Simulated Annealing and Tabu Search based Hybrid Algorithm for Multi Depot Vehicle Routing Problem with Time Windows and Split Delivery

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Abstract

Effective co-ordination of distribution operations of a manufacturing organization is paramount since it represents the major cost component of the logistic supply chain. In order to deal with high demand of orders with shorter lead time, distributed warehouse concept is introduced and this is currently being practiced. Furthermore, due to inherent deficiencies in VRP, rules have been extended to accommodate large scale orders by splitting the delivery. Artificial Intelligences (AI) based approaches widely used in the literature to solve VRP problems with extensions. In this research, hybrid algorithm (SATS) is developed based on Simulated Annealing (SA) and Tabu Search (TS) techniques to improve the solution quality of the complex Multi Depot Vehicle Routing Problem with Time Windows and Split Delivery (MDVRPTWSD). The simulation results reveal that SATS outperform in solution quality and the computational time.

Keywords
Artificial Intelligent, Split Delivery, Supply Chain.

1. Introduction

Co-ordination of transportation and distribution of consumer products is a vital issue in managing the Supply Chain efficiently and it is very important to deal with the modern concepts such as Lean, JIT etc. Supply Chain spans the all activities from material extraction stage to dispatching of finished product to customer in manufacturing industry. In the field of manufacturing, high transportation cost is one of the critical issues since it consumes significant amount from the overall production cost. Meeting with the company goals while overcoming the shortages in transportation is very challengeable due to the complexity of the problem. However, there are number of standard problems defined in the area of transportation and distribution such as VRP, TSP etc.

A typical VRP can be defined as an allocation of customers to feasible sets of routes, where route consists of a vehicle and each dealer is visited only once by exactly one vehicle and all vehicles start and end at the same single depot. Though the classical VRP deals with the single depot, it is very difficult to handle huge network with large number of customers varying demands. Therefore, in order to handle complicated networks effectively, intermediate depots have been introduced and the problem related to such a distribution network is known as Multi depot vehicle routing problem (MDVRP). Sometimes it is not realistic that a customer's demand must be delivered by a single vehicle. Therefore, by allowing deliveries to be split, customer can be served by more than one vehicle. Benefits of bringing up the concepts such as multiple warehouses and split delivery are very practical in current distribution operation and also are the strategies to reach the objective of this study such as minimize overall transportation cost while fully utilizing the vehicle space.

However, existence of above condition make the problem more complex and it would be difficult to solve within a reasonable time window. Due to computational complexity of the problem this is mathematically termed as NP hard problems. Heuristics such as Genetic Algorithm (GA), SA, TS are highly attractive for a complex problem to find a better solution within reasonably less computational time. In addition, integrating the properties of approximation algorithm leads to a better solution than applying one alone. It has been found in the literature, TS emerged as an effective algorithmic approach for the VRP. However, the quality of the solution found by the TS depends on the initial solution. To overcome this problem and to provide a robust and efficient methodology for MDVRPTWSD, the heuristic search approach combining SA and TS strategy is developed.

The remainder of the paper is organized as follows. In Section 2, summary of the literature review is presented and in Section 3, the definition of the problem is given with the mathematical representation. The method adopted is described in Section 4, at the beginning it explains the initial stage of the proposed method and thereafter improvement of the initial solution is described. In Section 5, the simulation results are presented. This is followed by conclusion.

2. Literature Review

Numbers of studies have been found in the literature related to MDVRP with many extensions, the objective of the studies of MDVRP found in the literature were similar with the objectives of VRP such as minimizing transportation cost, reducing the lateness, minimizing the number of vehicles used without being over capacitated etc. Due to high computational complexity of such problems, most researchers are adopted with AI techniques to find good solution within reasonable time.

Telfar (1994) presented an overview on applicable heuristic for global optimization while investigating the algorithm performance on TSP. Ho and Haugland (2004), Belfiore and Yoshizaki (2008) considered VRPTW with split delivery (VRPTWSD). Ho and Haugland (2004) have developed a tabu search heuristic for VRPTWSD and they showed that heuristic is favour in terms of number of vehicle and total distance traveled. Moreover, they emphasised the significance of having split the delivery. Belfiore and Yoshizaki (2008) studied the same problem with heterogeneous fleet of vehicle for real life application. They have presented a scatter search algorithm which is capable of offering better solution in terms of distribution cost and number of trucks used compared to current practice. William (2008) addressed Multi depot vehicle routing problem and introduced a hybrid genetic algorithm. They presented the hybrid algorithm in two stages concluding the second phase is more efficient than the first phase. Pisinger and Ropke (2005) presented an unified heuristic which is able to solve five different variant of VRP. Jeon et al. (2007) solved VRP using hybrid genetic algorithm suggesting a mathematical programming model with new numerical formulas. The process of hybrid genetic algorithm additionally considers the initial population by using both random generation and heuristic techniques. Bell and McMullen (2004) used Ant colony optimization techniques for VRP. Further it is compared with the other varities of AI techniques such as TS, SA, GA. It is conclude that the algorithm is successful in finding solutions within 1% of known optimal solutions. Nagy and Salhi (2005) presented a new heuristic for VRP with pickup and delivery system.

3. Problem Formulation

Problem formulation defines the problem and the notation used throughout the study.

- Customers: A distribution network consists of sets of customers $C = \{1,2,\ldots\ldots\ldots n\}$ residing at "n" different locations, catered by distributed ware houses (WH_j) where $j = 1\ldots\ldots m$ and $m<n$. A customer is defined with a demand (q_i) where $q_i>0$ with the time window i.e an interval $[a_i,b_i] \in R$ where a_i and b_i are the earliest and latest time to start to service the customer . Every pair of customers (i,j) where $i,j \in C$ and $i \neq j$ is associated with a traveling distance d_{ij} and traveling time t_{ij}.

- Vehicle: A fleet of homogeneous vehicle with the capacity of V_c scattered throughout the network, allowing to start from any depot and reach the place where it is requested. In this study, the demand of a customer may be fulfilled by more than one vehicle when the demand of the customer exceeds the vehicle capacity which known as split delivery.

The MDVRPTWSD can be stated as;
For each arc (i,j) where i,j € C , $i \neq j$ and for each vehicle k, we define X_{ijk} and Y_{ik}.

$$X_{ijk} = \begin{cases} 1 \text{ ; If the vehicle k travel to demand point j directly from demand point i} \\ 0 \text{ ; Otherwise} \end{cases}$$

$$Y_{ik} = \begin{cases} 1 \text{ ; If the demand point i is visited by vehicle k} \\ 0 \text{ ; Otherwise} \end{cases}$$

P_{ik} - Delivery amount at demand point i by vehicle k
q_i - Demand at customer i
a_i - Earliest arrival time of customer i
b_i - Latest arrival time of customer i

$$\text{Minimize } Z(d) = C \sum_{i=0}^{N} \sum_{j=0}^{N} d_{ij} \sum_{k=1}^{u} x_{ijk} \tag{1}$$

$$\sum_{k=1}^{u} p_{ik} = q_i \tag{2}$$

$$p_{ik} \leq q_i y_{ik} \tag{3}$$

$$a_j^k \leq a_j^k + t_{ij} \leq l_j^k \tag{4}$$

$$a_j^k \leq l_j^k + t_{ij} \leq l_j^k \tag{5}$$

$$\sum_{i,j=0}^{n} x_{ijk} q_i \leq v_c \tag{6}$$

Equation (1) provides the cost function of total distance traveled. Therefore (1) is selected as the objective function in the view of minimizing the total distance travelled. This mathematical model has several limitations and they are given as the constraints in the model. Constraint (2) ensures that a vehicle only serves the demand point visited by the vehicle. Constraint (3) guarantees that the demand of each demand point is totally satisfied. Constraint (4) and (5) refers the time window of a customer and (6) ensures that customer demand does not exceed the vehicle capacity.

4. Methodology

Proposed methodology is arranged in a two phases as shown in Figure 1. First phase describes the generation of initial solution and second phase optimizes the initial solution using promising heuristics such as TS and SA and SATS algorithm.

4.1 Initialization

The pseudocode of the initialization is illustrated in the Figure 2. At the very beginning of the process, it is required to get all the information about customers (i.e. location, distance in between each customers and warehouses, demand, etc). Based on the above, the shortest distance of each demand point is generated by using Dijsktra algorithm. All received tasks have to be processed in such a way that tasks are separated according to the date. Thereafter, they are sorted out by earliest arrival time in order to give the most priority to urgent task. The selection of warehouse (depot) is done considering the closeness to the first task of the prepared task list and the availability of the goods in the warehouse. If the vehicle is capable of loading more tasks, they are selected based on distance. Moreover, the selected first closest task should satisfy the time window or else algorithms consider the next closest task. This will repeat until the vehicle is fully loaded with the tasks and start with the next vehicle until the all tasks are assigned.

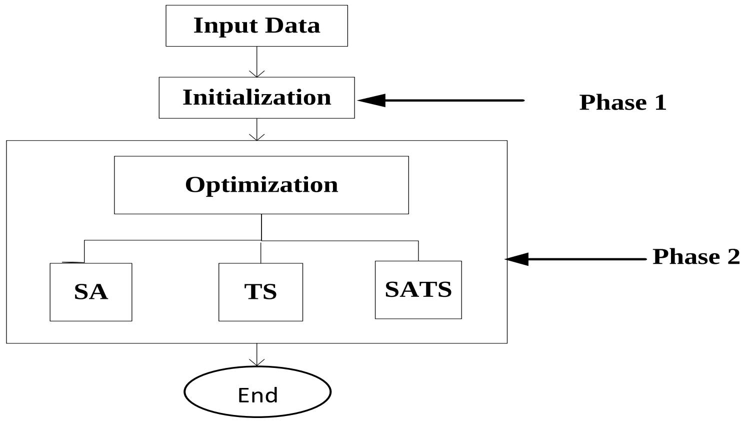

Figure 1: Basic steps of the proposed methodology

Step 1: Process all arrived tasks 1. Separate tasks day wise
2.Sort them by earliest arrival time
Step 2: Start from the first task of the sorted task list. (i=1)
Step 3: Find the tasks which exceed the vehicle capacity. Those deliveries are to be split.
Step 4: Calculate the remaining space after selecting the first task
Step 5: Do while until the remaining space >0
Select the tasks compatible with the remaining space
Find the shortest distance from the i^{th} task to all feasible tasks
Select the most closest task
If the tasks satisfy the time frame; select that tasks.
else go to next closest task
end
End
Step 6: Terminate the program after inserting all tasks to routes.

Figure 2: Pseudocode of Initialization

4.3 Implementation of Hybrid Algorithm

- SA: is developed analogous to physical process of annealing with high diversification search where diversification can be controlled by varying the cooling rate. Moreover, SA accepts the candidate solution probabilistically by the metropolis acceptance criterion, provides a procedure to find sufficient good solution over the solution space. Therefore, hybrid SATS algorithm is developed in such a way that SA is used to find the promising elite solution inside the solution space and TS intensifies search around these solution.

- TS: is a meta-heuristic that guides a local search procedure to explore the solution space beyond the local optimality. There are number of local search methodologies which can be used with TS. Local procedure is a search that uses move operation, to define the neighborhood of any given solution. One of the main components of TS is its adaptive memory. Thereby, many combinations can be checked in the second stage and capability of storing many solutions is possible with tabu search. Selection of best solution and replacing the worst by better solution are also advantages of TS.

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Step 1: Generate an initial solution by greedy approach. Initial solution consists of m routes.
Step 2: Get the first route Ri (I = 1). Assume route (Ri) consists of N customers (1…….N)
Step 3: For i = 1 : m
            For j = 1 : m
                If i ~= j
            Exchange each customer and calculate total distance that has to be travelled after exchanging
            If the current solution is greater than the initial one, do not make that change
            else
            Check the feasibility :  time window and total delivery quantity
            End
                End
            End
      End
Step 4: Terminate the program and print the final routes.
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Figure 3: Pseudocode of TS heuristic

- **SATS:** SA inherent with good convergence property but lacking with keeping previously generated solution. However, due to lack of memory, search may oscillate around the local minimum. Therefore, incorporating a memory facility; one of the characteristics of TS, leads to chance of getting prominent solution in the solution space.
  The solution generated by the greedy approach is optimized by the SATS algorithm. Interchange move operator is used in neighborhood search mechanism while checking the validity of time windows and vehicle capacity. If the current neighborhood solution is feasible and satisfies: (Current solution < Best solution) or probability exp (best solution – current solution)/ T > random [0,1], then the solution is pushed into memory. Temperature T of SA has an important influence on the selected solution and consequently affects the quality of the solution provided by the SATS algorithm. If the temperature is too low, the algorithm may be terminated. It has been seen that the temperature decreases as the best tour length being found.

```
Step 1: Generate an initial solution by greedy approach.
Step 2: Initialize parameters, Initial temperature T start = T, empty tabu list, Set cooling rate, ending
        temperature, iter = 0.
Step 3 : Do neighborhood search (interchange move operator).
Step 4: Check the feasibility (i.e: Time window and Vehicle capacity).
Step 5: If it is not satisfied: Select the best neighbor which is not tabu.
Step 6: If the best neighbor is feasible and satisfy aspiration criterion (current solution < best solution) OR
        (exp(best  solution - current solution)  / T ) >  rand [0,1].
Step 7: Update tabu list and set temperature, set the best tour length.
Step 8: Tabu list size exceed, Replace the worst solution by a better solution.
Step 9: If Temp < Tend | iter is terminated. Go to step2.
Step 10: If the termination criterion satisfied. Stop.
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Figure 4: Pseudocode of SATS heuristic

5. Simulation and Results

Randomly generated data sets with different problem sizes are used in the case studies. Numbers of simulations were carried out on MDVRPTWSD problem with the heuristics such as SATS, SA and TS. As reported in Table 1 SATS shows significant improvement in initial solution than other two algorithms. Moreover, newly developed SATS algorithm could generate a promising solution considerably less computational time. Therefore, it is convinced that combination of characteristics of two heuristics leads to promising solution. In addition, comparison between results from TS and SA reveals that, TS is outperforming in solution quality.

Problem size	Case	Parameters No. of vehicle /Collection	Heuristics (Distance/ Computational time(s))			
			Initial	TS	SA	SATS
20	1	4/150	103/7.99	72/154.39	94/79.97	84/73.31
	2	6/ 210	135/ 9.02	100/150	145/166.7	95/110
	3	5/195	150/ 9.12	135/232	142/243	120/134
35	1	7/249	79/9.73	69/396	117/486	74/289
	2	7/268	91/10.43	84/112.12	131/154	79/100.77
	3	8/300	110/11.11	100/132	111/168.68	90/122
50	1	8/318	142/13.12	130/134	135/189	122/102
	2	7/ 250	89/9.45	80/112	84/166	75/90
	3	10/355	105/12.13	93/98.15	102/102.56	89/96.77

Table 1 : Comparison between TS, SA and SATS heuristics

6. Conclusions

In this paper, MDVRPTWSD problem is solved using different heuristics such as TS, SA and SATS where SATS is a heuristic which consists of characteristics in TS and SA. Once initial solution is generated by greedy approach, based on traveling distance, time window and order size, the meta heuristic techniques were used to find near optimal solution. Numbers of experiments were conducted to investigate how each heuristic technique performs on the problem. The results reveal that the number of vehicle and total travelled distance can be reduced significantly by SATS compare to SA and TS. Thereby, it can be concluded that hybrid algorithms perform well in complex problems.

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