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BAYESIAN INFORMATION UPDATE FOR RETAILER'S TWO-STAGE INVENTORY MODEL UNDER DEMAND UNCERTAINTY

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ABSTRACT

This paper investigates demand forecast and order placement policy of a seasonal product for the retailers in an active business season. In a two-phase demand model, the market demand is uncertain, and decision-making for order placement is addressed for the second phase after observing the demand at the first phase during the active season. The model uses a Poisson process for customer demand and a gamma prior distribution for the demand rate variation. The method addresses the updating of the prior probability to a posterior one dictating by Bayesian process. The experts' opinion for the peak demand is incorporated in parameter updating process. The numerical analyses are discussed and conclusion is presented.

Key words: Seasonal products, Active business season; Bayesian process.

1. INTRODUCTION

This study is motivated by the decision-making problem that retailers face for ordering seasonal products from distributors during an active sale season. In this approach, the active sale season is partitioned into two phases. The demand information is gathered from a short sale period during the early phase of the active season and demand is anticipated for the later phase. In a multi-phase model, Bayes' rule is effective in decision making for later phase, after observing the activities at the earlier phase(s). Products such as winter jacket, woolen apparel have short shelf-life and unpredictable market demand is considered here. The parameters of the demand model are dependent on sales at early phase, past years' sales, and expert's opinions about the demand during the active season applying the Bayesian procedure. The decision-making plan for retailers' inventory replenishment is made, such that the revenue is maximized while a high level customer service is maintained during the active sale season.

Demand uncertainty often influences the decision of the retailer's inventory management system. Recently, a number of researches considered two-stage inventory stocking problems with demand information updates include Atamtürk and Zhang (2007), Lau and Lau (2004), Chio *et al.* (2003), Pertuzzi and Monahan (2003), , Teunter and Haneveld (2002). All decisions variables associated with the realization of demand uncertainty, prediction and products procurement plan are decided after observing the market demand. Researchers used a quick response policy in order to reduce ordering lead time and quick adjustment of stocking decision according to the market change (Iyer and Bergen, 1997). Agrawal and smith (2003) developed a linear programming model for retailers based on customer's preference on color, brands or set structure for fashion apparel. Atamtürk and Zhang (2007) proposed a two-stage robust optimization approach to inventory replenishment and location-transportation problems with uncertain demand. Other models contributed in developing retailer's fashion goods procurement decision model using up-stream and down-stream resources (Fisher and Raman, 1996; Gilbert and Ballou, 1999; Chio *et al.*, 2004).

The proposed model can be applied to develop decision making plans for many retail products and services. The model can be used to predict demand of the products with little market information and the cutting edge product such as computer hardware devices, CD writers, DVD burner. In addition, the model can be applied to non-industrial businesses such as tickets for art exhibition, or special sports events, airline tickets. The objective here is to predict the demand of the seasonal products for the active selling season and to demonstrate a replenishment plan to procure product in order to maintain a high customer service.

The outline of this paper is as follows. The concept of two-phase demand problem is described in Section 2. Experts' subjective judgment in the prior demand model is analyzed in Section 3. The optimal inventory replenishment policy and numerical examples are presented in Section 4. Conclusion is illustrated in last section.

2. PROBLEM DESCRIPTION

In a two-phase demand model during an active selling season, retailers acknowledge stochastic demand over a finite time horizon [0, *T*]. The current demand is realized for a short period at Phase-A, and predict demand for phase-B. It is assumed that the demand pattern follows a Poisson process with an unknown demand rate. The uncertainty of the demand rate is described by a probability distribution. The future demand rate is estimated in a Bayesian fashion based on the current observation at Phase-A, experts' view about the upcoming season sales, and the past data. The structure of the demand model is shown in Figure 1.

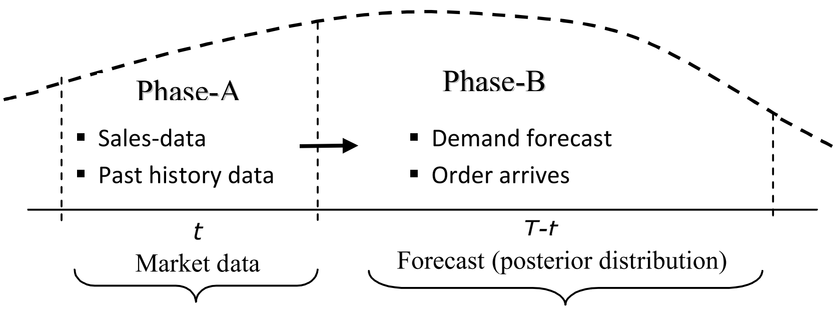

Figure 1: Two-phase Inventory Model for Retailers

In a two-phase model, the Bayes' technique plays a major role to forecast demand for the Phase-B. The analysis is concerned with the updating of data observed at Phase-A. Using the parameters vector θ, and observation demand at Phase-A (D_A), the probability of θ prior to collection data is updated to the posterior probability $p(\theta|D_A)$ by Bayes' rule as

$$(\mid \ ) = \frac{(\mid)\,( )}{\int \ (\mid)\,( )} \qquad (1)$$

The numerator is the product of the likelihood term $(\ \mid \ )$ and the prior probability $(\ )$. The denominator is the sum of this product over all hypothesis domain of θ. The benefits of using Bayesian approach is the ability to utilize the maximum available information such as current observation, historical demand and experts' knowledge in an integrated fashion to estimate the future demand through the posterior distribution.

3. DEMAND MODEL

The demand of the seasonal product at the retailer store is assumed a Poisson distribution process with unknown rate. For convenience, the gamma distribution is chosen as a prior distribution for the demand rate for two reasons: (i) gamma distribution is the conjugate member of exponential family, (ii) gamma distribution has a finite domain (unlike normal), flexible and reproduce a wide range of means or variances. A gamma distributed demand with parameters a, β has a density function $\frac{}{\Gamma( )} \, ^{(\ )}\exp(- \ )$. The estimated mean and variance are $/$ and $/$, respectively.

Parameters of the Prior Distribution:

Estimating parameters of the prior model is a critical and subjective part in this paper. The approach discussed here is the process to approximate unknown parameters (α , β) of the prior demand model using the weighted combination of observed demand at Phase-A, past years' sales D_h, and experts' opinion about the demand range at Phase-B with respect to demand at Phase-A. The weighted combination of demand is

$$D_w = w.D_A + (1-w)D_h, \qquad (2)$$

where k is the weighted value varies from 0 to 1.

In the first set of equations, the weighted combination D_w is assumed to be equivalent to the mean and mode of the prior gamma distribution (Aronis *et. al.*, 2004).

$$/ \ = \quad + (1 - \ ) \qquad (3.1)$$

$$(\ - 1)/ \ = \quad + (1 - \ ) \qquad (3.2)$$

The choice of weighted value k is a subjective decision; it may be defined as the following:

(i) **New products**: new product with no sale history, the value k = 1, leaves out the parameter D_h, the mean, $\alpha/\beta = k\,D_A$.

(ii) **In-market product**: a wide range of k value is possible, k = 0.5 gives a combined mean demand as in Eq. 3.1. For relatively new product, more weight should be at recent demand observation; k value can be 0.6 to 0.9. For known products, k value can be 0.1 to 0.4, so that more weight falls on history data.

The second set of equation may come from experts' opinion that 99% cases the demand at Phase-B fluctuates from 200% to 250% of the demand at Phase-A (i.e., $2D_A$ to $2.5D_A$). Summarizing the above, the expressions Gamma prior distribution gives

$$\int \frac{}{\Gamma(\ )} (- \quad) = 0.99 \qquad (4.1\ )$$

$$\int \frac{}{\Gamma(\ )} (- \quad) = 0.99 \qquad (4.2)$$

For convenience, four sets of equations can be arranged to determine the parameters (α , β) of the prior distribution described in Table 1.

Table 1: Approach to Estimate Prior Parameters

Sets	Equations	$k = 0.1$	$k = 0.5$	$k = 0.9$
Set-1	$/ \ = \ + (1- \ )$ $\frac{}{\Gamma(\ )} (- \quad) = 0.99$	$\alpha = 4.0$ $\beta = 0.59$	$\alpha = 4.0$ $\beta = 0.66$	$\alpha = 4.0$ $\beta = 0.57$
Set-2	$(\ -1)/ \ = \ + (1- \ )$ $\frac{}{\Gamma(\ )} (- \quad) = 0.99$	$\alpha = 4.0$ $\beta = 0.6$	$\alpha = 4.0$ $\beta = 0.50$	$\alpha = 4.0$ $\beta = 0.42$
Set-3	$/ \ = \ + (1- \ )$ $\frac{}{\Gamma(\ )} (- \quad) = 0.99$	$\alpha = 5.0$ $\beta = 1.0$	$\alpha = 5.0$ $\beta = 0.83$	$\alpha = 5.0$ $\beta = 0.71$
Set-4	$(\ -1)/ \ = \ + (1- \ )$ $\frac{}{\Gamma(\ )} (- \quad) = 0.99$	$\alpha = 5.0$ $\beta = 0.8$	$\alpha = 5.0$ $\beta = 0.66$	$\alpha = 5.0$ $\beta = 0.57$

Bayesian Demand Update

In Bayesian process, the ability to revise inventory policies by updating demand forecasts based on Phase-A sales observation significantly improve inventory management and customer service for retail chains. Observed demand at Phase-A for time period t, D_A follows the Poisson distribution. Uncertainty of the demand rate D_A is described by the gamma distribution. Using the Bayesian technique in Eq. 1, the updated parameters of the Posterior distribution are Gamma (A, B) where $A = (D_A + \alpha)$ and $B = (t + \beta)$. The expected mean demand using the posterior parameters are given by

$$A/B = (D_A+\alpha)/(t+\beta). \qquad (5)$$

4. INVENTORY REPLENISHMENT AT PHASE-B

A periodic review (*R*, *l*) stocking policy is considered for retailer's inventory replenishment system to minimize the inventory costs. In Phase-B the period (*T* - *t*) is divided into *n* equal intervals or cycles. At every *l* time intervals, a replenishment order is placed to raise the inventory to order-up-to level *R*. Retailers selling price is $\$p$, purchasing price is $\$C_p$ and holding cost is $\$h$ and fixed cost is $\$A$. If shortages occur, with the shortage cost $\$C_s$, the expected shortages cost per replenishment cycle is given by

$$-\int^{\infty} \quad - \ ) \qquad = --[1- \ (\ , \ )] = --[\bar{\ }(\ , \ )] \tag{6}$$

where D_B represents demand at Phase-B; and *F*(*R*; *t*) is the cumulative gamma distribution. Using shortage cost (Eq. 6), the profit function per cycle yields

$$(\ , \ ) = \quad - \qquad - h \quad -- \ --- \ -[\bar{\ }(\ , \ )] \tag{7}$$

After the first derivation and setting it to zero, retailer's expected profit function becomes

$$(\ , \ ) = (\quad - h \quad)/ \tag{8}$$

For gamma demand and decision variable *R* (order-up-to level), the Equation (8) gives

$$\int \ \frac{\ }{\Gamma(\ )} \qquad (- \quad) \quad = \frac{\ }{\ } \tag{9}$$

Numerical Example: (Inventory Replenishment)

A retailer is making decision to procure a branded winter jacket after observing the initial sales of 13 units during first two weeks period at the beginning of an active demand season. The product costs \$25.50 and holding cost rate is 30% per year. Reorder is placed at the start of each week with negligible transportation time. Stock out may occur at retailer's expenses. The estimated stock out costs is \$10 when shortages occur. The objective here is to find the required quantity for retailers to maintain an order-up-to level to satisfy 99% customers. Using (α, β) values illustrated in Table 1 and D_A = 13 units for 2 weeks period. The holding cost per week, *h* = 0.30/52(\$25.5) = \$0.147/week. With $\$Cs$ =\$10, right hand side of Eq. 9, $(\quad - h \quad)/ \quad = 0.985$. The order-up-to inventory level using the parameters of posterior distribution in Eq. 9 is shown in Table 2.

Table 2: Parameters of Prior and Posterior Distributions and Order-up-to Level

	weight	*Prior*		*Posterior*		*Updated*	R
	k	*a*	*β*	*A*	B	*Mean*	*units/week*
Set-1	0.1	4.0	0.59	17	2.59	6.56	19
	0.5	4.0	0.66	17	2.66	6.39	17
	0.9	4.0	0.57	17	2.57	6.61	19
Set-2	0.1	4.0	0.6	17	2.60	6.54	18
	0.5	4.0	0.5	17	2.50	6.80	22
	0.9	4.0	0.42	17	2.42	7.02	26
Set-3	0.1	5.0	0.90	18	2.90	6.00	12
	0.5	5.0	0.83	18	2.83	6.36	13
	0.9	5.0	0.71	18	2.71	6.64	15
Set-4	0.1	5.0	0.80	18	2.80	6.43	14
	0.5	5.0	0.66	18	2.66	6.77	17
	0.9	5.0	0.57	18	2.57	7.00	19

5. CONCLUSION

This paper documents the demand information update for seasonal retail products. The demand updating allows the retailers to make order placement decision for an active sale period based on the purchasing behavior at the early sales in the retailer stores. The approach discussed the updating of the prior distribution parameters using experts' judgment about the situation and customer's behavior during the sale periods using Bayesian process. The order-up-to level inventory replenishment policy is discussed, subsequently.

Most distributors tries to respond to the order requests of the retailers, but often encourage them to commit orders in advance to get the requested quantity in due time at wholesale prices. While making decisions of how many units to procure, retailers frequently face uncertainty of determining the demand rates and quantity. The proposed model is a decision-making illustration for the retailers to procure products with more accurate and updated forecast.

Bayesian approach is one of a number of ways to make inference about the uncertain demand. The updating process allows experts' judgment to incorporate into current and past data sets to develop the decision-making process. It balances uncertainty by utilizing potential benefit of experts' knowledge and most current observations for decision-making with data extrapolation decision. By doing so, it helps order placement decision- making of the seasonal or fashion merchandise now, to the future.

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