Proceedings of the 2011 International Conference on Industrial Engineering and Operations Management
Kuala Lumpur, Malaysia, January 22 – 24, 2011

Integrating Quality and Inspection for the Optimal Lot-sizing Problem with Rework, Minimal Repair and Inspection Time

Ya-Hui Lin
Department of Industrial Engineering and Management
Chin Min Institute of Technology, Toufen, Miaoli 35153, Taiwan

Yan-Chun Chen*
Department of Industrial Engineering and Management
Tungnan University, Shen-Keng, Taipei 22202, Taiwan
*Corresponding author

Abstract

This research is derived an integrated model of economic production quantity, inspection and preventive maintenance to incorporate rework, minimal repair and inspection time for an imperfect production system. Some proportions of non-conforming items are assumed scrap before the rework process start and others are reworked. The rework process is imperfect that is a proportion of the reworked items fail the repairing and become scrap items. We derive the optimal solutions for the inspection interval, inspection frequency and economic production quantity for a deteriorating process. And numerical analysis is provided to analyze the effect of rework, minimal repair and inspection time on profit.

Keywords
Imperfect process, imperfect rework, preventive maintenance, minimal repair, inspection time

1. Introduction

For most manufacturing industries, the batch mode of production is widely utilized. Therefore, how to determine the optimal economic production quantity (EPQ) is a pertinent inventory management issue. In practically, a production system begins with the in-control state; it may shift to the out-of-control state and later produce non-conforming items. When the type of situation outlined above develops, the process is interrupted; in this case, it can be described as an imperfect process. Hence, a number of studies have investigated the effects of process deterioration on the optimal EPQ [1-2]. They found that the resulting EPQ is smaller than that in the classical model. Darwish [3] investigated the relationship between the setup cost and the production run length in the EPQ model and found that this relationship has a significant impact on the optimal production quantity and total cost.

To obtain products with better quality, integrating inspection and maintenance of production system is essential. While numerous models with PM assume that a system will be in perfect condition after each instance of PM, in reality the form of failure rate changes following the implementation of PM. Ben-Daya [4] considered an imperfect process having a deterioration distribution with increasing hazard rate and imperfect maintenance to determine EPQ and PM level. Darwish and Ben-Daya [5] examine the effect of inspection errors and preventive maintenance on a production inventory system, and Liao *et al*. [6] integrated maintenance and production programs with EPQ model considered the impact of restoration action such as imperfect repair, rework and PM on the damage of a deteriorating production system.

In the actual production system, a percentage of non-conforming items can be changed into conforming items by reworking. Printed circuit board assembly manufacturing and plastic injection molding process are good examples. Hayek and Salameh [7] proposed a production lot sizing strategy when imperfect products can be reworked and determined the optimal policy that minimizes the total inventory cost per unit time where shortages are allowed. Chiu [8] extended the study of Hayek and Salameh [7] and proposed an EPQ model with the reworking of repairable items and disposal items for an imperfect production process where backorders are permitted. Chiu *et al*. [9] studied the optimal production quantity for a imperfect production system with imperfect rework process, random scrap rate, and service level constraint which backlogging is less than or equal to that of the same model without backlogging. Biswas and Sarker [10] proposed optimal batch quantity models for a lean production system with in-cycle rework and scrap.

To close the actual production system, this paper is to develop an integrated production, inventory and inspection model in order to study the joint effects of process deterioration, imperfect maintenance, imperfect reworking, minimal repair, and inspection time on the optimal lot sizing and inspection decisions that maximum the expected total profit. The rest of this paper is organized as follows. Section 2 describes the integrated model and introduces notations. Section 3 presents the model development and formulas. Section 4 proposes a procedure for gaining solutions, and the results for various numerical analyses are presented. The final section contains concluding remarks.

2. Notations and Model Description

The following notation is used to develop the model. The additional notations will be mentioned in the text as and when required:

D	:demand rate in units per unit time;
P	:production rate in units per unit time, where $P > D$;
P_r	:rework rate of non-conforming items in units per unit time;
T	:actual production time per cycle (production run);
T_r	:the time of reworking non-conforming items for each cycle;
CT	:the inventory cycle time for each cycle;
C_h	: holding cost per item per unit of time ($/ item/ unit time);
C_I	: inspection cost per unit ($/ unit);
C_r	: repair cost for each non-conforming item reworked ($/ item);
C_d	: disposal cost for each scrap item ($/ item);
C_{mr}	: the minimal repair cost per unit; ($/ unit);
P_u	:selling price of each conforming item ($/ item);
I_j	:inventory level after the jth inspection;
k	:number of inspections carried out during each production run;
h_j	:length of the jth inspection interval;
t_j	:time for the jth inspection, $t_j = \sum_{i=1}^{j} h_i + (j-1)s$;
$f(t)$	:probability density function of the time to shift to out-of-control state;
$F(t)$	:cumulative distribution of $f(t)$;
C_{pm}^{a}	:cost of the actual PM activities;
$C_{pm}^{\max}$	:cost of the maximum PM level;
p_j	:conditional probability that the process shifts to the out-of-control state during the time interval $(t_{j-1}+s, t_j)$ given that the process is in in-control state at time t_{j-1}.

This paper considers a production system producing a single product. At the beginning of a production cycle, the production system is assumed to be in the in-control state in which the process only produces acceptable items. After a period of production time, process may shift to out-of-control state to produce some non-conforming items. The elapsed time for a process to shift is a random variable that follows a general distribution with increasing failure rate. Assume s is the time length used for each inspection. The process is inspected at time t_1, t_2,…, t_k to observe the state of a process. If the system retains in the in-control state, PM is carried out right after each inspection at $t_j + s, j = 1,2,\cdots,k-1$. The reduction of the age of production system depends on the level of PM so that we select the PM cost, C_{pm}^{a} as a decision variable. The PM level is fixed in the entire production cycle as long as it is determined in the beginning. The length of each inspection interval is determined such that the integrated hazard rate over each interval is constant.

If the system is inspected in the out-of-control state, it can be either the first type or the second type. And the process produces non-conforming items at a constant rate d_I and d_{II} with first type out-of-control state and second type out-of-control state, respectively. The first type of out-of-control state is a relatively mild production process shift with probability $1-\theta$, where the process can be recovered with a minimal repair to the condition right before the breakdown. The minimal repair is assumed not to change the hazard rate of the process. Also, the time of a minimal repair is negligible. The second type of out-of-control state is a more serious shift with probability θ. In this case, the system has to be completely shut down for complete repair so that it is restored to an as-new state. Assuming that once the production system shifts to the out-of-control state it will stay in that state until the inspection is carry out, regardless of the type of out-of-control state.

In real production systems, some non-conforming items may be reworked. Assume that a percentage d_1 ($0 \le d_1 \le 1$) of non-conforming items that are scrapped and will not be reworked. They are discarded before the rework process starts. Other percentages $(1-d_1)$ of the non-conforming items are assumed to be reworked and the rework process starts immediately when the regular production ends at a rate P_r. The rework process is assumed to be imperfect, in which a percentage d_2 ($0 \le d_2 \le 1$) of non-conforming items fail the reworking and become scrapped items.

The production cycle ends either when the system is in the second type of out-of-control state or the last inspection is completed whichever occurs first. Then to renew a production cycle, the extra works might needed on the system and ensure the next cycle begin with in-control state. Therefore PM is not performed in the last inspection and when the system is down. There might be errors in inspections. For simplicity, we do not consider such errors. Demand rate and production rate in the system are assumed to be constant. Shortages are not allowed.

3. Model Formulation

According to the assumptions of the production system, an inventory cycle be given in Figure 1. Figure 1 represents the inventory cycle and the *j*th inspection and PM are performed at time t_j and t_j+s, respectively. The expected total cost of the integrated model consists of setup cost C_s which is fixed, inventory holding cost $E(HC)$, reworking cost for non-conforming items $E(RW)$, disposal cost for scrapped items $E(D)$, maintenance costs $E(M)$ which includes PM cost and minimal repair cost, inspection cost $E(IC)$, and restoration cost $E(RC)$. To derive the costs, we need to find out expected regular production cycle time $E(T)$, expected rework cycle time $E(T_r)$, expected inventory cycle time $E(CT)$ and the total expected number of non-conforming items $E(N)$.

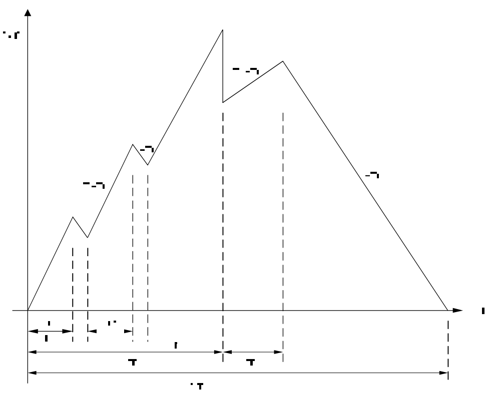

Figure 1. Inventory cycle

The expected regular production cycle time will consequently be $E(T)=\sum_{j=1}^{k-1}[(h_j+s)\prod_{i=1}^{j-1}(1-\theta p_i)]+h_k\prod_{i=1}^{k-1}(1-\theta p_i).$

When the general production cycle ends, the total expected number of non-conforming items is $E(N)$ given by

$$E(N)=\sum_{j=1}^{k} p_j[(1-\theta)E(N_j^I)+\theta E(N_j^{II})]\prod_{i=1}^{j-1}(1-\theta p_i). \tag{1}$$

where $E(N_j^I)$ ($E(N_j^{II})$) is the expected number of non-conforming items produced due to the first (second) type out-of-control during the period ($t_{j-1}+s$, t_j). These two expected numbers are $E(N_j^I)=\int_{a_{j-1}}^{b_j-s} d_I P(b_j-s-t)\frac{(1-\theta)f(t)[\bar{F}(t)]^{-\theta}}{[\bar{F}(a_{j-1})]^{1-\theta}}dt$ and $E(N_j^{II})=\int_{a_{j-1}}^{b_j-s} d_{II} P(b_j-s-t)\frac{\theta f(t)[\bar{F}(t)]^{\theta-1}}{[\bar{F}(a_{j-1})]^{\theta}}dt.$

Then the expected rework cycle time is obtained by $E(T_r)=(1-d_1)E(N)/P_r$.

Hence from Figure 1, the expected inventory cycle length is presented as follows:

$$E(CT)=\frac{1}{D}\{P[E(T)-(k-1)s]-E(N)+(1-d_1)(1-d_2)E(N)\}. \tag{2}$$

And the expected total production quantity is $E(Q)=P\cdot[E(T)-(k-1)s]$.

The expected inventory holding cost is calculated by multiplying the inventory holding cost per unit of time by the expected inventory over the course of the inventory cycle (area under the curve in Fig. 1). Hence, the expected inventory holding cost is determined by the following formulae.

$$E(HC)=C_h\int_0^{CT} I(t)dt = C_h E(H) \tag{3}$$

where $E(H)$ is the expected inventory in the production cycle under the function $I(t)$ which can be obtained as

$$E(H)=\frac{1}{2}\{\sum_{j=1}^{k}[h_j(I_{j-1}+I_j+Ds)[\prod_{i=1}^{j-1}(1-p_i)]+s(2I_j+Ds)\prod_{i=1}^{j-1}(1-p_i)]+\sum_{j=1}^{k-1}[\prod_{i=1}^{j-1}(1-p_i)]p_j\frac{I_j^2}{D}+\prod_{j=1}^{k-1}(1-p_j)\frac{I_k^2}{D}\},$$

where I_j is the inventory level at time t_j+s , $I_j=\sum_{i=1}^{j}\left[(P-D)h_i-D\cdot s\right]$ for $j=1,2,\cdots,k$ and $I_0=0$ and if $I_j<0$, we set it to be zero.

When the regular production ends, the quantity that can be reworked is $(1-d_1)E(N)$. Then the expected reworking cost is $E(RW)=C_r\cdot(1-d_1)E(N).$

The quantity of the reworked items that become scrap is $(1-d_1)d_2E(N)$ when the rework process ends. Therefore the expected disposal cost for scrap items is $E(DC)=C_d\cdot[d_1+(1-d_1)d_2]E(N).$

As mentioned earlier, the system can not be as good as new after implementing PM. The reduction of the age of the equipment is a function of the PM cost. Let γ_k be the imperfect coefficient at the *m*th PM, then $\gamma_m=\eta^{m-1}\cdot C_{PM}^a/C_{pm}^{\max}$, where the parameter η is a imperfectness factor, which impacts the influence of PM activities on the used age of the process, $0<\eta\le 1$.

Let b_m (a_m) is the actual age of system instantly before (after) the *m*th PM. Ben-Daya [11] considered both linear and non-linear relationship between the age reduction and PM cost. Here, the relationship is assumed linear which is $a_m=(1-\gamma_m)b_m$. Note that the actual age of a production system at time (t_j+s) is $b_j=a_{j-1}+h_j+s$ for $j=1,2,\cdots,k$ and $a_0=0$.

Then the maintenance cost should include the preventive maintenance cost (C_{pm}^a) and the minimal repair cost (C_{mr}). Hence the expected maintenance cost $E(M)$ is given by

$$E(M)=C_{pm}^a\sum_{j=1}^{k-1}\prod_{i=1}^{j}(1-\theta p_i)+C_{mr}\sum_{j=1}^{k-1}(1-\theta)p_j\prod_{i=1}^{j-1}(1-\theta p_i). \tag{4}$$

The expected number of inspection in the production cycle is one more than the expected number of PM because there is one inspection at the end of the production cycle. Therefore, the expected inspection cost is given by

$$E(IC)=C_I\{1+\sum_{j=1}^{k-1}\prod_{i=1}^{j}(1-\theta p_i)\}. \tag{5}$$

Assume that the second type out-of-control state occurs at the time *t* in the period (a_{j-1},b_j), and the detection delay time is b_j-t. According to Ben-Daya [3], the restoration cost is assumed a linear function of the detection delay. That is $RC_j=R(b_j-t)=r_0+r_1(b_j-t)$, where r_0 and r_1 are constants. The restoration cost per production cycle is given as the following:

$$E(RC)=\theta\sum_{j=1}^{k}p_j\prod_{i=1}^{j-1}(1-\theta p_i)\times\{(r_0+r_1b_j)[1-(\frac{\overline{F}(b_j)}{\overline{F}(a_{j-1})})^{\theta}]-r_1\int_{a_{j-1}}^{b_j}t\frac{\theta f(t)[\overline{F}(t)]^{\theta-1}}{[\overline{F}(a_{j-1})]^{\theta}}dt\}. \tag{6}$$

So the expected total cost per each cycle is $E(TC)=C_s+E(HC)+E(PM)+E(IC)+E(RW)+E(DC)+E(RC)$.

The expected total revenue is $E(TR)=P_u\cdot\{P[E(T)-(k-1)s]-[d_1+(1-d_1)d_2]E(N)\}$.

Then we can obtain the expected total profit per unit time given by

$$EU(\pi)=[E(TR)-E(TC)]/E(CT) \tag{7}$$

4. Solution procedure

The next problem is obtaining the optimal values from the decision variables for the above integrated model and discussing the optimization procedure used to determine the optimal solution which maximizes the total expected profit per unit time. First, the influence of C_{pm}^{a} on profit is specified and its optimal value is determined. Next, numerical analysis is used to identify how h_1 and k affect profit, and the maximum expected profit is determined which yields the optimum solution. The optimal Q value is then calculated after determining h_1 and k.

Applying the study of Banerjee and Rahim [12], most of the mechanical malfunctions follow the non-Markov shock model with increasing failure rate function. As the production process continues, the optimal inspection interval is progressively reduced. To balance the risks, each inspection interval has the same cumulative hazard rate. That is $\int_{t_{j-1}+s}^{t_j+s}r(t)\,dt=\int_0^{t_1+s}r(t)\,dt$, for $j=2,3,....,k$.

Because of the effect of performing PM, failure rate decreases progressively at the end of each arrival interval, and consequently $\int_{a_{j-1}}^{b_j}r(t)\,dt=\int_0^{h_1+s}r(t)\,dt$, for $j=2,3,....,k$.

Usually the time of process staying in the in-control state is to assume that follows a Weibull distribution, that is, its probability density function is $f(t)=\lambda\nu t^{\nu-1}e^{-\lambda t^{\nu}}$, $t>0,\nu\ge 1,\lambda>0$.

Then, applying hazard rate of the Weibull distribution which is $r(t)=\lambda\nu t^{\nu-1}$, the length of the inspection intervals is given by $h_j=[(a_{j-1})^{\nu}+(h_1+s)^{\nu}]^{1/\nu}-a_{j-1}-s,\ \ j=2,3,....,k$.

Therefore, to maximum expected total profit, the value of decision variables h_1, k and C_{pm}^{a} are needed to determine.

The optimal value of $k\ge 2$ could be determined by choosing k that satisfies two inequalities:

$EU(\pi)(k-1)\ge EU(\pi)(k)$ and $EU(\pi)(k+1)\ge EU(\pi)(k)$.

5. Numerical analysis

Numerical examples are presented to illustrate the important aspects of the integrated profit model. The time that the process stays in the in-control state is assumed to follow a Weibull distribution with scale and shape parameters $\lambda=5$ and $\nu=2.5$. The following parameters are fixed:

$D=500$, $P=1000$, $P_r=750$, $C_h=\$0.5$, $C_s=\$150$, $C_d=\$20$, $C_{pm}^{\max}=\$30$, $C_r=\$5$, $P_u=\$10$, $C_I=\$10$, $r_0=\$10$, $r_1=0.5$, $\eta=0.99$, $d_2=0.1$, $d_I=0.2$, $d_{II}=0.4$.

For the comparisons, we vary the percentage of non-conforming items that are scrap, the non-conforming rate when system is in second type out-of-control state and inspection time length. Numerical analysis is used to find the optimal number of inspections, the optimal length of first inspection interval, the EPQ and the expected total profit per unit time under different values of d_1 (d_1=0.0, 0.5 and 1.0), s (s=0.0, 0.05 and 0.1) and θ (θ=0.0, 0.5 and 1.0) values in Table 1. It shows that when the percentage of non-conforming items that are scrapped increases, the expected total profit decreases. Consequently, the expected profit from each cycle increases with reworking ratio. And the expected total profit increases with the increasing inspection time length. The $EU(\pi)$ when performing minimal repair (θ<1.0) is more than the $EU(\pi)$ without performing minimal repair (θ=1.0). The impact of s, d_1 and θ on the total profit are apparently.

Table 1. Optimal Values under Various Conditions

		$\theta = 0.0$				$\theta = 0.5$				$\theta = 1.0$			
		k^*	h_1^*	Q^*	$EU(\pi)$	k^*	h_1^*	Q^*	$EU(\pi)$	k^*	h_1^*	Q^*	$EU(\pi)$
s=0	d_1 =0.0	2	0.3304	661	4750.50	2	0.3406	632	4750.22	3	0.2669	672	4731.52
	d_1 =0.5	2	0.3010	602	4738.57	2	0.3106	585	4739.51	3	0.2448	636	4722.37
	d_1 =1.0	3	0.2535	758	4733.35	3	0.2623	723	4733.76	3	0.2320	613	4715.72
s=0.05	d_1 =0.0	2	0.3330	666	4754.53	2	0.3439	637	4753.84	3	0.2625	639	4734.83
	d_1 =0.5	3	0.2629	786	4747.22	3	0.2717	729	4744.88	3	0.2411	607	4725.41
	d_1 =1.0	3	0.2509	750	4741.77	3	0.2595	703	4740.15	3	0.2285	586	4718.38
s=0.1	d_1 =0.0	3	0.2790	833	4763.40	3	0.2868	743	4757.17	3	0.2563	604	4736.17
	d_1 =0.5	4	0.2300	910	4757.13	3	0.2675	705	4750.47	3	0.2353	574	4725.77
	d_1 =1.0	5	0.2018	991	4752.66	3	0.2550	679	4745.13	3	0.2225	554	4717.80

6. Conclusions

By integrating concerns dealing with to production, inventory, inspection and preventive maintenance, this study proposes a production and inspection strategy for maximizing the expected profit per unit time of imperfect process and maintenance with imperfect rework, minimal repair and inspection time. Deteriorating production systems are a reality in manufacturing. Regarding these systems, clarifying how production, inspection, preventive maintenance, and inventory are related can help managers perform operation control and quality assurance more effectively. In many practical production processes non-conforming items sometimes can be reworked and repaired. Therefore, the quantity that can be sold and the total profit can be increased significantly. Also, we considered the inspection time length in the production policy which is really effect the total profit. Finally, performing minimal repair caused the total profit increased. The optimization problem becomes more complicated with the consideration in the integrated model of EPQ. Future studies should be alerted to some other optimal techniques of this study.

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