

Cost Analysis for Used Products with Warranty and Preventive Repair

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Abstract

Technological advancement has paved way for a market for second-hand products. Owing to the uncertainty regarding the long-term performance of such products and extortionate product failure costs, warranty for second-hand products is gaining importance. This study focuses on second-hand warranty for a repairable deteriorating system modeled using quasi-renewal process. Two cost models: a fixed warranty model and a preventive repair warranty model have been developed through the expected warranty cost and the long-run average cost rate for the system respectively. Numerical illustrations have been provided to examine the effect of the degree of repair and the importance of preventive repair during warranty. The paper emphasizes on the incorporation of quasi-renewal processes and preventive repair within warranty for second-hand products.

Key words: Second-hand products, Quasi-renewal processes, Degree of repair, Warranty.

1. Introduction

Maintenance strategies play a crucial role in the analysis of deteriorating systems, as they help in improving the quality, reliability, cost and life times of systems. Traditional maintenance models assume perfect (as good as new) and minimal (as bad as it was) repairs. The renewal process and non-homogeneous Poisson processes can be viewed as corresponding to the two extremities of the perfect and minimal repairs respectively. After the seminal paper by Lam [12], in which successive working times and consecutive repair times form geometric processes, Wang and Pham [21, 22], relaxed the identically distributed assumption of the geometric process to assume only successive independent inter arrival times and called it as a quasi-renewal process. A quasi-renewal process models the deteriorating or improving behavior of a system by a parameter taking values from the interval $[0,1)$ or $(1,\infty)$ respectively.

Warranties have become a critical segment of the industrial environment. Often manufacturers use warranty as a marketing tool to advertise the quality of the products. A warranty policy is, in general a contract between the manufacturer and the consumer where in, the former agrees to repair/replace certain defects or failures in the product during an agreed period of time. These policies protect the customer against catastrophic failures while the manufacturers use it to protect themselves against excessive legal claims and for promotional purposes. Thus, warranty policies are vital tools for competitive success. Two common types of warranty policies used in the literature are Free replacement warranty (FRW) and Pro-Rata warranty (PRW) policies (Refer Blischke and Murthy [2]). In a FRW policy the manufacturer agrees to repair/replace the failed product at no cost to the customer during the warranty period. In the PRW policy replacements are done with a cost proportional to the operating time of the product. The main objective in product warranty analysis is to model and estimate the warranty cost. Mathematical convenience has prompted most of the existing warranty models to deal with replacement of the product on failure (Refer Chien [7]). However, a realistic option would be to consider a repair instead of a replacement. (Refer Yeo and Yuan [24]). In order to increase the availability and decrease the operating cost a preventive maintenance has been widely used (Refer Chun [9]).

New products released in the market are becoming more and more complex due to sophisticated technological advancements and are captivated with higher product life, inducing a market for second-hand products. In order to meet the customer expectation about satisfactory performance of such products or as a means of survival in an increasingly fierce market conditions, second-hand warranty has gained importance (Refer Chattopadhyay and Murthy [6]). The present work develops stochastic models based on the warranty policies employed and attempts to compute the expected warranty cost for the system using quasi-renewal process. Also, it incorporates preventive

repair within warranty for second-hand products and the long run average cost rate for the system is obtained. The lay out of the paper is as follows: brief literature review and notations are presented in Section 2 while model assumptions are made in Section 3. Explicit cost expressions are provided in Section 4. Numerical illustrations are discussed in Section 5. Finally Section 6 summarizes the work.

2. Literature Review

Maintenance using imperfect and minimal repairs has been studied by many authors. Brown and Proschan [3] explored one such imperfect maintenance in which at the time of each failure a perfect repair occurs with probability p and a minimal repair occurs with probability $(1-p)$. (Refer Pham and Wang [18]). Kijima et al. [11] explored a maintenance model with general repair, which brings the state of the system to a better state after a repair. (Refer Kahle[10]). Later geometric processes and quasi-renewal processes were considered appropriate for stochastically deteriorating systems (Refer Lam [13], Lam and Zhang [15], Leahu [16]). The cost analysis of such deteriorating systems would be incomplete without warranty. Blishcke and Murthy [1] provided taxonomy for various warranty policies. For a recent literature review see Murthy and Djameludin [17]. Chattopadhyay and Rahman [5] examined the development of lifetime warranty policies. Preventive repair is viewed as another important class of maintenance to enhance the system reliability (Refer Lam [14]). Yeh et al [23] focused on analyzing the impact of a free minimal repair warranty on the optimal periodic replacement policy. Recently Chukova and Hayakawa [8] considered instantaneous repairs during warranty modeled using quasi-renewal processes (Refer Samalti and Taner [20]). Chattopadhyay and Murthy [4] analyzed second-hand products while offering warranty. Saidi-Mehrabad et al [19] analyzed two effective ways for improving reliability of second-hand products sold with warranty.

2.1 Notations

X_i	:	time between $(i-1)^{th}$ and i^{th} repair; $i = 1, 2, 3, \dots$; independent random variables
r	:	quasi-renewal parameter
W	:	fixed warranty period
$F_i(x)$	:	Cdf $\{X_i\}; i = 1, 2, 3, \dots$
$N(W)$	:	no. of repairs in $[0, W)$
$K_i(x)$	:	convolution of $F_i(x)$
$C_1(W)$	:	expected warranty cost during W in Model 1
$C_2(W)$	:	expected warranty cost while using combination of parts
c_r	:	cost of each repair
C	:	running cost of the system
C_p	:	cost of preventive repair
c_f	:	cost of a failure
c_w	:	warranty cost
c_{rr}	:	reward rate
χ_1	:	expected cost due to catastrophic failure
χ_2	:	expected cost due to warranty expiration
χ_3	:	expected reward rate
$E[L]$	:	expected length of a cycle in Model 2
$C(\tau, W)$	:	long run average cost per unit time

3. Model Formulation

In this section two models: fixed and preventive repair warranty models are developed to analyze the warranty cost for second-hand products using quasi-renewal processes. In general, factors which affect the warranty cost for a second-hand product are age, usage and maintenance history. In this article we consider only the age of a system with instantaneous repairs. A quasi-renewal process is defined as follows:

A counting process $\{N(t), t \geq 0\}$ is a quasi-renewal process with parameter $r > 0$ and first operating time X_1 , if $X_i = r^{i-1}Z_i$, $i = 1, 2, 3, \dots$ and $\{Z_i\}_{i=1}^{\infty}$ forms a renewal process. The Cdf and pdf of X_n ; $n = 1, 2, 3, \dots$ are given respectively by $F_n(x) = F_1\left(\frac{x}{r^{n-1}}\right)$; $n = 1, 2, 3, \dots$ and $f_n(x) = \frac{1}{r^{n-1}} f_1\left(\frac{x}{r^{n-1}}\right)$; $n = 1, 2, 3, \dots$

3.1 Model 1: Fixed Warranty Model

In this model we consider a second-hand repairable deteriorating system, where deterioration occurs due to aging and accumulated wear. Failures occur at random instants of time, and are assumed to be independent. On each failure, a repair action takes place and the sequence of operating times follow a quasi-renewal process, with parameter r and first operating time X_1 . A free replacement warranty policy is offered for a fixed warranty period W . The objective is to analyze a fixed warranty model for second-hand products employing quasi-renewal processes, through the expected warranty cost for the system.

3.2 Model 2: Preventive Repair Model

A new system is considered at time $t = 0$, and a preventive repair, not as good as new, is carried out whenever the working time of the system reaches τ and the system is still working. The sequence of working times are assumed to be independent and constitute a quasi-renewal process with parameter r and first working time X_1 . The system is governed by a free replacement warranty policy, while a replacement is done whenever a catastrophic failure happens or the expiration of warranty, whichever is earlier. The objective is to find the optimal preventive repair time using the long run average cost per unit time for the system.

4. Cost Analysis

This section aims to formulate the cost equations for the models proposed in Section 2. The expected warranty cost and the long run average cost per unit time for a second-hand system are derived, when sold with free replacement warranty and while adopting preventive repair over a fixed warranty period W respectively.

4.1 Model 1

Let us suppose that the first failure time distribution for a second-hand system of age $A = a$ is given by

$$F_1(x) = \frac{F(x+a) - F(a)}{F(a)} \quad (1)$$

Since the sequence of operating times X_n 's are quasi-renewals, the Cdf of X_n is given by

$$F_n(x) = F_1\left(\frac{x}{r^{n-1}}\right) = \frac{F\left(\frac{x}{r^{n-1}} + a\right) - F(a)}{F(a)}, \quad n = 1, 2, 3, \dots \quad (2)$$

and the expected number of repairs in the fixed warranty period W is given by

$$M(W) = E[N(W)] = \sum_{n=0}^{\infty} nP\{N(W) = n\} = \sum_{n=0}^{\infty} n(K_n(W) - K_{n+1}(W)) = \sum_{n=1}^{\infty} K_n(W) \quad (3)$$

Subsequently the expected warranty cost for the second-hand system is given by

$$C_1(W) = c_r M(W) \quad (4)$$

where $M(\cdot)$ is the quasi-renewal function.

Further, we analyze the adoption of a combination of products for this model by making an assumption, that the manufacturer can perform a repair by replacing the failed parts by a new/used part. Let p be the probability of replacing a failed part by new and $(1-p)$ be the probability of replacing a failed part by a used part. Also c_n and c_u be the costs of replacing the new and used parts respectively. Then the expected warranty cost for the system is given by,

$$C_2(W) = pc_n M_1(W) + qc_u M_2(W) \quad (5)$$

where $M_1(W)$ and $M_2(W)$ are the quasi-renewal functions associated with the replacement of new and used products respectively. Equations (4) and (5) are further analyzed using a numerical example in Section 4.

4.2 Model 2

In this subsection we obtain the long run average cost per unit time for the preventive repair model. The long run average cost per unit time is given by

$$C(\tau, W) = \frac{\text{Expected cost in a cycle}}{\text{Expected length in a cycle}} = \frac{C + C_p + \chi_1 + \chi_2 + \chi_3}{E[L]} \quad (6)$$

A cycle terminates either due to expiration of warranty or a catastrophic failure whichever occurs first.

$$L = \begin{cases} U_{n+1} & I(U_{n+1} < W, X_1 > \tau, X_2 > \tau, \dots, X_n > \tau, X_{n+1} < \tau) \text{ when catastrophic failure occurs} \\ U_{n+1} & I(U_n < W < U_{n+1}, X_1 > \tau, X_2 > \tau, \dots, X_n > \tau) \text{ when warranty expires} \end{cases}$$

$$\begin{aligned} E[L] &= \sum_{n=1}^{\infty} E[U_{n+1} I(U_{n+1} < W, X_1 > \tau, X_2 > \tau, \dots, X_n > \tau, X_{n+1} < \tau)] \\ &\quad + \sum_{n=1}^{\infty} E[U_{n+1} I(U_n < W < U_{n+1}, X_1 > \tau, X_2 > \tau, \dots, X_n > \tau)] \\ &= \sum_{n=1}^{\infty} n\tau F_{n+1}(\tau) + \sum_{n=1}^{\infty} \int_{\tau}^{W-n\tau} t dF_{n+1}(t) + \sum_{n=1}^{\infty} n\tau \bar{F}_{n+1}(W-n\tau) + \sum_{n=1}^{\infty} \int_{W-n\tau}^{\infty} t dF_{n+1}(t) \end{aligned}$$

4.2.1 Expected Cost due to Catastrophic Failure

$$\text{Let } \chi_1 = \sum_{n=1}^{\infty} E\left[c_f I(U_{n+1} < W, X_1 > \tau, X_2 > \tau, \dots, X_n > \tau, X_{n+1} < \tau)\right] = c_f F_{n+1}(W-n\tau) \prod_{i=1}^n \bar{F}_1\left(\frac{\tau}{r^{n-1}}\right) F_{n+1}(\tau)$$

4.2.2 Expected Cost due to Warranty Expiration

$$\text{Let } \chi_2 = \sum_{n=1}^{\infty} c_w I(U_n < W < U_{n+1}, X_1 > \tau, X_2 > \tau, \dots, X_n > \tau) = c_w \sum_{n=1}^{\infty} \bar{F}_{n+1}(W-n\tau) \prod_{i=1}^n \bar{F}_1\left(\frac{\tau}{r^{n-1}}\right)$$

4.2.3 Expected Reward Rate

$$\begin{aligned} \text{Let } \chi_3 &= \sum_{n=1}^{\infty} (-c_{rr}) E[U_{n+1} I(U_{n+1} < W, X_1 > \tau, X_2 > \tau, \dots, X_n > \tau, X_{n+1} < \tau)] \\ &\quad + \sum_{n=1}^{\infty} (-c_{rr}) E[U_{n+1} I(U_n < W < U_{n+1}, X_1 > \tau, X_2 > \tau, \dots, X_n > \tau)] \\ &= -c_{rr} \sum_{n=1}^{\infty} n\tau F_{n+1}(\tau) - c_{rr} \sum_{n=1}^{\infty} \int_{\tau}^{W-n\tau} t dF_{n+1}(t) - c_{rr} \sum_{n=1}^{\infty} n\tau \bar{F}_{n+1}(W-n\tau) - c_{rr} \sum_{n=1}^{\infty} \int_{W-n\tau}^{\infty} t dF_{n+1}(t) \end{aligned}$$

The objective is to obtain τ^* using Equation (6).

5. Numerical Illustrations

This section is devoted to analyze the various cost equations obtained in Section 3. Suppose that the operating times are exponentially distributed with means λ_n : $f_n(x) = \lambda_n \exp\{-\lambda_n x\}$; $x \geq 0$; $\lambda_n > 0$, $n = 1, 2, 3, \dots$. Since X_i 's are

quasi-renewals we have $f_n(x) = \frac{\lambda_1}{r^{n-1}} \exp\left\{-\frac{\lambda_1 x}{r^{n-1}}\right\}$, $x \geq 0$, $\lambda_1 > 0$; $n = 1, 2, 3, \dots$ and $E[X_n] = \frac{r^{n-1}}{\lambda_1}$, $n = 1, 2, 3, \dots$.

When the parameter $\lambda_1 = 0.06$ and the repair cost $c_r = 100$, Figure 1 illustrates that the cost of an improved repair is lesser when compared to an imperfect repair. Consequently Equation (5) is examined for the various cases when repairs are performed by new or used parts.

Case 1: $p = 1; q = 0$: This is the case when the system, on failure is replaced by new parts only. In Figure 2, the bold line depicts the highest expected warranty cost for the system as expected.

Case 2: $p > q$: This corresponds to the case in which the manufacturer does a few repairs, by using new parts while the others with used parts. As, the probability of a replacement by new parts is more, the expected warranty cost is higher as seen in Figure 2 (refer smooth line with dots).

Case 3: $p = q$: The smooth line in Figure 2, demonstrates the effect of repair action carried out being equally likely.

Case 4: $p < q$: In this case the probability of replacement by used parts is more when compared to new parts and hence the dotted line in Figure 2 illustrates lesser expected warranty cost when compared to the former three cases.

Case 5: $p = 0; q = 1$: The present case occurs when a manufacturer employs used parts. The fact that the cost of repairing a second-hand system by a used part will be relatively cheaper is reflected in the least expected warranty cost as depicted in Figure 2.

Hence we conclude that the consideration of used parts while repairing a warranted second-hand product would be cost-effective.

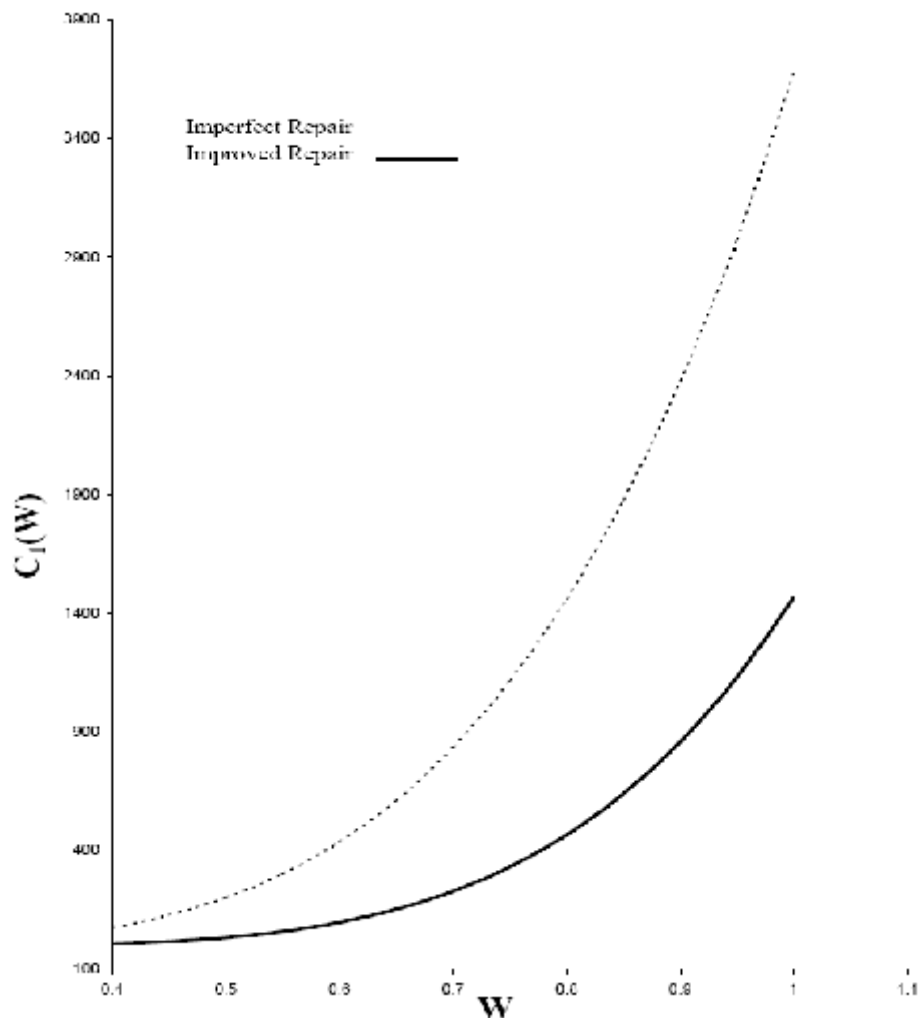


Figure 1: Comparison of improved and imperfect repairs for Model 1

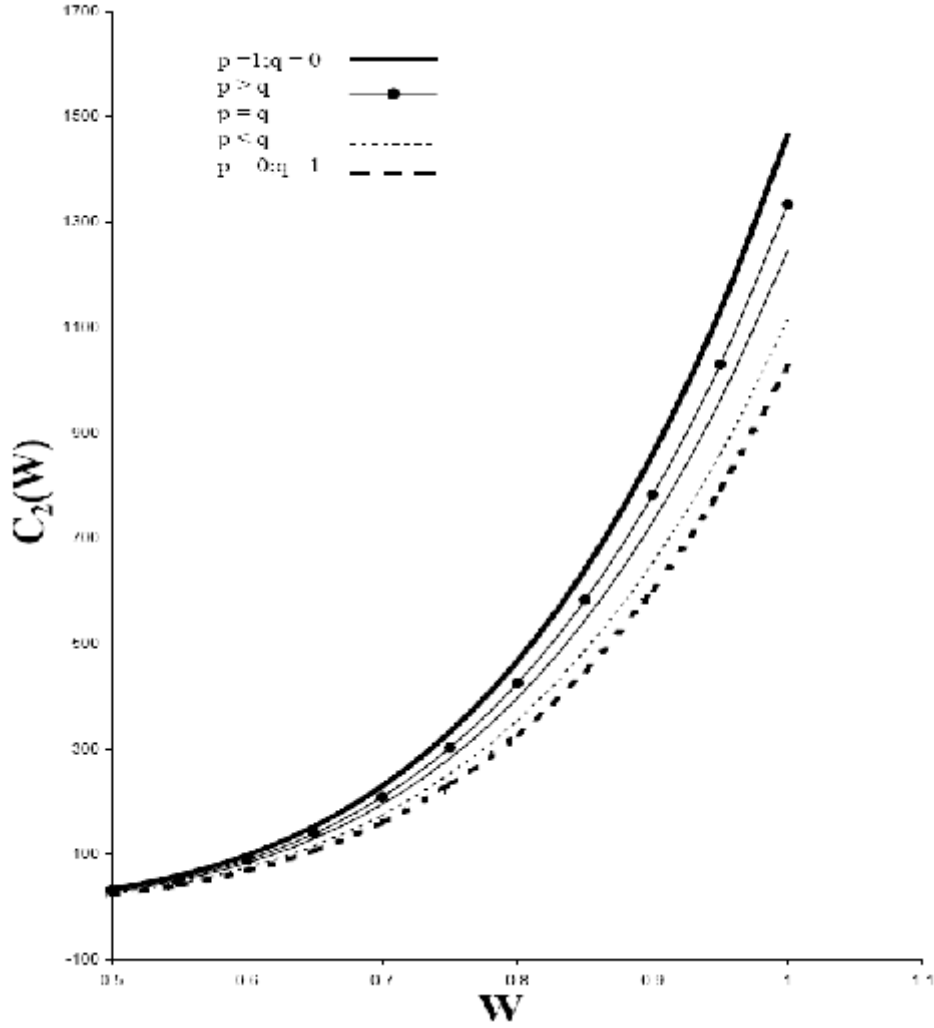


Figure 2: Combination of products for Model 1

Next, we examine $C(\tau, W)$ when $\lambda_1 = 0.06, r = 1.6, W = 2, a = 3, c_w = 5, c_f = 3, c_{rr} = 2.1, C = 1000$ and $C_p = 500$ for which the optimal preventive repair time is obtained as $\tau^* = 600$. The expected cost in a cycle for optimal (τ^*, W) is -6.8995×10^6 and $C_1(W) = 0.3479$. We observe that the expected cost in a cycle with optimal preventive repair time is less, depicting the importance of preventive repair for a second-hand product with warranty.

6. Concluding Remarks

In this paper we have analyzed two cost models for second-hand products using quasi-renewal process and warranty. In the first model, we have obtained the expected warranty cost for the system and also analyzed the importance of using a new or used product, while performing a repair action. Further in the second model, the optimal preventive repair time is obtained. Also, from the numerical illustrations we conclude that the quasi-renewal improved repairs and preventive repairs during warranty are appropriate while modeling second-hand products. The analysis in this paper will be helpful for manufacturers while making decisions regarding the quality of the repair and performing the preventive repair during warranty for such products. However, the complexity in obtaining an explicit expression for the quasi-renewal function analytically led to the adoption of a computational procedure. Future research directions may include various other warranty policies, and the consideration of past history of a second-hand

product. The authors have analyzed two dimensional warranties for these products and have been reported elsewhere.

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