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A hybrid Markov system dynamics approach for availability analysis of degraded systems

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Abstract

Time dependent availability is an important index used for evaluation of operational performance of repairable industrial systems. Traditionally, Markov analysis procedures are carried out for such studies. This becomes mathematically intensive when applied for larger systems. Moreover, this method does not indicate at what time a system reaches its steady state. However, in practice it is important to know at what time the steady state actually begins. This paper proposes a hybrid approach called as Markov System Dynamics (MSD) approach which combines the Markov model with system dynamics simulation for the time dependent availability analysis. The proposed method is also capable to compute the repairable system steady state time. The new approach is illustrated for a three state degraded system with repair. The results are compared with that obtained by traditional Markov analysis to validate the Markov System Dynamics (MSD) approach as an alternative for the time dependent availability analysis.

Key words
System dynamics simulation, availability analysis, Markov approach, degraded repairable systems.

1. Introduction
Availability is one of the most important indices used for evaluation of the effectiveness of any industrial plant, where most of the machines are repairable systems. Point, interval, and steady state availabilities are some availability indices used in literature. Point availability is the probability that a system in an operational state at a given time. Similarly, interval availability is the probability that a system is in operational state during a specified interval. Steady state availability is the probability that s system is in an operational state after the system reaches steady state.

Among several available methods, Markov method is widely used for reliability and availability analysis. A Markov chain analysis looks at a sequence of events, defined as transitions between states, and calculates the relative probability of encountering these events in both the short run and the long run. Markov chain models provide accurate long run availability and failure characterization calculations, and can sometimes be solved analytically, but are hard to formulate and involve high computational effort particularly as the number of states grows large. The solution procedure of these models is also mathematically intensive. The existing literature shows that if the failure time and/or the repair time distributions are not exponential, the analytical expression for the availability becomes difficult. The complexity of the modern engineering systems besides the need for realistic considerations when modelling their availability/reliability renders analytical methods very difficult to be used. There is extensive literature on availability characteristics of degraded repairable systems with two or three components under varying assumptions on the failures and repairs [1-4]. In these papers, authors used either Laplace transforms method or Lagrange's method to solve Chapman Kolmogorov differential equations associated with a particular problem. It has been observed that these methods involve complex computations and it is very difficult to calculate time dependent and steady state availability of the systems by these methods.

Therefore, several authors have suggested simulation techniques for estimating the availability [5-7]. Studies have also been performed to improve and optimise the availability of a system through different methods and techniques [8-12]. Many researchers have been searching for alternate methodologies for more practical and realistic availability analysis [13-14].

The present work proposes a hybrid approach called as Markov System Dynamics (MSD) simulation approach which combines the Markov model and system dynamics simulation for time dependent availability analysis and to study the dynamic behavior of degraded repairable systems. The proposed methodology is illustrated for a three state degraded system with repair. This methodology can be extended easily for more complex multi state systems.

The remaining sections of the paper are structured as follows. Section 2 describes the modeling aims and approach. Section 3 presents the Markov system dynamics approach to system availability assessment. Section 4 illustrates the availability assessment of a three state degraded system with repair using the proposed approach and compares the results with traditional techniques. Section 5 presents conclusions of this paper.

2. Modeling Aims and Approach

Simulation has been used as an approximation tool to eliminate the limitations of analytical Markov chains. It has been shown [15-16] that the stationary, continuous time Markov models are algebraically equivalent to linear system dynamics models. From this, a system dynamics representation of Markov models opens up the possibility of numerical solution and of avoiding the tedium of analytical solution. Another advantage of system dynamics modelling is that it is easy to experiment with alternative values of parameters. Hence sensitivity analysis can be performed easily during reliability/availability estimation and prediction. Finally, the steady state solutions for these problems can be computed by visual inspection of the flow diagrams and by making use of the fact that the net flow into a level is zero in the steady state. This has motivated us to propose a novel hybrid approach called as Markov System Dynamics (MSD) approach which combines the Markov model with system dynamics simulation in a simple and efficient way for time dependent availability analysis.

3. The proposed MSD approach

The approach of system dynamics was created and developed in the late 1950s by a group of researchers led by Forrester at the Massachusetts Institute of Technology (MIT), Cambridge, MA [17]. It is a methodology for modelling and redesigning manufacturing, business, and similar systems that are part man, part machine [18]. In the present work, initially the Markov analysis procedure is presented through the use of an example such as a three state degraded system with repair to derive and calculate its time dependent availability. Thereafter, the same system is modeled by the proposed approach.

Step 1: System's states description and assumptions

A three state degraded system with repair has been considered and analyzed through Markov analysis as follows.

3.1. Conventional approach

In this system the following assumptions are used to analyze the system through Markov analysis.

- The system is in one of three states: operating, degraded, or failed.
- The system can be repaired to a fully operational status only after it fails.

The transition rate diagram of this system according to Markov analysis is shown in Figure 1. In this diagram, state1 indicates that the system is fully operational; state 2 indicates that the system operates in a degraded mode and state 3 indicates the system failed state. Treating this as a Markov process, the corresponding equations are obtained.

$$\frac{dP_1(t)}{dt} = rP_3(t) - (\lambda_1 + \lambda_3)P_1(t) \tag{1}$$

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - \lambda_2 P_2(t) \tag{2}$$

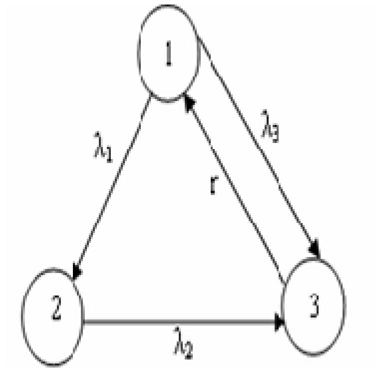

Figure 1: Rate diagram of the three-state system

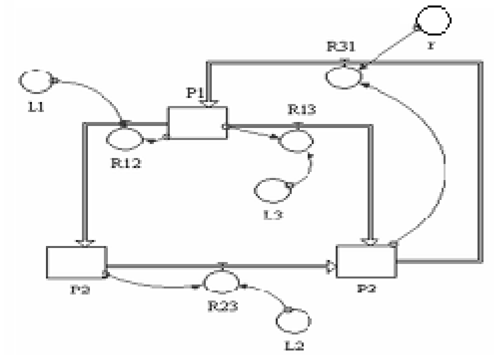

Figure 2: MSD model of the three-state system

$$\frac{d\,P_3(t)}{dt} = \lambda_3 P_1(t) + \lambda_2 P_2(t) - r\,P_3(t) \tag{3}$$

$$P_1(t) + P_2(t) + P_3(t) = 1 \tag{4}$$

And the point availability which provides the probability that this system is operating at time t, is given by

$$A(t) = P_1(t) + P_2(t) \tag{5}$$

The solution of the above Kolmogorov system of differential equations is required to calculate the time dependent availability. There is considerable mathematical difficulty in solving these equations. Therefore, steady state (instead of time dependent) availability is evaluated by assuming time equals to infinity in the equations (eq.1 to 5). However, this analysis does not give the point availability and the exact time at which a system reaches its steady state. In many practical situations we require these quantities for planning maintenance activities.

3.2. The MSD Approach

In the proposed MSD approach, the system state probabilities are evaluated by observing the dynamic behavior of the system over its entire simulated mission period. This method is capable of finding the point and steady state availabilities and the steady state time. This approach is discussed in the following sections.

Step 2: Data collection

The proposed MSD methodology starts after identification of the system states as mentioned in the previous section. Thereafter, adequate data collection is needed to compute the failure and repair rates of each state. For example, let us consider a system having three states: operating, degraded, or failed. When operating, it fails at the constant rate of 2.0 failures per day and becomes degraded at the rate of 3.0 failures per day. If degraded, its failure rate changes to 1.0 failure per day. Repair is carried out only in the failed mode (to the operating state), with repair rate of 10.0 failures per day. The operating and degraded states can be considered as the system available states.

Step 3: Building the MSD simulation model

The next step in the modelling process is to convert the transition rate diagram of system in to the rate and level diagram. The rate diagram (Figure1) of a three state degraded system with repair is now converted into a comprehensive system dynamic model. This is presented in Figure 2.

In the model depicted in Figure 2, the three states of a degraded system with repair are indicated with level variables P_1(t), P_2(t), P_3(t) and the state transitions are indicated with rate variables (R_{12}, $R_{13,}$ R_{23}, R_{31}) with the corresponding transition rates L_1, L_2, L_3, r (L_1 indicates failure rate of system when operating, L_2 indicates degraded failure rate of the system, L_3 indicates increased failure rate of the system after it is degraded and 'r' indicates its repair rate only in the failed mode and it leads to the operating state). The initial value of system availability (as indicated at the level variable P_1(t)) is assumed as unity. On simulation, the level of system availability decreases due the rate of failures of the system in its three states: operating, degraded, or failed and it recovers with the repair. These can be measured as probability density functions (pdf) of the system. The auxiliary variables (failure rate and repair rates of the system) influence the rate variables along the entire mission or operating time. Additionally, the level of the failure state of the system (i.e., level variable $P_3(t)$) is increases after the system degradation, leading to the declining availability of system.

Step 4: MSD simulation
The next stage of MSD approach is to simulate the comprehensive MSD model of the system. The following algorithm explains the proposed Markov System dynamics simulation procedural steps. For simulating the Markov system dynamics model, several available system dynamics software can be used.

Proposed algorithm:
Step1: The values of transition rates (L_1, L_2, L_3, r) *and the time interval dt are taken as inputs. Also the total time T is taken as input, i.e. the time for which the system has to be simulated.*
Step2: Initially set $P_1(t)$ *equal to one.*
Step3: Set P_i *equal to zero for i= 2, 3.*
Step4: A conditional loop is formed with the condition, $t<T$.
Step5: in each execution of the loop, the time is increased by DT, i.e. $t = t + dt$.
So, the loop will continue till time T with each step taken at time difference dt.
Step6: As assumed $P_1(t)$ *initially has unit probability, and the system fails when it becomes zero. Run a conditional loop as long as the condition i.e. the probability of* $P_1(t)>0$ *is satisfied.*
Step7: Within the loop all the rate variables are calculated. The probabilities of the states are calculated first and the outflow rates are calculated according to the logic as follows.

$$P_1(t)= P_1 (t-dt) + (R_{31}-R_{12}-R_{13})* dt \quad (6)$$

$$P_2 (t) = P_2 (t-dt) + (R_{12}--R_{23}) * dt \quad (7)$$

$$P_3 (t) = P_3 (t-dt) + (R_{13}+R_{23}-R_{31}) * dt \quad (8)$$

Step8: Then all the required values are displayed and the required graphs can be drawn by using these values to study the dynamic behavior.

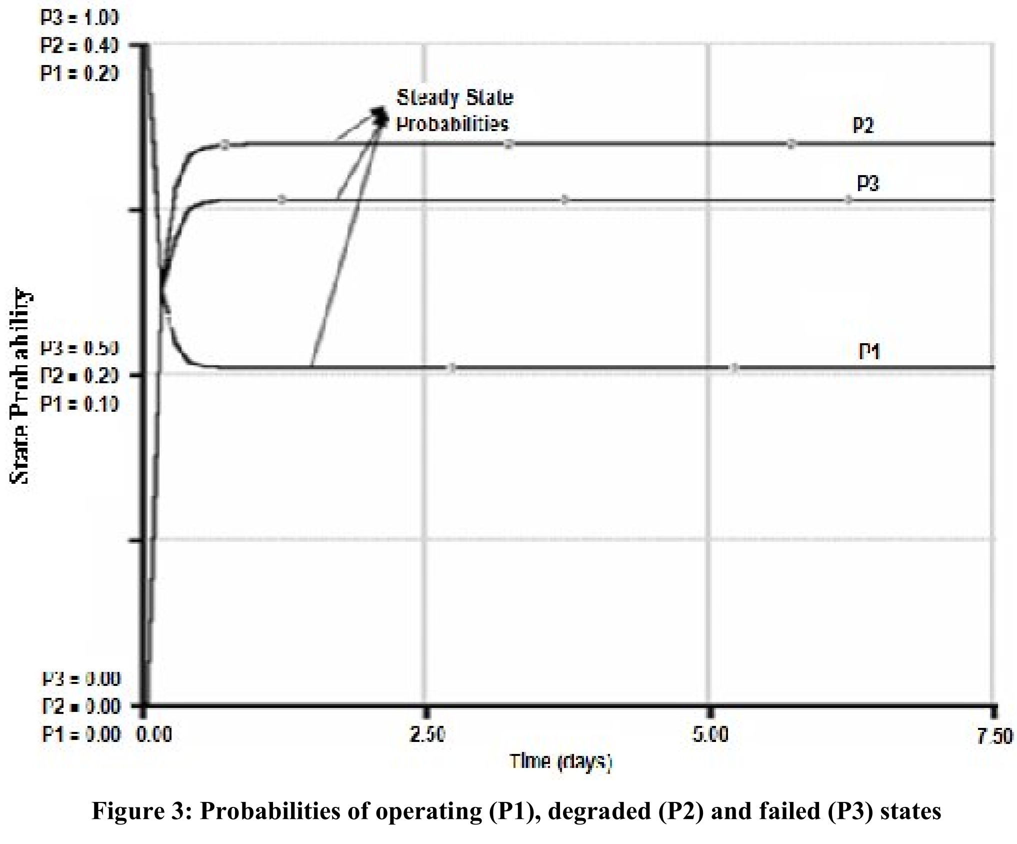

Figure 3: Probabilities of operating (P1), degraded (P2) and failed (P3) states

3.3 The Model Experimentation

By implementing this algorithm in the software Stella, the system is simulated by considering all the three states to analyze its time dependent state probabilities $P_1(t)$, $P_2(t)$, and $P_3(t)$. The simulation results are shown in the Figure 3 and also presented in the Table 1. The system availability is computed from these probabilities. In this case, the sum of $P_1(t)$ and $P_2(t)$ gives the system time dependent availability. It can be inferred from the Figure 3 that the probability of the operating state of the system decreases and reaches its steady state with increase in time due to degradation and subsequent failure. The time at which the system reaches its steady state can also be established from these results.

4. Results and Discussion

The example considered here can also be solved by the conventional Markov analysis. But it requires many Kolmogorov systems of differential equations to compute the availability. As discussed earlier, there is much difficulty in solving these equations. The system time dependent availability is thus computed from system state probabilities which are evaluated using a rigorous mathematical treatment. The steady state availability of the system is evaluated by assuming time equals to infinity in the Kolmogorov system of differential equations. However, this analysis does not give the exact time at which a system reaches its steady state. In many practical situations we require to find the time at which our system reaches steady state conditions for planning maintenance activities. The proposed MSD method in this paper is capable of finding this steady state point very easily. The worked out example shows this steady state point and also it gives the point, interval and steady state availabilities. The results obtained are presented in Table1, and Figure 3. Table 1 clearly indicates that the simulated results very closely match with computed values. It is found that the system reaches its steady state at 1.375 days and its steady state availability is 0.847458.

Table 1: State probabilities and system availability

Time (days)	P1	P2	P3	Availability	
				By MSD approach	By conventional approach
0	1.000000	0.000000	0.000000	1.000000	1.000000
½	0.511475	0.336914	0.151611	0.848389	0.848391
1	0.508493	0.338970	0.152537	0.847463	0.847493
1.375	**0.508475**	**0.338983**	**0.152542**	**0.847458**	0.847448
2	0.508475	0.338983	0.152542	0.847458	0.847448
3	0.508475	0.338983	0.152542	0.847458	0.847448
5	0.508475	0.338983	0.152542	0.847458	0.847448
10	0.508475	0.338983	0.152542	0.847458	0.847448

5. Conclusions

In this paper, a hybrid Markov System Dynamics (MSD) approach which combines the Markov model with system dynamics simulation has been proposed for time dependent availability analysis and to study the dynamic behavior of systems. The proposed framework is described in detail and illustrated for a three state degraded system with repair with a numerical example. The results of the simulation when compared with that obtained by traditional Markov analysis clearly validate the Markov System Dynamics (MSD) approach as an alternative approach for time dependent availability analysis. It is also shown that the MSD approach clearly indicates the time at which the system reaches its steady state. It is worth mentioning here that finding the steady state condition is extremely difficult or impossible using traditional approaches. The proposed MSD approach has done this very easily.

The only limitation of the MSD approach is that it requires development of the equivalent system dynamic model (rate and flow diagram) from that of the state transition diagram. However, this will not be a serious issue with adequate knowledge of System Dynamics and Markov approaches.

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