Proceedings of the 2011 International Conference on Industrial Engineering and Operations Management
Kuala Lumpur, Malaysia, January 22 – 24, 2011

An Optimal Maintenance/Production Planning for a Manufacturing System Under Random Failure Rate and a Subcontracting Constraint

Hajej Zied
Dellagi Sofiene
Rezg Nidhal
INRIA Nancy Grand Est /LGIPM
University of Metz, Ile du Saulcy, 57045 Metz Cedex – FRANCE

Abstract
This paper deals with a problem of production and maintenance of manufacturing system under subcontracting constraint. We have developed an integrated production/maintenance policy for a manufacturing system subjected to a random failure and calling up on subcontractor machine.
The problem consists on a machine, unable to satisfy a random demand. That's why it called upon another machine. In order to assure simultaneously an economical production planning and optimal maintenance strategy, a conjoint optimization is made which minimize the total production, holding and maintenance cost. An analytical study and a numerical example are presented in order to prove the developed approach.

Keywords
Manufacturing system, subcontractor, random demand, Failure rate, maintenance strategies and production plan.

1. Introduction
Production and maintenance strategies of manufacturing systems subject to uncertainties such as random demand, machine failures and the material availability variation. In order to limits these uncertainties, the relationships between enterprises are getting improved towards more cooperation and collaboration. In this context, many companies have recourse to the industrial subcontracting which became a very widespread practice to face these uncertainties. A number of approaches have been proposed in the literature and in most cases concern the determination of the economic manufacturing quantity for different products on a single or multiple manufacturing machines.
An integrated approach of maintenance policy and production planning and control has recently become an important research area. In this context, Rezg et al presented a common optimization of the preventive maintenance and stock control in a production line made up of *N* machines [1]. Van der Duyn Schouen and Vanneste addressed a production line of two machines separated by a buffer and proposed a preventive maintenance policy based not only on the age of the machine but also on the size of the buffer, both of which are used to determine when to perform a preventive maintenance action [2]. The best time at which a preventive maintenance action in a manufacturing system must be carried out is very important for minimizing the total cost of maintenance and production. A number of studies have tackled the performance evaluation of production systems that are subject to random failure [3, 4].
In this paper, we will study a problem of an integrated maintenance policy for a manufacturing system calling upon subcontractor. In the literature various studies treating subcontracting in manufacturing. New maintenance/production strategies by taking into account the context of subcontractor are studied by [5], developed and optimize a new maintenance policy with taking into account machine subcontractor constraints.
Still with the objective of minimizing the total cost of maintenance and production, the present paper extends a study presented in [6] tackled a production/ maintenance problem of a manufacturing system in a subcontractor machine. These authors presented a new production and maintenance policies. These policies take into account the influence of the production rate on the material failure rate in order to establish the optimal maintenance plan.
The remainder of the paper is organized as follows. Section 2 states and formulates the problem and defines the equations of production and maintenance policies. The analytical studies are developed for evaluating maintenance and production strategies in section 3. In Section 4 we present a simple numerical example in order to illustrate the analytical results. Finally, Section 5 provides the conclusions

2. Problem Setting

The production system studied is composed of one machine *M* which produces a single product, working at a rate *u*, to feed a stock S_1, in order to meet a random demand law characterized by a Normal distribution. The Normal mean and standard deviation parameters are respectively denoted by $\hat{d}$ and σ_d^2.

The system also composed of a second stock S_2 where the customer products are returned. After inspection, the part of this stock S_2 will be recycled. For the recycling the system uses a subcontractor composed of a machine M_2 which produces the same type of product.

Points of view reliability, machine *M* is subject to a random failure. The probability degradation law of machine *M* is described by the probability density function of time to failure *f(t)* and for which the failure rate $\lambda(t)$ increases with time and according to the production rate. Failure of machine *M* can be prevented by preventive maintenance actions. Concerning subcontractor machine we have estimated its availability rate β_s.

In this model we are concerned with the problem of the jointly optimal production and maintenance planning problem formulation. Our objective is to establish simultaneously an economical production plan and an optimal preventive maintenance period satisfying the randomly demand and taking into account the subcontractor constraint. The use of the optimal production plan in maintenance study is justified by the fact of taking into account the natural influence of the production rate on the evolution of the failure rate of the machine.

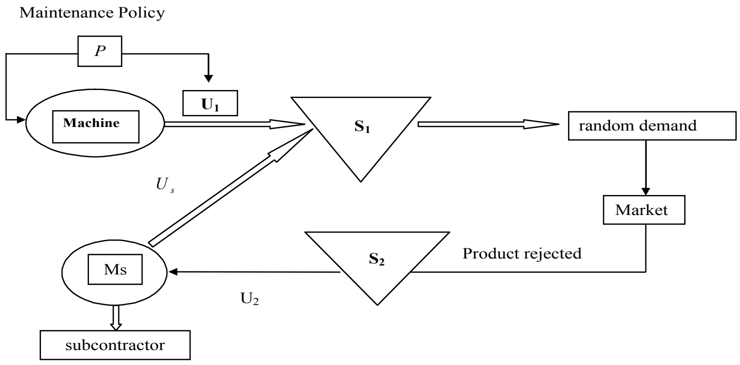


Fig.1. Problem description

2.1 Notation

H : finite production horizon
Δt: period length of production
$S_1(k)$: inventory level at the end of the period k ($k=1,\ldots\ldots,H/\Delta t$)
$S_2(k)$: inventory level at the end of the period k ($k=1,\ldots\ldots,H/\Delta t$)
$u(k)$: production level at period k
$u_s(k)$: production level of subcontractor machine at period k
$d(k)$: demand level at period k
βs: availability of subcontractor machine
C_{pr} : unit production cost
C_{ps} : unit production cost of subcontractor machine
C_s: holding cost of a product unit during the period k for S_1 and S_2
$f(t)$: probability density function of time to failure for the machine
$R(t)$: reliability function
Cp: preventive maintenance action cost
C_c: corrective maintenance action cost
mu: monetary unit
U_{max}: maximal production rate
F: total expected cost of production and inventory over the finite horizon H.
C: total expected maintenance cost per time unit
θ: probabilistic index (related to customer satisfaction)
χ: the average number of failure

2.2 Problem formulation

To establish an economical production plan and optimal maintenance strategy, we define a stochastic model that minimizes the total costs over a finite horizon. The goal of the production/maintenance plan is to determine the greatest combination of production, inventory levels and failure rate that minimizes the total cost over a planning horizon. In our model the customer satisfaction is made at the end of each period. The maintenance policy is taking into account the production rate in determining the optimal number N of partitions of preventive maintenance actions .

Formally, the problem is defined as follows:

$$\underset{(u,N)}{Min}\; F = C_s \times \left(E\left\{S_1(H)^2\right\} + E\left\{S_2(H)^2\right\}\right) + \sum_{k=1}^{H-1}\left(C_{pr} \times E\left\{u(k)^2\right\} + C_{ps} \times E\left\{u_s(k)^2\right\}\right)$$
$$+\sum_{k=1}^{H-1} C_s \times \left(E\left\{S_1(k)^2\right\} + E\left\{S_2(k)^2\right\}\right) + (N-1)\times M_p + M_c \times \chi(u,t)$$

Under the following constraints:

$$s_1(k+1) = s_1(k) + u_s(k) + u(k) - d(k) \qquad k = 0,1,....., H-1 \tag{1}$$
$$\text{Prob}\left[s_1(k+1) \geq 0\right] \geq \theta \qquad k = 0,1,\ldots\ldots, H-1 \tag{2}$$
$$s_2(k+1) = s_2(k) + r(k) - u_2(k) \qquad k = 0,1,....., H-1 \tag{3}$$
$$u_s(k) = \beta_s \cdot u_2(k) \tag{4}$$
$$u_2(k) = \alpha_2 \cdot r(k) \tag{5}$$
$$r(k) = \alpha_1 \cdot d(k-l) \tag{6}$$
$$0 \leq u_k \leq u_{\max} \qquad k = 0,1,....., H-1 \tag{7}$$

Where F denotes costs function of production, holding and maintenance. S_1 denotes the vector of states of the system where information about principal stock witch satisfies the demand; S_2 presented the vector of states of the second stock; u denotes the control variable of the system which contains information about the production rate of machine; u_s denotes the control variable of the capacity of production of subcontractor machine. The study set defines the inventory balance equation for each time period (1) take into account the production of principal and subcontractor machines. The equation (3) defines the inventory balance equation of second stock S_2. The constraint (2) imposes the service level requirement for each period and denotes the lower physical limit of inventory variable. The probabilistic constraint of inventory is taken as a chance-constraint in order to ensure that the inventory level is greater than zero with conditional probability of at least θ at each time period k.

2.3 Developed model

The idea is to develop and optimize the expected production and holding costs and the maintenance cost over the finite time horizon. It's assumed that the horizon is portioned equally into H periods. As mentioned above, the demand is satisfied at the end of each period. We recall that our objective is to determine the economical production plan and the optimal maintenance strategy over a time horizon H.

Formally our problem is presented as following:

$$(u,N)^* = \underset{(u,N)}{\min}\left(F(u,N)\right)$$
$$\text{with} \quad u = \left(u(1), u(2), \ldots\ldots u(k) \ldots u(H)\right) \text{ and } N = (1,2,\ldots)$$

The total cost composed by the production expected cost and the holding expected cost

- The expected production cost for period k is given by:

$$\xi_k(u,u_s) = C_{pr} \times E\left\{u(k)^2\right\} + C_{ps} \times E\left\{u_s(k)^2\right\} \tag{8}$$

- The expected holding cost of period k is given by:

$$\zeta_k(S_1,S_2) = C_s \times \left(E\left\{S_1(k)^2\right\} + E\left\{S_2(k)^2\right\}\right) \tag{9}$$

Since that the total expected cost including production and inventory over the finite H periods is expressed by:

$$\Upsilon(u) = C_s \times \left(E\left\{S_1(H)^2\right\} + E\left\{S_2(H)^2\right\}\right) + \sum_{k=1}^{H-1}\left(C_{p_r} \times E\left\{u(k)^2\right\} + C_{ps} \times E\left\{u_s(k)^2\right\}\right) + \sum_{k=1}^{H-1} C_s \times \left(E\left\{S_1(k)^2\right\} + E\left\{S_2(k)^2\right\}\right) \tag{10}$$

Remark:

$u(H)^2$ is not included in the cost formulation because we don't consider the production command at the end of the horizon H.

- the maintenance cost :

We seek to optimize the cost model associated with the preventive maintenance with minimal repair policy derived above. Note that the production rate over the horizon H has an impact on the failure rate $\lambda(t)$. Consequently, the objective here is to take into account the production rate in determining the optimal number of partitions N^* of preventive maintenance actions to be carried out, which in turn means that the preventive maintenance action takes place at $T^* = H/N^*$ tu(time unit).

Thus, the total expected cost until time H is:

$$\varphi(N,U) = (N-1)\times M_p + M_c \times \chi(U,t) \tag{11}$$

With

$\chi(U,t)$: the average number of failure

3. Analytical studies

3.1 Production policy

- Production and holding cost

This section focuses on transforming the total cost into an analytical expression deterministic which will then be easier to solve. Thus the production and holding costs simplified as:

Lemme1:

$$\Rightarrow G(u) = C_s \times \left[\hat{S}_1(H)^2 + \hat{S}_2(H)^2\right] + \sum_{k=0}^{H-1} C_{pr} \times u_1(k)^2 + \sum_{k=0}^{H-1} C_s \times \begin{pmatrix} k\times\sigma_d^2 + (k-1)\times(\beta_s\times\alpha_2\times\alpha_1)^2\times\sigma_d^2 + \hat{S}_1(k)^2 \\ +(k-1)\times\alpha_1^2\times(1+\alpha_2^2)\times\sigma_d^2 + \hat{S}_2(k)^2 \end{pmatrix} + \sum_{k=0}^{H-1} C_{p_s}\times u_s(k)^2 \tag{12}$$

Where $\hat{S}_1(k)$ represents level of mean principle stock at the end of period k

Where $\hat{S}_2(k)$ represents level of mean second stock at the end of period k

- The inventory balance equation:

Letting $d_k = \hat{d}_k$, the state balance equation of stock $S_1(k)$ can be converted to:

$$\hat{S}_1(k+1) = \hat{S}_1(k) + \hat{u}_s(k) + \hat{u}_1(k) - \hat{d}(k) \qquad k = 0,1,\ldots\ldots,H-1 \tag{13}$$

$$\hat{u}_s(k) = \beta_s \cdot \hat{u}_2(k);\ \hat{u}_2(k) = \alpha_2 \cdot \hat{r}(k);\ \hat{r}(k) = \alpha_1 \cdot \hat{d}(k-l)$$

And so the state balance equation of stock $S_2(k)$ becomes:

$$\hat{S}_2(k+1) = \hat{S}_2(k) + \hat{r}(k) - \hat{u}_2(k) \qquad k = 0,1,\ldots\ldots,H-1 \tag{14}$$

- The service level constrain:

Another step toward transforming the problem into a deterministic equivalent is to cast the service level constraint (equation 2) in a deterministic form by specifying certain minimum cumulative production quantities that depend on the service level requirements.

Lemme 2:

$$\Pr ob\left(S(k+1)\geq 0\right)\geq\theta \;\Rightarrow\; u(k)\geq\left(V_d\times V_{d,k-l}\right)\times\varphi^{-1}(\theta)-\beta_s\cdot\alpha_2\cdot\alpha_1\cdot\hat{d}(k-l)+\hat{d}(k)-S_1(k) \qquad k=1,2,\ldots,H-1$$

With:

$U_\theta(\,)$: Minimum cumulative production quantity

$$U_\theta\left(s(k),\theta\right) = \left(V_{d,k}\times V_{d,k-l}\right)\times\varphi^{-1}(\theta) + \hat{d}(k) - \delta\times\hat{d}(k-l) - S(k) \qquad k = 0,1,\ldots,H-1$$

$V_{d,k}$: Variance of demand d at period k

$V_{d,k-l}$: Variance of demand d at period k-l

φ : Cumulative Gaussian distribution function with mean $\frac{1}{V_{d,k-l}} \times \hat{d}(k) - \left(\frac{\beta_s \times \alpha_2 \times \alpha_1}{V_d}\right) \times \hat{d}(k-l)$ and finite variance $\left(\frac{1}{V_{d,k-l}}\right)^2 \times V_{d,k} + \left(-\frac{\beta_s \times \alpha_2 \times \alpha_1}{V_d}\right)^2 \times V_{d,k-l} \geq 0$

φ^{-1} : Inverse distribution function

3.2 Maintenance cost

Our maintenance policy adopted in the problem is a periodic preventive maintenance policy with minimal repair. More precisely, the machine will operate over a given horizon *H*. the maintenance policy adopted is as follows: the *H* production periods is divided equally into *N* parts of duration *T*. Perfect preventive maintenance actions are is performed periodically at times *k.T*, *k*=0,1,…,*N*, and *N.T*=*H* following which the unit is as good as new. Whenever a failure occurs between preventive maintenance actions, the system undergoes a minimal repair. It is assumed that the repair and replacement times are negligible.
The analytic expression of the total expected maintenance cost is as follows, with *T*=*H/N*, $N \in \{1,2,3 \ldots\}$

$$\Gamma_p(U, N) = (N-1) \cdot M_p + M_c \times \chi(u, T)$$

If we assume that $\lambda(T)$ represents the linear failure rate function at period *T* is expressed as following :

$$\lambda_i(t) = \lambda_{i-1}(\Delta t) + \frac{u(i)}{U_{max}} \cdot \lambda_0(t) \quad \forall t \in [0, \Delta t]$$

$$\Rightarrow \lambda_i(t) = \lambda_0(t_0) + B_i + \frac{u(i)}{U_{max}} \cdot \lambda_0(t) \text{ With } B_i = \sum_{l=1}^{i-1} \frac{u(l)}{U_{max}} \cdot \lambda_0(\Delta t);\ B_1 = 0 \text{ et } \lambda_0(t_0) = \lambda_0$$

We noticed that the maintenance policy is tightly related to the system degradation. That is why we adopted the production level in order to take into account the influence of the production rate on the failure rate *λ(t)*. Letting $L(T) = \int_0^T \lambda_i(t)dt$ denotes the expected failure number incurred over the interval [0,T] , the average failure number over the horizon *H* is:

$$\chi(u,T) = \sum_{j=0}^{N-1} \left(\sum_{i=In\left(j \times \frac{T}{\Delta t}\right)+1}^{In\left((j+1)\times\frac{T}{\Delta t}\right)} \int_0^{\Delta t} \lambda_i(t)dt + \int_0^{(j+1)\times T - In\left(\frac{(j+1)\times T}{\Delta t}\right)\times\Delta t} \lambda_{In\left(\frac{(j+1)\times T}{\Delta t}\right)+1}(t)dt + \int_{(j+1)\times T}^{\left(In\left(\frac{(j+1)\times T}{\Delta t}\right)+1\right)\times\Delta t} \frac{u\left(In\left(\frac{(j+1)\times T}{\Delta t}\right)+1\right)}{U_{max}} \times \lambda_0(t)dt \right)$$

with *T*=*H*/*N*

4. Numerical example

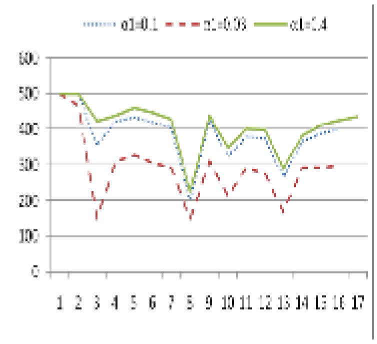
An example of a multi-period, single product, an aggregated production/maintenance planning problem is formulated by our model, which minimizes total costs over a finite planning horizon: *H*=72 months. The information required to run this model is given in sequence.

(i) the monthly mean demand $\hat{d}_k$ given by the sequence

{d_1=**350**, d_2= **420**, d_3= **340**, d_4= **392**, d_5=**431**, d_6=**444**, d_7= **442**, d_8=**340**, d_9=**392**, d_{10}= **375**, d_{11}= **392**, d_{12}= **400**, d_{13}= **350**, d_{14}= **370**, d_{15}= **395**, d_{16}= **415**, d_{17}= **431**, d_{18}= **444**, d_{19}= **442**, d_{20}= **340**, d_{21}= **392**, d_{22}= **375**, d_{23}= **400**, d_{24}= **420**, d_{25}= **350**, d_{26}= **340**, d_{27}= **392**, d_{28}= **370**, d_{29}= **431**, d_{30}= **392**, d_{31}= **500**, d_{32}= **350**, d_{33}= **320**, d_{34}= **420**, d_{35}=**365**, d_{36}= **480**, d_{37}= **300**, d_{38}= **290**, d_{39}= **360**, d_{40}= **370**, d_{41}=460, d_{42}= **460**, d_{43}=358, d_{44}= **400**, d_{45}= **442**, d_{46}= **340**, d_{47}= **392**, d_{48}= **375**, d_{49}=**400**, d_{50}=**431**, d_{51}=**400**, d_{52}= **390**, d_{53}=**350**, d_{54}= **290**, d_{55}=**358**, d_{56}=**360**, d_{57}= **460**, d_{58}= **442**, d_{59}= **392**, d_{60}=**450**, d_{61}=**400**, d_{62}=**450**, d_{63}=**360**, d_{64}=**350**, d_{65}=**300**, d_{66}=**392**, d_{67}=**375**, d_{68}=**400**, d_{69}=**431**, d_{70}=**400**, d_{71}=**390**, d_{72}=**450**}

(ii) lower and upper boundaries of production capacities: u_{min}=150 and u_{max}=500
the following data are used for the other parameters:
C_{pr} =3 *mu*, C_s =2 *mu/k* ,*Cps*=5 mu
(iii) The demand is assumed Gaussian with the standard deviation is σ_d=*4.06*
The customer satisfaction degree, associated with the stock constraint, is equal to 90% (θ=0.9). The rate of return $\alpha_1 \in [0, 0.5]$ and the time of the return of products τ=1. The second part of stock which will be recycled after an inspection, is characterized by α_2 = 0.9. Finally we assume that the subcontracting availability defined by β_s = 0.9.

We suppose that the failure time of machine *M* has a degradation law characterized by a Weibull distribution. The Weibull scale and shape parameters are respectively $\beta=100$ and $\alpha=2$. The cost associated with a corrective and preventive maintenance action are respectively $Mc=1500$ mu $Mp=100$ mu monetary.
Applying the numerical procedure we obtained the optimal production plan and the optimal maintenance period.



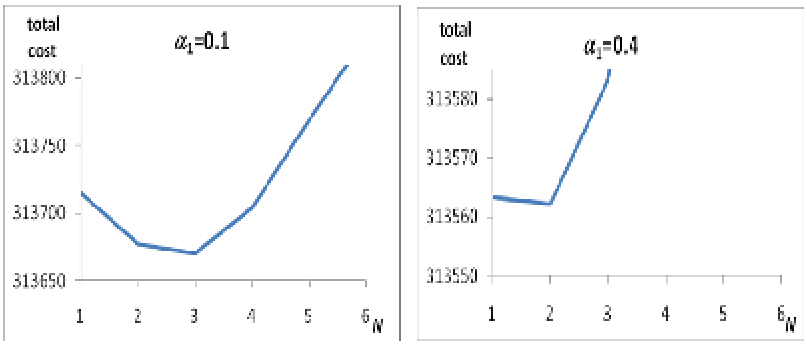


Fig.2. production plan

Fig.3. Curve of average total cost of maintenance according to *N*

According to previous figures, we notice the influence of product returns in terms of production plan and the optimal preventive maintenance period.
Figure 3 shows the production plans of different values of α_1(α_1=0.03, α_1=0.1, α_1=0.4). When α_1 increases, the production plan decreases, this is logical because we have more products returned by the customer to the stock
Figure 3 show the total cost of production, stock and maintenance, $F(U,N)$,, according to *N*. The optimal number of partitions N^* Maintenance is:
For α_1=0.1: N^*=3, F^*=313669, 6*um* and the optimal period of maintenance $T^*=H/N^*=24\Delta t$
For α_1=0.4: N^*=2, F^*=313582,8*um* and the optimal period of maintenance $T^*=H/N^*=36\Delta t$
In the same way as the previous comment about the production plan, the period of maintenance has become increasingly important when $\alpha_1$value increases. This is logical α_1 grows, so that *u* is decreasing, thus the preventive maintenance actions are less.

4. Conclusion

In this work, the result of a general class of stochastic production planning and maintenance scheduling problems via optimal procedure was investigated. The objective is to satisfy economically a random demand under some constraints like random failure rate and a subcontracting constraint. The manufacturing system considered is prone to random failures. Minimal repairs are adapted at every failure. So as to reduce the failure frequency, preventive maintenance actions are programmed according to the production rate.
In order to obtain simultaneously an optimal production and maintenance scheduling, we have formulated and solved a stochastic production/maintenance problem. The optimal production/maintenance plan is obtained with taking into account the influence of the subcontracting on the production plan and on the evolution of manufacturing system failure rate.

References

1. Rezg,N.,Xie,X.,and Mati,Y., 2004 ,"Joint optimization of preventive maintenance and inventory control in a production line using simulation"; International Journal of Production Research,44,2029-2046.
2. Van der Duyn Schouen, F.A., and Vanneste, S.G.,1995,"Maintenance optimization of a production system with buffer capacity",European Journal of Operational Research, 82,323-338.
3. Buzacott, J.A., and Shanthikumar, 1993, "Stochastic Models of manufacturing systems"; Prentice Hall,Englewood Cliffs,NJ.
4. Dallery, Y. and Gershwin, S.B., 1992, "Manufacturing flow line systems: a review of models and analytical results". Queue. Sys. 12, 3–94.
5. S. Dellagi, N. Rezg, X. Xie, 2007, "Preventive Maintenance of Manufacturing Systems Under Environmental Constraints", International Journal of Production Research, vol. 45, Issue 5, p1233-1254.
6. Hajjej, Z., S. Dellagi and N. Rezg. 2009,"An optimal production/maintenance planning under stochastic random demand,service level and failure rate". IEEE International Conference on Automation Science and Engineering, pp. 292 -297, Bangalore.