Proceedings of the 2011 International Conference on Industrial Engineering and Operations Management
Kuala Lumpur, Malaysia, January 22 – 24, 2011

Cyclic Scheduling in a Job shop with Multiple Assembly Firms

Tetsuya Kanja and Koichi Nakade
Department of Industrial Engineering and Management
Nagoya Institute of Technology, Nagoya 466-8555, JAPAN

Abstract

This paper deals with a job shop in a part production system with multiple assembly firms. The part manufacturer produces multiple types of products in a job shop. The cyclic schedule is repeatedly applied. From the inventory of processed parts, assembly factories take multiple types of finished products with their own fixed intervals. This paper first gives an effective numerical procedure which gives optimal schedules and cycle time minimizing the inventory and setup costs of the part factory when the receipt intervals to the assembly factories are given. Then the optimal schedules, cycle time and receipt intervals of assembly firms minimizing the total average cost in a supply chain are derived, which is the sum of inventory and setup costs in the part factory and the transportation costs and inventory costs in the assembly firms. This schedule is compared with the other individual optimal schedule and intervals and its superiority on performance is shown.

Keywords
Job shop, Cyclic scheduling, EOQ, Lot sizing, Supply Chain Optimization.

1. Introduction

Small amounts of production for many kinds of products have come to be requested and its efficient production method has become important. Scheduling and lot sizing problems in a factory are the basis of such optimization. The economic order quantity (Economic Order Quantity: EOQ) is well-known as a method of minimizing the inventory and order costs of the factory. In many cases, the assembly firms decide the interval and the amount of orders to the parts plant based on the economic order quantity. On the other hand, the form of a cyclic scheduling that repeats the production of lots with a constant it is according to the demand for the post-processing is often taken, and the production schedule of the parts plant has received the restriction to the parts plant where parts are supplied by the receipt interval of two or more assembly halls in the downstream.

The cyclic scheduling problem in the job shop environment has been taken up in Ouenniche and Boctor[1] and Matsushita and Nakade[2]. Nakade et al. [3] have treated the case that the final product of each kind is received at a constant interval time. Trabi et al. [4] treated a flow shop problem with cyclic scheduling. Ouyang and Zhu[5] developed an economic lot scheduling in manufacturing/remanufacturing systems. Chatfield [6] considered an economic lot scheduling problem in a single machine by the genetic lot scheduling procedure.

In decades, the supply chain optimization becomes also important, under which total costs in the part production and the assemble factory are minimized. Recently, Clausen and Ju [7] have considered a single machine lot sizing and scheduling problem in a part factory with an assembly factory using the parts, which discusses optimal scheduling minimizing the total long-run average inventory and transportation costs in a supply chain.

In this paper, the job shop is considered in a part production factory with multiple assembly factories, which receive parts. The part manufacturer produces multiple types of products in a job shop. The cyclic schedule is repeatedly applied. From the inventory of processed parts, assembly factories take multiple types of finished products in their own fixed intervals. An effective numerical procedure is developed for deriving optimal schedules and cycle time minimizing the inventory and setup costs of the part factory when the receipt intervals are given. Numerical examples show the performance of the algorithm. The optimal schedules, cycle time and receipt intervals of assembly factories are also derived minimizing the total average cost in a supply chain. This schedule is compared with the other individual optimal schedule and intervals and its superiority on performance is shown.

2. Model

2.1 Flow of Products

The model treated in this paper is illustrated in Fig. 2.1. A Job shop is in a part factory and each type of items are processed in machines with a fixed order. Finished products processed at their machines are put in inventory places for assembly factories. They are received with a fixed time interval o_k by assembly factory k. In the assembly factory items are processed continuously and the inventory decreases in time.

In this study, a cyclic schedule is applied in a finite horizon H. Continuous deterministic demand is assumed to assemble items, and the demand must be satisfied in assembly firms, and the assembler's demand for parts must also be satisfied in the inventory places of the part factory. The cycle time is given by a finite horizon H divided by some positive integer, so the cycle time T is given by $T = H/\xi$ for some positive integer ξ. We also assume that in each cycle each kind of items are produced as a batch at a time.

Assembly factory k produces assembly items, and it consumes part i at a deterministic rate $r_{i,k}$. The part receipt intervals are also assumed by $o_k = H/\xi'_k$ with integer ξ'_k.

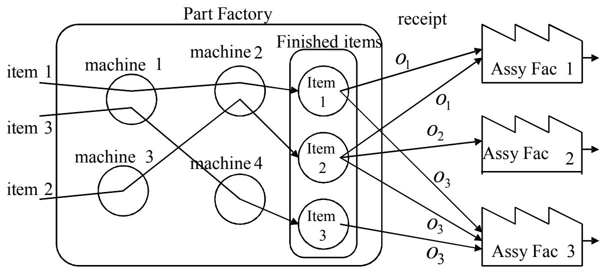

Figure 1: Flow of products

The cost incurred in the part factory consists of a holding cost for work-in-process, an inventory cost for the finished items, a setup cost for the machine. The cost incurred in the assembly factory includes the transportation cost and inventory cost.

3. Formulation

3.1 Notations

Given notations are the followings:

m : the number of machines, $M = \{1,\ldots,m\}$: a set of machines,

n : the number of items, $N = \{1,\ldots,n\}$: a set of kinds of items,

l : the number of assembly factories, $L = \{1,\ldots,l\}$: a set of assembly factories,

m_i : the number of machines in which item i is processed,

n_j : the number of items which are processed at machine j, $P(j)$: a set of items which are processed at machine j,

$\mu(i,k)$: the index of the machine which processes item i in the k th order,

$r_{i,k}$: the demand rate of item i at assembly factory k, $r_i = \sum_{k\in L} r_{i,k}$,

$p_{i,j}$: a production rate of item i at machine j, $s_{i_1,i_2,j}$: a setup time from item i_1 to item i_2 at machine j,

$c_{i_1,i_2,j}$: a setup cost from item i_1 to item i_2 at machine j,

$h_{i,j}$: a holding cost rate of item i at machine j, h_i : an inventory cost rate per unit item of item i,

$v_{i,k}$: a transportation cost of item i to assembly factory k,

$t_{i,j}$: carrying time of item i from machine j to the next machine,

H : the length of finite planning horizon, $\hat{e}_k$: the first receiving epoch of assembly factory k.

Next we give the variable determined by fixing ξ and ξ'_k ($k \in L$).

ξ : the number of production cycles in horizon H,

ξ'_k: the number of receipts by assembly factory k in horizon H,

T : the length of one production cycle time,

o_k : the interval between successive receipts by assembly factory k,

$\pi_{i,j}$: the processing time of item i at machine j in one cycle, given by $\pi_{i,j} = r_i T / p_{i,j}$,

$\alpha_k = \max\{a;\ \text{both } o_k/a \text{ and } T/a \text{ are integers.}\}$,

e_k : the remainder obtained when $\hat{e}_k$ is divided by T.

Next we show the decision variables for a scheduling problem under fixed cycle time and receipt intervals.,

$$b_{i_1,i_2,j} = \begin{cases} 1 & \text{if item } i_2 \text{ is operated after processing item } i_1 \text{ at machine } j \\ 0 & \text{otherwise} \end{cases}$$

$$x_{i_1,i_2,j} = \begin{cases} 1 & \text{if the last item } i_1 \text{ is and the first item is } i_2 \text{ in one cycle at machine } j \\ 0 & \text{otherwise} \end{cases}$$

$d_{i,j}$: the epoch when machine j starts processing of item i,

$\omega_{i,k}$: an integer valuable making the difference between receiving epoch of factory k and arriving epoch at the inventory of finished item i less than or equal to T,

$\tau_{i,k}$: an integer valuable making the difference between receiving epoch of factory k and arriving epoch at the inventory of finished item i less than or equal to α_k.

3.2 Formulation

The model studied in this paper can be formulated into the following program.

Minimize

$$\frac{1}{T}\sum_{j\in M}\sum_{i_1\in P(j)}\sum_{i_2\in P(j),i_1\neq i_2} c_{i_1,i_2,j} b_{i_1,i_2,j} + \sum_{i\in N}\sum_{k=1}^{m_i-1} h_{i,\mu(i,k)} r_i \left(d_{i,\mu(i,k+1)} + \frac{1}{2}\pi_{i,\mu(i,k+1)} - d_{i,\mu(i,k)} - \frac{1}{2}\pi_{i,\mu(i,k)} \right) + \sum_{i\in N} h_i r_i \pi_{i,\mu(i,m_i)}$$
$$+\sum_{k\in L}\sum_{i\in N} h_i r_{i,k} \left\{ \frac{T+o_k}{2} - d_{i,\mu(i,m_i)} - \pi_{i,\mu(i,m_i)} - t_{i,\mu(i,m_i)} + e_k + \omega_{i,k} T - \tau_{i,k}\alpha_k \right\} + \sum_{k\in L}\sum_{i\in N} \frac{v_{i,k}}{o_k} + \sum_{k\in L}\sum_{i\in N} h_i \frac{o_k r_{i,k}}{2}, \tag{1}$$

subject to

$$d_{i,\mu(i,k)} + \pi_{i,\mu(i,k)} + t_{i,\mu(i,k)} \le d_{i,\mu(i,k+1)},\ k = 1,\dots,m_i - 1, i \in N, \tag{2}$$
$$d_{i_1,j} + \pi_{i_1,j} + s_{i_1,i_2,j} - d_{i_2,j} \le M(1 - b_{i_1,i_2,j} + x_{i_1,i_2,j}),\ i_1,i_2 \in P(j), i_1 \neq i_2, j \in M, \tag{3}$$
$$d_{i_1,j} + \pi_{i_1,j} + s_{i_1,i_2,j} - d_{i_2,j} \le T,\ i_1,i_2 \in P(j), i_1 \neq i_2, j \in M, \tag{4}$$
$$0 < d_{i,\mu(i,m_i)} + \pi_{i,\mu(i,m_i)} + t_{i,\mu(i,m_i)} - (e_k + \omega_{i,k}T - \tau_{i,k}\alpha_k) \le \alpha_k, i \in N, k \in L, \tag{5}$$
$$\sum_{i_2\in P(j),i_1\neq i_2} b_{i_1,i_2,j} = 1, i_1 \in P(j),, \quad \sum_{i_1\in P(j),i_1\neq i_2} b_{i_1,i_2,j} = 1, i_2 \in P(j), j \in M, \tag{6}$$
$$\sum_{i_1\in P(j)}\sum_{i_2\in P(j),i_1\neq i_2} x_{i_1,i_2,j} = 1, j \in M, \quad b_{i_1,i_2,j} \ge x_{i_1,i_2,j}, i_1,i_2 \in P(j), i_1 \neq i_2, j \in M, \tag{7}$$
$$b_{i_1,i_2,j} \in \{0,1\}, \quad x_{i_1,i_2,j} \in \{0,1\},\ i_1,i_2 \in P(j), i_1 \neq i_2, j \in M, \tag{8}$$
$$d_{i,\mu(i,k)} \ge 0, i \in N, k = 1,\dots,m_i, \tag{9}$$
$$\omega_{i,k} \in \{0,1,\dots,[\omega_{i,k}^{\max}]\}, \tau_{i,k} \in \{0,1,2,\dots,T/\alpha_k - 1\}, i \in N, k \in L. \tag{10}$$

Objective function gives a cost rate of the supply chain at unit time. The first term of the objective function is the sum of setup costs at all machines. The second is the sum of inventory costs on work-in-processes at all machines. The third term is the total inventory costs of finished products after the final processing to the inventory place. The fourth is the total cost of inventory on finished cost at all inventory places, which has been discussed in [2] and [3]. The fifth is the transportation cost, and the last is the total inventory costs at all recipients..

Constraints show the followings. The constraint (2) is on the transportation epochs between two successive machines, and (3) is on the setup times, where M' is constant and has a big value. Constraint (4) means that processing for one item at each machine is once in each cycle, and (5) means that the difference between the receiving epochs and the arriving epochs at machines is less than α_k, which decides the average cost on the finished products, which are discussed in [2]. Constraints (6) to (10) show the constraints on values that decision variables take.

3.3 Formulation under Fixed Cycle Time and Receipt Intervals

The formulation described in section 3.2 includes the non-linear objective function and constraints. Thus we fix the cycle time T and time interval between successive receiving epoch o_k, that also makes the variables $\pi_{i,j}$ and α_k fixed, and the problem in section 3.2 becomes a mixed integer linear program.

Variable ξ has an upper bound for the problem having solutions. Since the sum of total processing and setup times is less than or equal to the cycle time. Let the order of processing at machine j denoted by σ_j and $\sigma_j(k)$ denote the item which is processed in the k th order at machine j in one cycle. Then ξ must satisfy

$$\xi \le \min_j \frac{\left(1 - \sum_{i \in P(j)} \frac{r_i}{p_{i,j}}\right) H}{\min_{\sigma_j} \left(\sum_{k=1}^{n_j - 1} s_{\sigma_j(k), \sigma_j(k+1), j} + s_{\sigma_j(n_j), \sigma_j(1), j} \right)}. \tag{11}$$

If the cycle time T is fixed, then M in (3) can be replaced by T, and constraint (4) can be removed.

3.4 Economic Order Quantities on the Assembly Firms

When the assembly factories decide the inter-receipt intervals o_k (ξ'_k) in advance, then they decide the values by using EOQ, The optimal inter-receipt interval o_k^* at assembly factory k is given by

$$o_k^* = \sqrt{\sum_{i \in N} h_i r_{i,k} \Big/ \left(2 \sum_{i \in N} v_{i,k}\right)}.$$

Note that this value does not usually take the value obtained by dividing the planning horizon H by some integer, and so the optimal interval in practice is $\lfloor o_k^* \rfloor$ or $\lceil o_k^* \rceil$ taking the smaller average cost of assembly factory k.

4. Numerical Algorithms

Under the time intervals between successive receipts fixed, the algorithm for finding the optimal cycle time and the schedule in the job shop are discussed. Since deriving optimal solutions for all possible cycle times leads to too many computation times, we use the upper and lower bounds on the optimal average costs for each cycle time $T = H/\xi$ for integer ξ, and we solve these bounds efficiently for several necessary ξ, which makes the necessary computation time small. We have tested several types of bounds, and we use the following lower and upper bounds.

As lower bounds

$LP(\xi)$: an objective value for the optimal solution of linear relaxation problem,

$LB_1(\xi)$: an objective value for the optimal solution of the problem with the cost at the inventory of finished items at the receipt space estimated as the smallest value.

As upper bounds

$UB_1(\xi)$: an objective value for the optimal solution of the problem with the cost at the inventory of finished items at the receipt space estimated as the largest value,

$UB_2(\xi)$: the objective value for the optimal solution of the problem with the schedule as the one in optimal solution obtained by deriving $UB_1(\xi)$.

$UB_2(\xi)$ and $LB_1(\xi)$ can be obtained with a few time by computing the solution on $UB_1(\xi)$. We also denote the optimal objective value by $SOL(\xi)$.

The numerical algorithm is given for deriving optimal cycle time and schedule when the time intervals between successive receipts are given.

Step 1: The upper bound on ξ, denoted by $\xi_{\max}$, is computed by (11). Let $X = \{1, \ldots, \xi_{\max}\}$.

Step 2: For all $\xi \in X$, derive $LP(\xi)$ and set $LB(\xi)$ by this value.

Step 3: For $\xi \in X$ which has minimal $LB(\xi)$, compute $UB_1(\xi)$, $UB_2(\xi)$ and $LB_1(\xi)$. Set the smaller upper bound and the greater lower bound as $UB(\xi)$ and $LB(\xi)$, respectively. If there is $\hat{\xi}$ such $UB(\xi) < LB(\hat{\xi})$ then $\hat{\xi}$ is removed from X, and if for some $\tilde{\xi}$ $UB(\tilde{\xi}) < LB(\xi)$, then ξ is excluded from X. Then for $\xi \in X$ which has the second minimal $LB(\xi)$ the bounds are computed in the similar way, and continue the procedure until for all elements ξ in X bounds are computed.

Step 4: For $\xi \in X$ which has minimal $UB(\xi)$ derive $SOL(\xi)$ by the original problem with the constraint that objective function is greater than or equal to $LB(\xi)$ and less than or equal to $UB(\xi)$. If $SOL(\xi) < LB(\hat{\xi})$ for some $\hat{\xi}$, then $\hat{\xi}$ is excluded from X. Then for $\xi \in X$ which has second minimal $UB(\xi)$, $SOL(\xi)$ is computed in the similar way, and continue the procedure until $SOL(\xi)$ are computed .for all elements ξ in X

Step 5: Output $SOL(\xi)$ which attains minimum among $\xi \in X$.

5. Numerical Experiments

5.1 Computation Times of the Algorithm

The numerical experiments are conducted in the case where a job shop has 5 machines and 5 to 9 items. The time interval between successive receipts are set by (11) in section 3.4. In Table 1, the results of the numbers of solving problems and computation times are shown. In this example, 20 experiments have been conducted for each number of items on PC with Core2 Quad CPU (2.83GHz), and the averages of their results are given. For individual combinatorial problems appearing in the algorithm software Xpress 7 is used. From this table, the numbers on deriving bounds on the objective functions or $SOL(\xi)$ are much smaller than the number of deriving $LP(\xi)$. The proposed method in section 4 decreases computation times for deriving optimal cycle times and job shop schedules.

Table 1: Computation times

# of items	$LP(\xi)$		$UB_1(\xi)$, $UB_2(\xi)$, $LB_1(\xi)$		$SOL(\xi)$		Total Computation Times
	# of solving	Computation Times	# of deriving	Computation times	# of solving	Computation times	
5	73.95	0.35	2.05	0.22	1.70	0.47	1.07
6	63.80	0.36	2.60	0.50	1.70	0.90	1.76
7	54.10	0.37	2.60	1.01	2.20	3.96	5.35
8	41.35	0.35	2.45	14.46	2.05	205.17	219.98
9	31.50	0.33	3.60	936.06	2.70	2759.51	3695.90

5.2 Comparison on Receipt Intervals

Optimal schedules and cycle time in a job shop are calculated for several combinations on the time intervals between successive receipts (by changing ξ'_k). Here it is assumed that H=240, and ξ'_k=10, 20, 30 or 40. This means that the planning horizon is 240 hours (10 days) and the numbers of receipts are once, twice, three times or four times in one day. The results are given in the case where there are 5 machines, 5 items and 3 assembly factories. The same performance is also evaluated in the case that their intervals are computed by EOQ (in this case, the corresponding ξ'_k is (27, 22, 17)).

From the table, even among the optimal intervals restricted to combinations of 4 cases (ξ'_k=10, 20, 30 or 40), optimal intervals (20, 20, 20) give minimal total cost in a supply chain. Note that in the case (30, 20, 20), which is the combination of the nearest intervals to the EOQ intervals (27, 22, 17), the total cost in assembly factories is smaller than that under (20, 20, 20), but the total cost in the part factory is much greater.

The combination (10, 10, 10) minimizes the total cost in a part factory, but in this case the total cost in assembly factories is much greater. Thus, by setting appropriately the receipt intervals to the assembly factories and cycle time in a part factory, the total cost in a supply chain is minimized.

Table 2: Comparison among multiple combinations on receipt intervals

ξ'_k	Total costs in assembly factories	Optimal ξ	Total cost in the part factories	Total costs
(27,22,17)	732.8	17	1379.2	2112.0
(10,10,10)	957.0	10	787.4	1744.4
(20,20,20)	747.6	20	918.4	1666.0
(30,20,20)	742.7	20	1072.8	1815.5

6. Conclusion

In this study a production and inventory system consisting of a job shop in a part factory and multiple assembly factories are developed. The cyclic schedule is repeatedly applied. From the inventory of processed parts, assemblers take multiple types of finished products to their own factories in their own fixed intervals. An effective procedure is developed for deriving optimal schedules and cycle time minimizing the inventory and setup costs of the part factory when the receipt intervals are given. Then the optimal schedules, cycle time and receipt intervals of assembly factories are derived minimizing the total average cost in a supply chain. Numerical examples show the effectiveness of the procedure and the superiority of derived supply-chain optimal schedule, cycle time and receipt intervals to the individual optimal schedules. A numerical algorithm for deriving near-optimal schedules and cycle times in practical scale problems should be developed and it is left for future research.

Acknowledgement

This research is supported by JSPS Grand-in-Aid for Scientific Research (C) 22510145.

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