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Construction and Selection of Tightened-Normal-Tightened Schemes of Type TNT-(n_1, n_2; c) indexed through Six Sigma Quality Levels

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Abstract

"Six Sigma is a tool used to convert management problem into a statistical problem and to find a statistical solution then convert it to a management solution". Six Sigma is a set of practices originally developed by Motorola to systematically improve processes by eliminating defects. In particular processes that operate with Six Sigma quality initiatives produce 3.4 defects or below per (one) million opportunities (dpmo). In this paper a new procedure for the construction and selection of Tightened-Normal-Tightened sampling scheme of type TNT-(n_1, n_2; c) indexed through Six Sigma Quality Level-1(SSQL-1) and Six Sigma Quality Level-2 (SSQL-2) are presented. Tables are also constructed and presented for the easy selection of the plans.

Keywords
Six Sigma Quality Levels, Tightened-Normal-Tightened sampling scheme, Poisson distribution and Operating Characteristic curve.

1. Introduction

A single defective unit of the sample, in a compliance testing, calls for rejection of the entire lot. To overcome this undesirability, Tightened-Normal-Tightened (TNT) sampling scheme was developed by Calvin [1]. This scheme utilizes two single sampling plans (with c = 0) of different sample sizes together with switching rules to create swelling in the upper portion of the OC curve. Soundararajan and Vijayaraghavan [2] studied the TNT scheme by assuming 'c' to take values other than zero, the scheme can be designated as TNT-(n_1, n_2; c) which refers to a TNT scheme where the normal and tightened single sampling plans have the acceptance number c, but on tightened inspection the sample size is n_1 and on normal inspection the sample size is n_2 ($< n_1$). Radhakrishnan and Sivakumaran [3-7] constructed sampling plans based on Six Sigma Quality Levels by taking Poisson distribution as the base distribution. Further Radhakrishnan [8] constructed Six Sigma based Single Sampling Plans indexed through Six Sigma Quality Levels with Poisson Distribution, Weighted Poisson Distribution and IRPD as the base line distributions.

In this section a Tightened-Normal-Tightened sampling scheme of type TNT-(n_1, n_2; c) is constructed by assuming the probability of acceptance of the lot, $P_a(p)$ as 1-3.4 x 10^{-6}, the concept of Six Sigma quality suggested by Motorola [9]. The proportion defective corresponding to this probability in the OC curve is termed as Six Sigma Quality Level-1 (SSQL-1). This new sampling plan is constructed with a point on the OC curve (SSQL-1, 1-α_1), where $\alpha_1 = 3.4 \times 10^{-6}$ is similar to (AQL, 1-α). The proportion defective corresponding to the probability $2\alpha_1$ in the OC curve is termed as Six Sigma Quality Level-2 (SSQL-2). Sampling plans can also be constructed with a point on the OC curve (SSQL-2, β_1), where $\beta_1 = 2\alpha_1$ is similar to (LQL, β). The tables are also provided for the easy selection of the plans using the above concepts.

2. Operating Procedure

Step 1: Inspect under tightened inspection, using the single sampling plan with sample size n_1 and acceptance number c. If t lots in a row are accepted, switch to normal inspection (Step 2).

Step 2: Inspect under normal inspection using the single sampling with sample size n_2 (< n_1) and acceptance number c. Switch to tightened inspection, after a rejection, if an additional lot is rejected in the next s lots.

Thus, the TNT-(n_1, n_2; c) sampling scheme is specified by the parameters n_1, n_2, c, s and t, constitute the criteria for switching to tightened and normal inspection respectively.

3. Conditions for Application

- The production is steady so that results of past, present and future lots are broadly indicative of a continuing process.
- Lots are submitted sequentially in the order of production.
- Inspection is attributes with stable quality between lots for calculation of the OC curves.
- The sample units are selected from a big lot and production is continuous.
- Human involvement should be less in the manufacturing process.
- The Company adopts Six Sigma quality initiatives in its processes.

4. Glossary of symbols

The symbols used in this paper are as follows:

n_1 - Sample size under tightened inspection plan
n_2 - Sample size under normal inspection plan
c - Acceptance number
s - Criterion for switching to tightened inspection
t - Criterion for switching to normal inspection
P_1 - Probability of acceptance under tightened inspection
P_2 - Probability of acceptance under normal inspection
AQL - Acceptable Quality Level
LQL - Limiting Quality Level
SSQL-1 - Six Sigma Quality Level-1
SSQL-2 - Six Sigma Quality Level-2
α - Producer's risk
β - Consumer's risk
α_1 - Modified producer's risk
β_1 - $2\alpha_1$ (Modified consumer's risk)

5. Operating Characteristic (OC) function

Under Poisson model the OC function of the TNT scheme given by Calvin (1977) is

$$P_a(p) = \frac{P_1(1-P_2^s)(1-P_1^t)(1-P_2) + P_2P_1^t(1-P_1)(2-P_2^s)}{(1-P_2^s)(1-P_1^t)(1-P_2) + P_1^t(1-P_1)(2-P_2^s)} \qquad (1)$$

Under the conditions for application of Poisson model for the OC curve, P_1 and P_2 are defined as

Where $P_1 = \sum_{x=0}^{c} \frac{e^{-n_1 p}(n_1 p)^x}{x!}$, $P_2 = \sum_{x=0}^{c} \frac{e^{-n_2 p}(n_2 p)^x}{x!}$

The OC function of TNT scheme corresponds to the scheme OC function of MIL-STD-105D for s =4 and t =5 [Hald and Thyregod [10]; Dodge [11]; Calvin [1]].

6. Construction of TNT-(n_1, n_2; c) schemes for a specified SSQL-1

By fixing the probability of acceptance $P_a(p)$ of the lot provided in equation 1, as 1-3.4 x 10^{-6} with Poisson Distribution as the basic distribution, the values of nSSQL-1 are obtained for various combinations of 'c' and 'k' using a Visual Basic program and are presented in Table 1. The sample size 'n = n_2'of the normal plan is obtained as n_2 = nSSQL-1/SSQL-1 and then the sample size n_1 of the tightened plan is found as $n_1 = kn_2$ (k > 1). Hence the parameters of the TNT-(n_1, n_2; c) schemes n_1, n_2 and c are obtained for various values of SSQL-1.

The sigma levels of the process are calculated using the Process Sigma Calculator [12] by providing the sample size and the acceptance number.

Example 1

For a given SSQL-1 = 0.000002, c = 1 and k = 2.0 the value of nSSQL-1 is selected from Table 1 as 0.0026088 and the corresponding sample size of normal plan 'n_2' is computed as n_2 = 0.0026088/0.000002 = 1305 and the sample size of tightened plan n_1 is computed as n_1 = (2.0)(1305) = 2610 which are associated with 4.7 and 4.9 sigma levels respectively. Hence the parameters of TNT-(n_1, n_2; c) are n_1 = 2610, n_2 = 1305 and c =1 are obtained for a specified SSQL-1 = 0.000002.

Practical application

In a cell phone manufacturing company, if the manufacturer of cell phones fixes the quality of cell phones as SSQL-1 = 0.000002 (2 non conforming cell phones out of 1 million items), then inspect under tightened inspection with sample of size 2610 cell phones and acceptance number c = 1 from the manufactured lot of a particular month. If 5 lots in a row are accepted under tightened inspection, then switch to normal inspection. Again inspect under normal inspection with a sample of 1305 cell phones and acceptance number c = 1 from the manufactured lot of a particular month. Switch to tightened inspection, if an additional lot is rejected in the next 4 lots and inform the management for corrective action.

Example 2

For a given SSQL-1 = 0.000025, c = 2 and k = 2.25 the value of nSSQL-1 is selected from Table 1 as 0.0275100 and the corresponding sample size of normal plan'n_2' is computed as n_2 = 0.0275100/0.000025 = 1101 and the sample size of the tightened plan n_1 is computed as n_1 = (2.25)(1101) = 2478 which are associated with 4.4 and 4.7 sigma levels respectively. Hence the parameters of TNT-(n_1, n_2; c) are n_1 = 2478, n_2 = 1101 and c =2 are obtained for a specified SSQL-1 = 0.000025.

The OC curves of the schemes provided in Example 1 and Example 2 are presented in Figure 1.

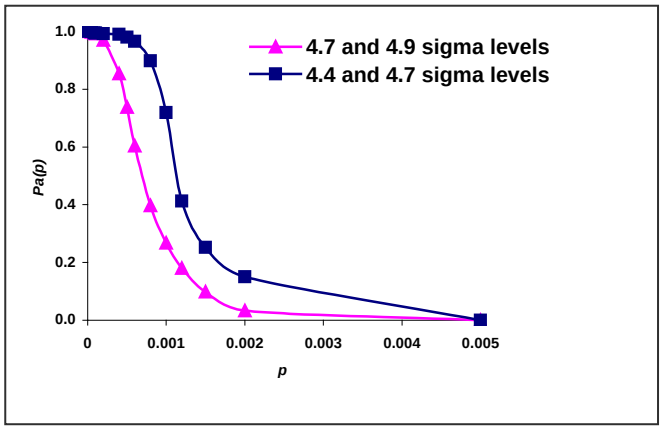

Figure 1: OC curves for the schemes n_1 = 2610, n_2 = 1305, c = 1; n_1 = 2478, n_2 = 1101, c = 2;

7. Construction of TNT-(n_1, n_2; c) schemes for a specified SSQL-2

By fixing the probability of acceptance $P_a(p)$ of the lot provided in equation 1, as $\beta_1 = 2\alpha_1$ where $\alpha_1 = 3.4 \times 10^{-6}$ with Poisson Distribution as the basic distribution, the values of nSSQL-2 are obtained for various combinations of 'c' and 'k' using a Visual Basic program and are presented in Table 2. The sample size 'n = n_2' of the normal plan is obtained as n_2 = nSSQL-2/SSQL-2 and then the sample size 'n_1' of the tightened plan is found as $n_1 = kn_2$ (k >1). Hence the parameters of the TNT-(n_1, n_2; c) schemes n_1, n_2 and c are obtained for various values of SSQL-2.

The sigma levels of the process are calculated using the Process Sigma Calculator [12] by providing the sample size and acceptance number.

Example 3

For a given SSQL-2 = 0.008, c = 2 and k = 2.0 the value of nSSQL-2 is selected from Table 2 as 8.4940 and the corresponding sample size 'n_2' is computed as n_2 = 8.4940/0.008 = 1062 and n_1 = (2.0)(1062) = 2124 which are associated with 4.4 and 4.6 sigma levels respectively. Hence the parameters of TNT-(n_1, n_2; c) are n_1 = 2124, n_2 = 1062 and c = 2 are obtained for a specified SSQL-2 = 0.008.

Practical application

In a cell phone manufacturing company, if the distributor of cell phones fixes the quality of cell phones as SSQL-2 = 0.008 (8 non conforming cell phones out of 1 thousand), then inspect under tightened inspection with sample of size 2124 cell phones and acceptance number c = 2 from the manufactured lot of a particular day/week. If 5 lots in a row are accepted under tightened inspection, then switch to normal inspection. Again inspect under normal inspection with a sample of 1062 cell phones and acceptance number c = 2 from the manufactured lot of a particular day/week. Switch to tightened inspection, if an additional lot is rejected in the next 4 lots and inform the management for corrective action.

Example 4

For a given SSQL-2 = 0.009, c = 1 and k = 1.50 the value of nSSQL-2 is selected from Table 2 as 9.7660 and the corresponding sample size 'n_2' is computed as n_2 = 9.7660/0.009 = 1086 and n_1 = (1.50) (1086) = 1629 which are associated with 4.6 and 4.7 sigma levels respectively. Hence the parameters of TNT-(n_1, n_2; c) are n_1 = 1629, n_2 = 1086 and c = 1 are obtained for a specified SSQL-2 = 0.009. The OC curves of the schemes provided in Example 3 and Example 4 are presented in Figure 2.

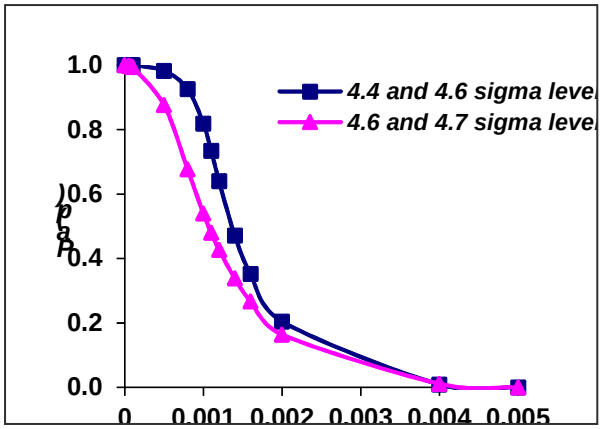

Figure 2: OC curves for the schemes n_1 = 2124, n_2 =1062, c = 2; n_1 = 1629, n_2 = 1086, c =1.

8. Conclusion

In this chapter a new procedure for the construction and selection of Tightened-Normal-Tightened sampling scheme of types TNT-(n_1, n_2; c) and TNT-(n; c_1, c_2) indexed through Six Sigma Quality Level-1(SSQL-1) and Six Sigma Quality Level-2 (SSQL-2) single points are presented. Tables are also constructed and presented for the easy selection of the plans. The Plans suggested in this paper can replace the existing sampling plans, because those plans are constructed based on three sigma levels only. If these sampling plans are used, all the lots submitted for inspection will be accepted (including the bad lots). As more and more companies started adopting Six Sigma initiatives, it is absolutely necessary to use the plans suggested in this paper for inspection. This will not only help the producers in providing better quality but also increase the satisfaction and confidence of the consumers. The sampling plans based on Six Sigma Quality Levels are designed not only to the manufacturing industries alone, but also to the other industries such as Health care, Software, Courier service, Transport service, Insurance, Financial / Investment Services, E-commerce industry and so on. If all the companies practicing with Six Sigma quality initiatives started using the plans suggested in this paper, then not only the quality and confidence on their product will reach its new height but also enhance the satisfaction of the consumer. These plans are very useful to the companies because the producer's risk and consumer's risk are less, which results in more satisfaction for the consumers and higher profit for the manufacturers.

Table 1: Parameters of TNT-(n_1, n_2; c) schemes for a specified SSQL-1

c	k	nSSQL-1
0	1.25	0.0000034
	1.50	0.0000034
1	1.25	0.0026000
	1.50	0.0026100
	1.75	0.0026090
	2.00	0.0026088
	2.25	0.0026086
	2.50	0.0026084
	2.75	0.0026082
	3.00	0.0026081
2	2.00	0.0275200
	2.25	0.0275100
	2.50	0.0275100
	2.75	0.0275000
	3.00	0.0274990
3	2.25	0.0968900
	2.50	0.0969000
	2.75	0.0968800
	3.00	0.0968700
4	2.50	0.2177000
	2.75	0.2176500
	3.00	0.2176300
5	2.50	0.3880000
	2.75	0.3879000
	3.00	0.3878000

Table 2: Parameters of TNT-(n_1, n_2; c) schemes for a specified SSQL-2

c	k	nSSQL-2
0	1.25	9.5190
	1.50	7.9320
1	1.25	7.6750
	1.50	9.7660
	1.75	8.3712
	2.00	7.3245
	2.25	6.5105
	2.50	5.8595
	2.75	5.3270
	3.00	4.8830
2	2.00	8.4940
	2.25	7.5500
	2.50	6.7950
	2.75	6.1775
	3.00	5.6625
3	2.25	8.4975
	2.50	7.6480
	2.75	6.9525
	3.00	6.3730
4	2.50	8.4480
	2.75	7.6800
	3.00	7.0400
5	2.50	9.2100
	2.75	8.3730
	3.00	7.6750

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