Monte Carlo simulations. Zhu et al. (2013) carried out time-dependent reliability analysis for railway axle steels by evaluating the nonlinear damage accumulation model while considering the number of cycles following log-normal distribution. Rathod et al. (2011) derived a degradation model in fatigue loading by the principle of damage accumulation considering normal distribution for strength with decreasing mean and linearly increasing variability over time along with reliability analysis.

The maximum likelihood estimation method has been used by many researchers for getting the best estimates of the fit model. Feng et al. (2018) presented a methodology to predict probabilistic S-N curves using the maximum likelihood method and carried out simulations to validate the results. D'Anna et al. (2017) proposed a model for estimating the structural reliability considering the fatigue life to follow Birnbaum- Saunders model distribution and both the parameters depending on stress. The model parameters were estimated using the maximum likelihood estimation method and it was observed that the model gave a good fit for the fatigue failure data of aluminum coupons under cyclic stress obtained by accelerated life tests. Jaya Algorithm is an effective optimization technique that continuously works towards taking you closer to the best solution and away from the worst solution. Du et al., 2018 used Jaya algorithm in solving optimization-based structural damage identification problem. The author presented the effectiveness of the technique even in high noise levels. R. V. Rao et al., 2017 presented the application of the algorithm in optimizing the process parameters in machining while handling multiple objectives. Similarly, the technique has been used by other researchers for obtaining solutions in optimization problems with effectiveness and at a faster rate(Caldeira & Gnanavelbabu, 2019; R. V. Rao et al., 2017).

3. Method

Most of the mechanical components are subjected to fatigue due to varying/cyclic loads during their lifetime. Failure of many mechanical components has been traced down to fatigue as the major cause. Thus, it is very important to predict the failure of such components and improve the product life by reliability-based design. Even though a large

amount of work has been carried out in the field of fatigue, many challenges can still be seen in this area. The current study carries out reliability studies for a component whose strength decreases with time or number of cycles.
Consider the conceptual monotonically decreasing degradation path depicted in Figure 1(Coit et al., 2005). The presumed distribution of strength is depicted with increasing time i.e., at t_1, t_2 and t_3 respectively. As can be seen, the distribution of strength is degrading over a period of time which in this case is considered to be nonlinear. At any given time, the degradation of strength will follow a particular distribution. In this research, the distribution is assumed to be normal with a non-linearly varying mean and linearly varying standard deviation. In Figure 1, it can be seen that the mean of strength decreases and the standard deviation increases over time. A failure will occur when the strength reaches below the threshold value D. Note that this may not be the case for all failure mechanisms but this behavior is often observed in mechanical components. For any particular time, reliability can then be estimated as the probability that the degradation measure is greater than a critical threshold value.

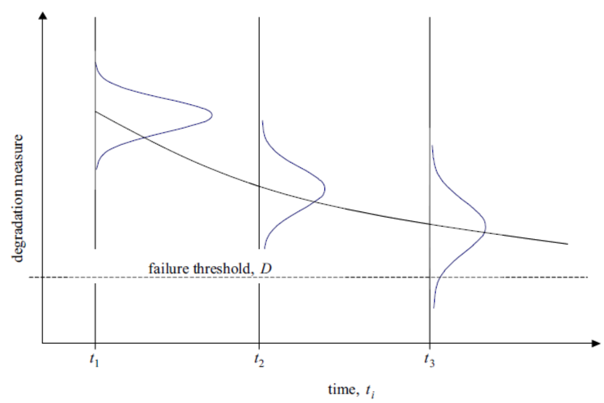

Figure 1. Degradation path example

If the strength is normally distributed,

$$f(x/t) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu(t)}{\sigma(t)}\right)^2\right) \tag{1}$$

$$F_x(x/t) = \int_0^\infty \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{u-\mu(t)}{\sigma(t)}\right)^2\right) du \tag{2}$$

$$= \phi\left(\frac{x-\mu(t)}{\sigma(t)}\right) \tag{3}$$

Where $f(x/t)$ and $F_x(x/t)$ are defined as the probability density function and cumulative distribution function for random strengths conditional on time or number of cycles.

If N components are connected in series, the failure time of a component, T, is the minimum of N individual components failure times.

$$T_i = \text{minimum}\{t; x_i(t) \le D\}\ i = 1,2,3, \dots\dots N$$

$$T = \text{minimum}\{T_1, T_2, T_3 \dots\dots T_N\}$$

Reliability can be given as

$$R(t) = 1 - F_T(t) = \Pr(T > t) = \Pr\{\text{minimum } T_i > t\}$$

$$= \Pr(T_1 > t \cap T_2 > t \cap T_3 > t \dots T_N > t)$$

$$= \Pr(T_1 > t)\Pr(T_2 > t)\ \Pr(T_3 > t)\dots\ \Pr(T_N > t)$$

$$=\left(1-F_x(D/t)\right)\left(1-F_x(D/t)\right)\left(1-F_x(D/t)\right)\ldots\left(1-F_x(D/t)\right)$$

$$R=\left(1-F_x(D/t)\right)^N \tag{4}$$

As studied for various data of SN curves, it has been observed that the quadratic model gives a better fit than a linear model(Sheikh & Khan, 2020; Yan et al., 2000). The parameters μ(t) & σ(t) which are dependent on time are modelled as

$$\mu(t) = a_1 - a_2 t - a_3 t^2 \tag{5}$$

$$\sigma(t) = a_4 + a_5 t \tag{6}$$

The reliability thus becomes time dependent and can be given as

$$R(t) = \Pr(y > D)$$

$$= 1 - F_y(D/t) \qquad \text{- for a single component}$$

$$= 1 - F_y(D/t)^N \qquad \text{-for N components}$$

$$R=\left(1-\phi\left(\frac{D-(a_1-a_2t-a_3t^2)}{a_4+a_5t}\right)\right) \quad \text{- for a single component} \tag{7}$$

$$R=\left(1-\phi\left(\frac{D-(a_1-a_2t-a_3t^2)}{a_4+a_5t}\right)\right)^N \quad \text{- for N components} \tag{8}$$

For the normal distribution example, the conditional probability density function and reliability can be evaluated by estimating the parameters a_1, a_2, a_3 and a_4. If normal distribution is not suitable, other distributions (e.g. Weibull, gamma, exponential, etc.) can be considered depending on the application and past data.

3.1 Maximum likelihood estimation

The method of maximum likelihood has been earlier used by researchers to find the best estimates of the parameters of a probability distribution by maximizing the likelihood function. The likelihood function can be obtained as:

$$L(a/t) = \prod_{i=1}^{m}\prod_{j=1}^{n_i} f_x\left(x_{ij}/t_i\right) \tag{9}$$

$$L(a/x,t) = \prod_{i=1}^{m}\prod_{j=1}^{n_i}\frac{1}{(a_4+a_5t)\sqrt{2\pi}}\exp\left(-\left(\frac{x_{ij}-(a_1-a_2t-a_3t^2)}{a_4+a_5t}\right)^2\right) \tag{10}$$

$$\log(L) = \sum_{i=1}^{m}\log\left(\frac{1}{a_4+a_5t_i}\right) + m\log\left(\frac{1}{\sqrt{2\pi}}\right) - \sum_{i=1}^{m}\sum_{j=1}^{ni}\left(\frac{x_{ij}-(a_1-a_2t-a_3t^2)}{a_4+a_5t}\right)^2 \tag{11}$$

3.2 Jaya Algorithm

Jaya algorithm is one of the recent optimization techniques having the ability to obtain very close estimates of the parameters in a function. The algorithm is simple and is seen to perform very effectively. In this study, Jaya algorithm has been used to maximize the likelihood function 11. Figure 2 below shows the steps for optimization using Jaya algorithm. The number of design variables is taken as 5 and the population size is taken as 10. The iteration number of 500 is considered as the termination criteria.

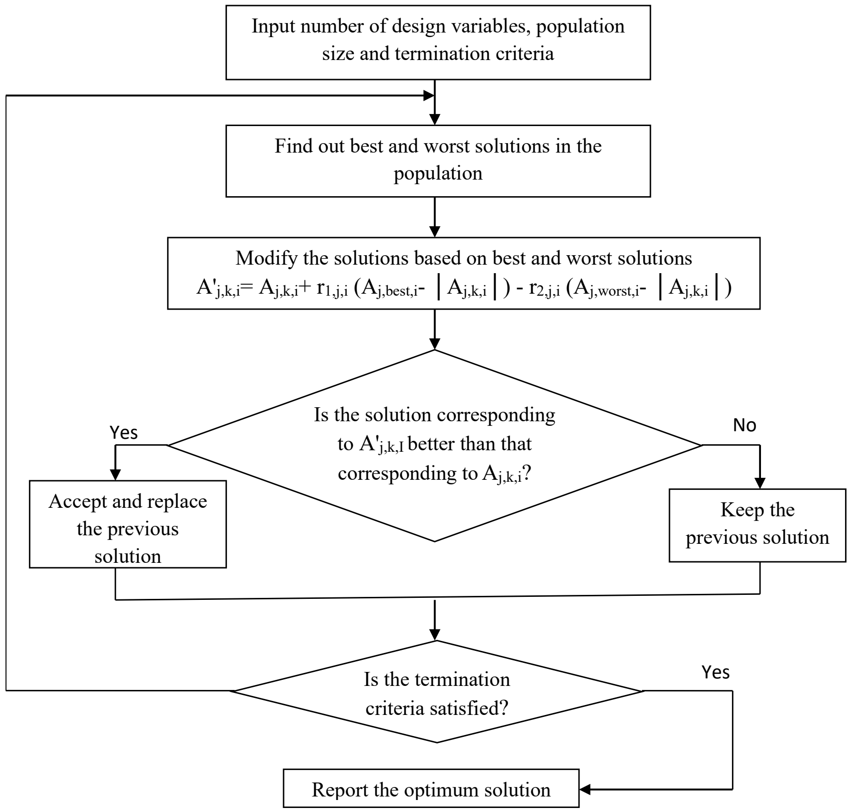

Figure 2. Flowchart for Jaya algorithm

4. Application to Shearing of Solder Joints in Automotive circuits

As we know that Automotive systems have to operate under different conditions like temperature, humidity, etc. in its lifetime. The automotive electrical circuits tend to deteriorate over the number of miles run as a result of working in severe environmental conditions. Thus, the solders in automotive electronic circuits tend to shear over a period of time thus undergoing a failure because of fatigue, creep, etc. A solder connection failure will occur when its strength (measured by a shear strength test) reduces to a critical threshold. The circuit card assembly consisting of a number of solder connections fails when any one of the solder connections deteriorates to the critical level. Therefore, reliability can be defined as the probability that the minimum shear strength, at a particular time, exceeds the critical value. Failure time of circuit card is the minimum of N individual solder connection failure times. Prediction based on reliability is crucial in degradation modeling and thus creating suitable designs for safety and quality. Degradation modeling is based on probability that the predicted distribution of strength intersects the predefined failure threshold limit. The proposed methodology has been applied to the data for shearing of solder joints in automotive circuits with 7 solder resistors in series which was first mentioned by Coit et al. (2005). The time t_i is considered as the number of miles the car travels before the occurrence of a failure and x_{ij} is the shear strength. It was also shown that the normal distribution gives a suitable fit for the strength data of individual circuit boards.

Using the proposed methodology, the estimates for the parameters obtained are shown in Table 2. It can be noted that at time t_i=0, the mean equals to a_1 i.e., 6.029325. Parameters a2, a3, a4 and a5 are found out to be 0.0000067502, 4.675394 x 10^{-11}, 0.479823 and 4.56611 x 10^{-06} respectively. The maximum function value is obtained as -205.1022 and the time taken for the compilation is 0.78237 seconds. It can be observed that the function converges to real roots after around 80 iterations. The scatter plot of the data along with the fitted line plot is shown in Figure 3. As can be seen in the figure, as the mileage increases, the mean of strength decreases. Also, it can be observed that the spread increases with an increase in mileage. The convergence behavior of Jaya algorithm is shown in Figure 4.

Table 2. Estimation using proposed methodology

a_1	6.029325
a_2	0.0000067502
a_3	4.675394×10^{-11}
a_4	0.479823
a_5	4.56611×10^{-06}
fmax	-205.1022
Compilation time (s)	0.78237

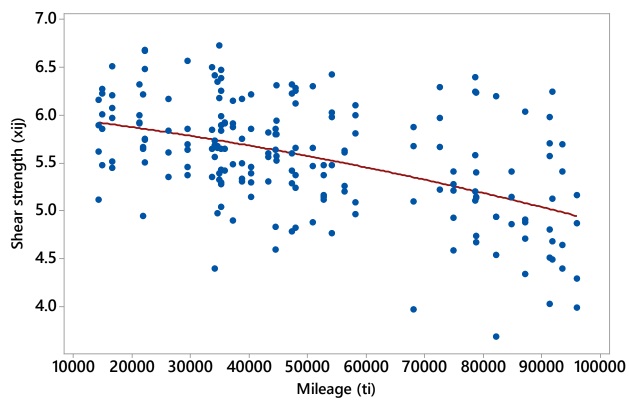

Figure 3. Scatter and fitted line plot for shearing strength vs time to shear

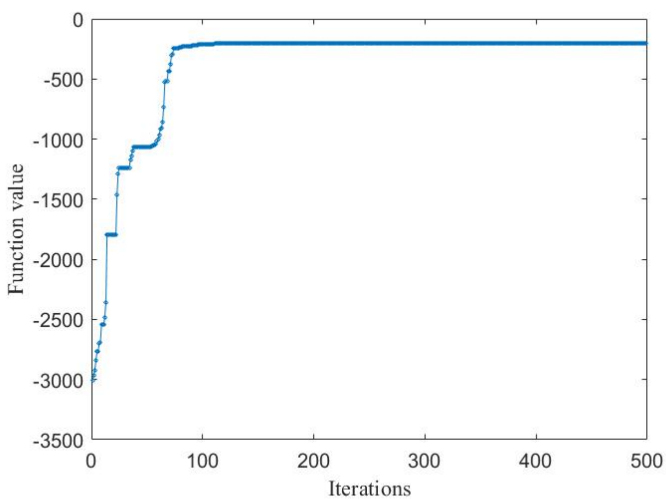

Figure 4 Convergence behaviour of Jaya algorithm

As per the literature by experimental examination, a failure threshold of 4.0 was determined. The time-dependent reliability thus, can be given as

$$R(t) = \left(1 - \phi\left(\frac{4 - (6.029325 - 0.0000067502t - 4.675394 \times 10^{-11}t^2)}{0.479823 + 4.56611 \times 10^{-6}t}\right)\right)^N$$

The plot for reliability vs mileage is shown in Figure 5. The reliability starts decreasing significantly after around 20000 cycles. A steep fall can be observed for reliability after around 50000 cycles. This figure shows that the reliability in the existing literature may be overestimated.

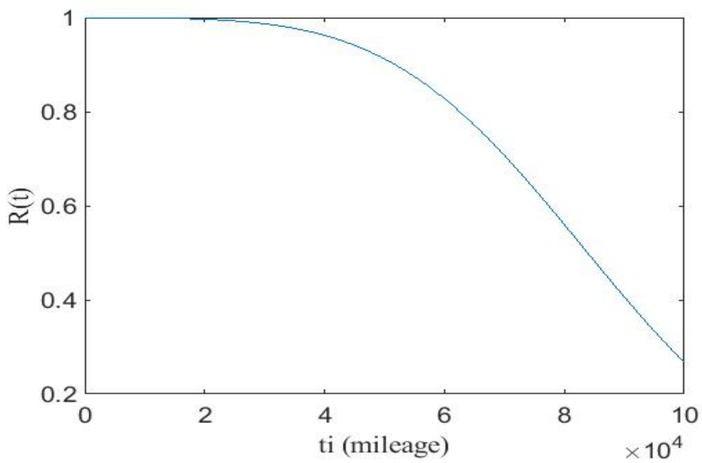

Figure 5. Reliability vs mileage

5. Conclusion

In this study, a methodology has been proposed for evaluating the time-dependent reliability of a component subjected to deterministic stress. A nonlinear model for the mean and linear model for standard deviation has been considered. The maximum likelihood estimation via Jaya algorithm has been used to obtain the estimates of the parameters. The method has been applied to the shear strength data of automotive circuits and a plot for reliability vs time has been obtained. It has been observed that the proposed methodology is simple, fast and gives very good results.

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