# A Model for Optimal Pricing and Capacity Allocation for a Firm Facing Market Cannibalization

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## Abstract
This paper studies a firm's profitability problem offering its products into different market segments at differentiated prices. In order to improve the firms' profitability the firm needs to decide the prices and order quantities allocations for each market segment. In a perfect market segmentation, it is assumed that the customers do not cannibalize between market segments. However, it is not uncommon to observe an imperfect market segmentation, in which the customers cannibalize from a high price market segment to a lower price segment and vice versa. Models to determine the optimal strategies for pricing and order quantity for a firm experiencing customers' cannibalization and facing both the deterministic and stochastic price dependent demand. Numerical examples are presented to demonstrate the effect of cannibalization using the proposed models.

## Keywords
Revenue Management; Pricing; Market Segmentation; Cannibalization; Inventory Control

## 1. Introduction:
Revenue Management (RM) has been well recognized as an essential practice in many businesses. RM is loosely defined as the set of strategies adopted by a business to improve its profitability (Philips, 2005). According to a detailed literature review presented in McGill and Van Ryzin (1999) it is notified that among many businesses, the airline industry is perhaps the major user of RM tools, however, most RM tools find applications in many other industries, such as, Retail stores, Hotels, Car rentals, etc. An airline RM practices are categorized into four: forecasting; overbooking; seat inventory control (booking control); and pricing. All of these categories are well researched, however, in experts' opinion the integration of pricing and inventory control is expected to improve firms' revenues significantly (Cote et al., 2003). In addition to this in marketing and strategy, the cannibalization refers to a reduction in the sales volume or market share of one product as a result of the introduction of a new product by the same producer. For example, if Pepsi, were to introduce a similar product such as Diet Pepsi, then this new product could take some of the sales away from the original Pepsi. Cannibalization is a key consideration in product portfolio analysis and product line design. A second commonly noticed example of cannibalization is when companies, particularly retail companies, open outlets too close to each other. Much of the customers for the new outlet could have come from the old outlet. Thus, the potential for cannibalization is often discussed when considering companies with many outlets in an area, such as Emirates airlines, Wal-Mart, etc. Another example of cannibalization is when a firm creates a promotion like 20% discount for one item (for example Pepsi). The tendency of consumers is to buy the discounted item (Pepsi) rather than the other items with a higher price. However, when the promotion event is over, the regular drinker of Coke will resume buying Coke. By this behaviour, there is a temporary cannibalization happening due to a promotion event. Although business cannibalization may seem inherently negative, but it can be a positive thing. It sometimes involves a carefully planned strategy, and it also forces a company to think outside the box in order to evolve with the changing needs of both the marketplace and the consumer. In the world of e-commerce for example, some companies intentionally cannibalize their retail sales through lower prices on their online product offerings. Seeing these discounts on items, more number of consumers than usual would buy these items. Even though their in-store sales might decline, the company may see overall positive gains. The paper is organized in the following sections. In Section 2, we outline a brief literature review about RM and pricing problem. In Section 3, we discuss the background of a firm's RM problem offering same product into a segmented market. The firms needs to exercise a integrated control strategy that determines both the price and order quantity for market segment in order to maximize the generated revenue. In Section 4, we present

the models proposed and explain our approach in solving the problem in details. Finally, in Section 5, we provide a numerical example and discuss the findings .

## 2. Literature Review:

A single period newsvendor problem is a building block in stochastic inventory control. It incorporates fundamental techniques of stochastic decision-making and can be applied to a much broader scope. The problem is well researched that its history traces back to Edgeworth (1888), where it first appeared in the banking context. During the 1950's, war effects enabled the expansion of research in this area, leading to the formulation of this problem as the inventory control problem. Arrow et al. (1951) showed that it is critical to have optimal buffer stocks in an inventory control system. Porteus (1990), and Lee and Nahmias (1990) presented although review of the newsvendor problem using a stochastic demand. In most studies, the pricing is considered as a fixed parameter rather than a decision variable. Whitin (1955) was the first to discuss the pricing issues in the inventory control theory. Mills (1959) extended Whitin's work by modelling the uncertainty of the price sensitive demand. He suggested an additive form for the study and assumed that the stochastic demand was a summation of the price-dependent risk-less demand and of the random factor. The risk-less demand is considered a deterministic function of the price. The most evident benefit of such modelling is that the random behaviour of the demand is captured using standard distributions independent of the pricing. Karlin and Carr (1962) presented a multiplicative form of the demand. In their model, the random demand is considered as the product of the riskless demand function and of the random factor. Both the additive and multiplicative models are fundamental to the pricing problem. Some subsequent contributions to the additive model are due to Ernst (1970), Young (1978), Lau and Lau (1996) and Petruzzi and Dada (1999). The contributions to the multiplicative model include Nevins (1966), Zabel (1970), Young (1978) and Petruzzi and Dada (1999). Mieghem and Dada (1999) studied the quantity and pricing of the price versus the production postponement in the competitive market. A coordination of the dynamic joint pricing and production in a supply chain is studied in Zhao and Wang (2002) using a leader/follower game. Optimal control policies are identified for the channel coordination. Bish and Wang (2004) studied the optimal resource investment decision on a two-product, price-setting firm that operates in a monopolistic market and that employs a postponed pricing scheme. The principles on the firm's optimal resource investment decision are provided. Gupta et al. (2006) developed a pricing model and heuristic solution procedures for clearing end-of-season inventory. Yao et al. (2006) revisited the standard newsvendor problem and its extension with pricing. The work generalizes the problem under both the additive and multiplicative modelling approaches and shows quasi-concavity of the revenue function of the problem under various stochastic demand distributions. Chen et al. (2006) addressed dynamic adjustment of the production rate and of the sale price to maximize the long run discounted profit. Bell and Zhang (2006) examined the different decisions surrounding the implementation of an aggressive RM pricing in a firm facing a single period stochastic pricing and a stocking problem. They also identified decisions that have large financial effects. Bhargava et al. (2006) studied the optimal stock out compensation in the electronic retail industry using price as the decision tool. Raza and Akgunduz (2008) studied the an airline RM problem of optimal seat allocation and fare pricing in non-cooperative game setting and suggested the competitive fare pricing strategies for an airline competing in a duopoly market. Raza and Akgunduz (2010) also investigated the code sharing (cooperation) strategies for airlines using Nash Bargain game.

Differentiated pricing is among essential RM techniques to improve the profitability of a firm. Firms offer their products to their customers in different markets who may have different willingness to pay. There are several papers discussing the price discrimination. Narasimhan (1984) discussed the price discrimination theory of coupon. Smith et al. (1991) studied the issue of fairness in consumer pricing. Philips (2005) presented a riskless model for revenue optimization with differentiated pricing of a single product offered into two market segments, full (high) and discounted (low) price markets. The market segmentation was assumed imperfect and thus the customers who belong to the group with high willingness to pay could opt to purchase the product from lower price market segment. This issue of imperfect market segmentation is known in marketing literature as Cannibalization.

## 3. Motivation and Problem Definition:

As identified in the literature review, there are several n airline is offering two or more fare classes to customers in a monopolistic market. The market is also segmented is assumed to be perfect and is same segmented such that each fare class offering belongs to a distinct market segment.

Many firms, such as airlines, car rentals, hotels, restaurants, etc., usually face difficulties in forecasting the customers' future demand while planning for their pricing strategy since the historical information represents only the sales and not the customers demand. This concept is explained in Figure 1, where the total quantity demand for a product is the area (0AC), but the revenue generated at the selling price ($p_1$) and demand ($q_1$) is actually the area under ($0p_1Bq_1$). Thus, the difference between the two areas is surplus demand, or the lost sale opportunity.

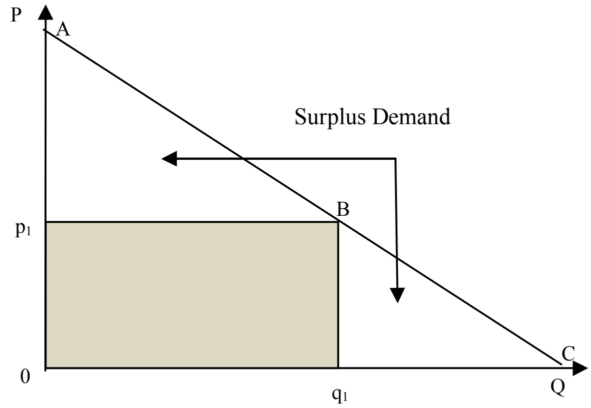


Figure 1: Product Demand vs. Actual Sales

Since firms usually try to maximize their generated revenues from their products, they tend to group their customer into different segments based on the customers' price willingness-to-pay attitude, and by offering each segment tailored products with different prices. Figure 2 shows an example of two-customer segments were the high-price segment is offered a product with selling price $p_1$ and the relevant demand is $q_1$, and the low-price segment product price is $p_2$ and the related customers' demand is $q_2$. Because of the market segmentation, the total revenue is increased and it is now equal to both areas (p1bfg) and (hdef). Although the customers' surplus demand is now reduced, yet more revenue is captured. The unsatisfied demand level still can be noticed, and more revenue could be generated.

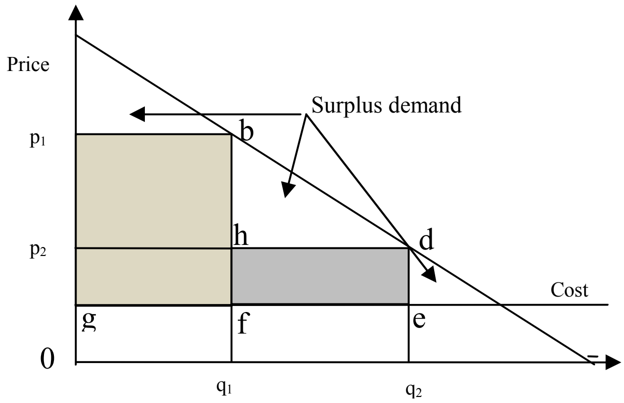


Figure 2: Two-Product Perfect Market Segmentation

Thus, it is in the best interest of a firm to segment the market demand. If the market segments are completely independent and the firm faces no capacity constraints, then the determination of the differentiated prices is quite simple. However, when the market segmentation is incompletely independent (imperfect) then there exists a phenomenon called cannibalization. Unfortunately, in most market situations perfect market price differentiation is rare to observe. Cannibalization is likely to happen whenever the customers cannot be perfectly segmented based on their willingness to pay and rationalities. Arbitrage is likely whenever a third party can purchase the product at a low price and resale it at a high price. Regional pricing is subject to arbitrage whenever a product can be purchased in a low-price region and transported cheaply to be resold at a higher price elsewhere.

## 4. The Models:

In this section, we first discuss the riskless model. The model is presented in Phillips (2005) which considers the effect of cannibalization under price dependent riskless demand. Later in this study, the models presented in Phillips (2005) are extend to the case in which the market demand is stochastic and price dependent, and thus the model enables consideration of market demand risk.

**4.1. The riskless model:**

To analyze differentiated pricing with cannibalization. First, we briefly discuss the problem of riskless pricing of the product that is offered to two market segments. We also assume that cannibalization exists, and customer cannibalize from high priced market segment to a lower priced segment. Under cannibalization, we consider a product offered by a firm to two marketing segments. Customers' segments are assumed to have cannibalization due to imperfect market segments. The product has a cost *c*, and it is offered to two markets segments with two different prices $p_1$ and $p_2$. Market segment 1 is restricted to the customers who are willing to pay high price $p_1$, and market segment 2 is priced at lower price $p_2$. Since the first market segment has customers with more willingness to pay compared to the second segment, thus $\quad > \quad > \quad$.

The customers' riskless price dependent demand, $\ (\ )$ related to the high price market segment follows increasing price elasticity (IPE) property. Similarly, the customers' riskless price dependent demand related to the lower prices market segment is $\ (\ )$, and it follows the IPE property. Both the demands are assumed linear, thus $\ (\ ) = [\ \ -\ \ ]$, where $[\ ]\ =\ \{\ ,0\}$. Likewise, for the lower price segment, the riskless demand is assumed to be $\ (\ ) = [\ \ -\ \ ]$. Thus, the maximum price which a firm can set for market segments 1 and 2 are $\ = —$ and $\ = —$ respectively. Given that $\beta_1$ and $\beta_2$ are equal, $\alpha_1$- $\alpha_2$ is the maximum demand that attributes to the high price customer segment. Assuming there is market cannibalization represented by a factor $\theta$, the adjusted demand for each market segment would be:

$$(\ ,\ ) = (1 -\ )\ \ (\ ) \qquad (1)$$

$$(\ ,\ ) =\ \ (\ ) + (\ \ -\ \ ) \qquad (2)$$

With the conditions: $\ \geq\ \ >\ $, and $0 \leq\ \ \leq 1$

Thus, the pricing optimization problem would be:

$$(\ ,\ ) = \max_{,}\ (\ -\ )\ \ (\ ,\ ) + (\ -\ )\ \ (\ ,\ ) \qquad (3)$$

Subject to: $\ \geq\ \ >\ $, and $0 \leq\ \ \leq 1$

**4.2. The risk based model:**

Now, we consider the problem in which the firm is facing stochastic price dependent demand in both market segments, and with the affect of cannibalization. It is assumed that $RD_1$ is the stochastic price dependent demand experienced in full price market segment 1, and likewise $RD_2$ is the stochastic price dependent demand for the discounted price market segment 2. Additive approach is used to model both $RD_1$ and $RD_2$.

$RD_1$ has an additive random factor $\xi_1$ such that $\xi\ \in [\underline{\xi 1}, \overline{\xi 1}]$, and similarly, $RD_2$ has an additive random factor $\xi_2$ such that $\xi\ \in [\underline{\xi 2}, \overline{\xi 2}]$. For simplification in the analysis, the expected for both the $\xi_1$ and $\xi_2$ is zero.

Thus, $RD_1$ and $RD_2$ are given by:

$$=\ \ (\ ,\ ) + \xi \qquad (4)$$

$$=\ \ (\ ,\ ) + \xi \qquad (5)$$

The total revenue for the two products is written as:

$$(\ ,\ ) = \max_{,}\ (\ -\ )\ \ \{\ ,\ \ \} + (\ -\ )\ \ \{\ ,\ \ \} \qquad (6)$$

Subject to: $\ \geq\ \ >\ $, and $0 \leq\ \ \leq 1$

Where: $\ = (\ ,\ \ )$ and $\ = (\ ,\ \ )$

Assuming that each of the two random factors $\xi$ and $\xi$ follow a continuous Probability Distribution Functions (PDFs) $f_1(.)$, and $f_2(.)$ respectively. Whereas, their cumulative Probability Distribution Functions (CFDs) are $F_1(.)$, and $F_2(.)$ respectively.

Upon simplification, the following expression is derived for the revenue:

$$(\ ,\ ) =\ \ +\ \ -\ \int_{\underline{\xi}}^{(\ )}\ \xi\ \ \xi\ -\ \int_{\underline{\xi}}^{(\ )}\ \xi\ \ \xi\ -\ (\ \ +\ \ ) \quad (7)$$

Subject to: $\ \geq\ \ >\ $, and $0 \leq\ \ \leq 1$

The revenue function presented in Equation 7 is the expected revenue. The objective is the find the prices and order quantities, $\ ,\ $ that maximize the total revenue $\pi\ (\ ,\ )$. This is a non-linear optimization problem, in order to solve the problem the Karush-Kuhn Tuker (KKT) optimality conditions could be explored (Bertsekas, 1999). The other possibilities are to use the commercially available software such as MATHEMATICA or MATLAB in order to solve the problem. This study uses a MATHEMATICA based built-in numerical optimization procedure, NMaximize to achieve this task. The Nelder & Mead method (http://mathworld.wolfram.com/Nelder-MeadMethod.html) is selected as the search algorithm for the NMaximize procedure of the MATHEMATICA.

## 5. Numerical Experimentation and Discussion:

We provide a numerical example to illustrate our model and explain the results. The example is adapted from Phillips (2005). First we consider the riskless analysis which studies the riskless model, when the firm is trying to find out the optimal price for a product offered to a single market segment. Assuming linear riskless price dependent demand function, $( ) = ( - )$ where $\alpha$= 9000, and $\beta$=800. The cost of product per unit c is \$5. In this case, the maximum selling price is $-$. The optimal price p* which yields the maximum revenue must satisfy the first order optimality condition is $^* = \$8.13$. Thus, when the firm sets an optimal selling price $^* = \$8.13$, the total unit sales would be equal to 2,500 units, and the total revenue is equal to \$7,812.50. Now, when the firm applies market segmentation and offers the same product into two market segments. When the firm applies two-market segments, for illustration purpose, it is assumed that: c = 5; $\alpha_1$ =9000; $\alpha_2$ =5600; $\beta_1$=$\beta_2$=800;

The prices that maximize the revenue in this example would be:

$$^* = \$8.13 \text{ and } ^* = \min(6 + 2.12 \; , 7)$$

As a result, if $\theta$=0, then $^* = \$6$, and the total revenue generated increases by 10% and it is equal to \$8,612.50. As $\theta$ increases, the cannibalization increases and $^*$ increases as well. When $\theta$=0.22, the optimal low price is increased to 6.47, and the total revenue is equal to \$7,816.60, which is still competitive to the revenue generated from no-market segmentation. However, if the cannibalization rate exceeds 22%, then it is better for the firm to sell its product without market segmentation as segmentation is weak in this case. Figure 3 illustrates the impact of cannibalization under an assumption the demand in both marker segments is riskless. If the firm does not apply market segmentation and offers one price to all customers, the maximum revenue would be equal to \$7,751.99 when the quantity demand is normally distributed.

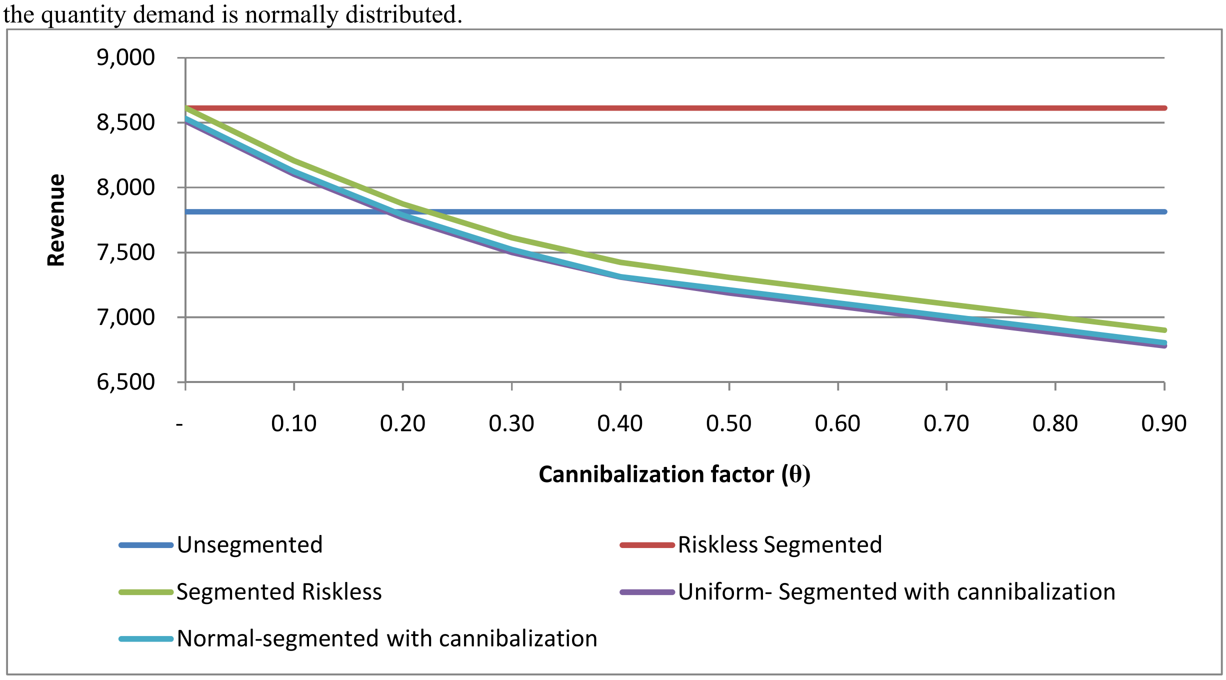


Figure 3: Effect of Cannibalization

Now we consider the risk based model presented in Section 4.2. The random demand factor, $\xi_1$ and $\xi_2$ follow uniform and normal distributions such that $\xi_1$ is bounded in [-40, 40], and $\xi_2$ is bounded in [-30, 30]. Again referring to Figure 3 which graphically shows the impact of cannibalization. If the firm offers one price for all customers while assuming no risk associated with the quantity demand, the maximum revenue will be 7,812.50. Conversely, we consider risk based model in which the customer demand is price dependent stochastic and follows normal distribution. If the firm divides the customers into two segments based on their willingness-to-pay, the total revenue would increase by 10% and reaches \$8,530.96 compared to un-segmented revenue for the situation in which the price dependent stochastic demand is normally distributed. When cannibalization exists between the two market segments, a firm could still generate higher revenues with the market segmentation as long as the cannibalization factor $\theta \leq 0.22$ as noticed in this numerical example. When cannibalization factor (segmentation imperfection ) increases further, it will have negative impact on the total revenue. In such a situation, it is better for the firm to apply non-segmented market strategy and offer the product with the same price for all customers in order to generate higher revenues. Following this analysis, we consider the price dependent stochastic demand is uniformly distributed. The finding are consistent and comparable to normal distribution, the firm will still generate higher revenue as long as cannibalization factor $\theta \leq 0.21$.

## References:


1. Arrow, K., Harris T., & Marschak J. (1951). "Optimal Inventory policy". Econometrica, Vol. 19, pp. 250-272
2. Bell, P. C. & Zhang M. (2006)." Managerial decision-making in the single period optimum pricing problem". The journal of operational research society, Vol. 57, pp. 377-388
3. Bish, E. K. and Wang, Q. (2004),"Optimal investment strategies for flexible resources,considering pricing and correlated demands", Operations Research, Vol. 52, 954-964.
4. Chen, Y. H., Ray, S. and Song, Y. Y. (2006), "Optimal pricing and inventory control policy in periodic-review systems with fixed ordering cost and lost sales", *Naval Research Logistics*, Vol. 53, 117- 136.
5. Cote, J.P., Marcotte, P. and Savard, G. (2003) "A bi-level modeling approach to pricing and fare optimisation in the airline industry", Journal of Revenue and Pricing Management, Vol. 2, pp.23–36.
6. Ernst R.L. (1970). Linear inventory models for a monopolistic firm. PhD thesis. University of California, Berkeley
7. Gupta, D., Hill, A. and Bouzdine-Chameeva, T. (2006), "A pricing model for clearing end-of-season retail inventory", European Journal of Operational Research, Vol. 170, pp. 518-540.
8. Karlin, S. and Carr, C. R. (1962), Studies in Applied Probability and Management Science, Stanford University Press, Stanford, CA, chapter Prices and Optimal Inventory Policies.
9. Lau, A. H. & Lau, H. S. (1996), "The multi-product multi-constraint newsboy problem: applications, formulations, and solutions", Journal of operations management, Vol. 13, pp. 153-162
10. Lee, H. and Nahmias, S. (1990), Handbooks in Operations Research and Management Science, North-Holland, chapter Single-Product, Single Location Models, pp. 1-58.
11. McGill, J. I. (1995), `Censored regression analysis of multiclass demand data subject to joint capacity constraints', Annals of Operations Research, Vol. 60, pp. 209-240.
12. McGill, J. I. and Ryzin, G. J. V. (1999), "Revenue Management: Research overview and prospects', Transportation Science", Vol. 33, pp. 233-256.
13. Mieghem, J. A. V. and Dada, M. (1999), "Price versus production postponement: Capacity and Competition", Management Science, Vol. 45, pp. 1631-1649.
14. Mills, E. S. (1959), "Uncertainity and price theory", Quarterly Journal of Economics, Vol. 73, pp. 116-130.
15. Mills, E. S. (1962), Price, Output and Inventory Policy, John Wiley, New York.
16. Nevins, A. J. (1966), "Some effects of uncertainity: Simulation of a model of price", Quarterly Journal of Economics, Vol. 80, pp. 73-87.
17. Petruzzi, N. C. and Dada, M. (1999), "Pricing and the news vendor problem: A review with extensions'" Operations Research, Vol. 47, pp. 183-194.
18. Philips R. L. (2005), 'Pricing and revenue optimization', Standford business book, Standford university press
19. Porteus, E. (1990), Handbooks in Operations Research and Management Science, North-Holland, chapter Stochastic Inventory Theory, pp. 605-652.
20. Raza, S. A., Akgunduz, A. (2008), "An airline revenue management fare pricing game with seats allocation", International Journal of Revenue Management, Vol.2, No. 1, pp. 42-62
21. Raza, S. A., Akgunduz, A. (2010), "The impact of fare pricing cooperation in airline revenue management", International Journal of Operational Research, Vol.7, No. 3, pp. 277-296
22. Whitin, T. M. (1955), "Inventroy control and price theory", Management Science, Vol. 2, pp. 61-68.
23. Yao, L., Chen, Y. F. and Yan, H. (2006), "The newsvendor problem with pricing: Extension", International Journal of Management Science and Engineering Management, Vol. 1, pp. 3-16.
24. Young, L. (1978), "Price, inventory and the structure of uncertain demand", New Zealand Operations Research, Vol. 5, pp. 157-177.
25. Zabel, E. (1970), "Monopoly and uncertainity", Review of Economic Studies, Vol. 37, pp. 205-219.
26. Zhao, W. and Wang, Y. Z. (2002), "Coordination of joint pricing-production decisions in a supply Chain", IIE Transactions, Vol. 34, pp. 701-715.