Developing a new variables sampling plan for products with multiple quality characteristics based on process yield

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Abstract

In a real world, according to the tendency of today's modern quality improvement, the manufactured products involve more than one quality characteristics is quite common. The conventional process capability analysis and sampling technique may be inappropriately used to judge the performance of the inspected items. Therefore, in this paper, a new variables sampling plan is developed for lot sentencing when the product is inspected with multiple quality characteristics (MQCs). The proposed method is developed based on the advanced process yield index S_{pk}^{T}, which can exactly reflect the overall process yield rather than considering each quality characteristic individually and the plan parameters (n, c_0) are solved by the classical two-point condition (AQL and LQL) on the operating characteristic (OC) curve. Table of plan parameters is constructed based on selected quality requirements and risks, hence the users can easily determine the required sample size and corresponding critical value for lot sentencing. In addition, a numerical example is presented to demonstrate the proposed approach for its applicability.

Keywords

Multiple quality characteristics, acceptance sampling plans, process yield index, lot sentencing.

1. Introduction

Acceptance sampling, a typical and practical tool in the field of statistical quality control and quality assurance. Several sampling plans have been developed for making a decision whether the submitted lot should be accepted or rejected based on the collected sample information. Two kinds of acceptance sampling plan are classified by data type, attributes and variables. However, as a rapid advancement of manufacturing technology, many industries require their products to be of high quality with a very low number of nonconformities (measured in the units of parts per million), called NCPPM. It seems that the attributes sampling plan for calculating the NCPPM no longer work since any sample of reasonable size will probably contain no defectives. Hence, some investigations (see Hamaker (1979), Duncan (1986) and Montgomery (2009)) have shown that the variables inspection can conquer this deficiency of sampling. In the past decade, several variables sampling plans have been developed based on different sampling strategies and perspectives, including Balamurali *et al*. (2005), Pearn and Wu (2007), Senthilkumar and Muthuraj (2010), Balamurali and Usha (2012), Wu and Liu (2014) and Liu *et al*. (2014), just to name a few.

However, many existing variables sampling plans are only focused on single quality characteristic. Nowadays, many electronic manufacturing industries have been transferred its character, OEM (original equipment manufactures) towards to ODM (original design manufactures), which means that the produced items should be completely satisfied by consumers' expectation, are nothing but high quality and functionality of products. In general, these specialized products involve more than one quality characteristic. Hence, the variables sampling plans for single quality characteristic may be inappropriately used for this situation. Therefore, in this paper, we propose a new variables sampling plan for multiple quality characteristics based on the advanced process yield index S_{pk}^{T}, which can exactly reflect the overall process yield. By implementing the proposed method, we may avoid the overestimation of overall process yield and consequently make a more accurate and reliable lot determination.

2. Process yield and multiple quality characteristics

The process yield index S_{pk} was firstly proposed by Boyles (1994) to specially reflect the exact process yield (a one-to-one relationship between index value and process yield or NCPPM) that can be used as an important criterion for judging the process performance. The process yield index can be calculated by the Eq. (1) as below. Moreover, the S_{pk} can be rewritten and expressed as a function of two basic process capability indices $C_p = (USL - LSL)/6\sigma$ (for reflecting the process precision only) and $C_a = 1 - |\mu - M|/d$ (for measuring the process centering only). Furthermore, if $S_{pk} = S_0$, then the process yield can be calculated by $yield = 2\Phi(3S_0) - 1$ and the NCPPM can be obtained by $\text{NCPPM} = 2[1 - \Phi(3S_0)] \times 10^6$.

$$S_{pk} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(\frac{USL-\mu}{\sigma}\right) + \frac{1}{2}\Phi\left(\frac{\mu - LSL}{\sigma}\right)\right] = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(3C_pC_a\right) + \frac{1}{2}\Phi\left(3C_p(2-C_a)\right)\right] \tag{1}$$

where *USL* and *LSL* are the upper and lower specification limits, respectively, $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution and $\Phi^{-1}(\cdot)$ denotes the inverse function of $\Phi(\cdot)$. In practice, we may use the sample mean $\bar{x} = \sum_{i=1}^{n} x_i / n$ and sample standard deviation $S = \left[\sum_{i=1}^{n}(x_i - \bar{x})^2 / (n-1)\right]^{1/2}$ to be the point estimators for the population mean μ and σ, respectively. Then, the natural estimator $\hat{S}_{pk}$ can be defined as:

$$S_{pk} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(\frac{USL-\bar{x}}{S}\right) + \frac{1}{2}\Phi\left(\frac{\bar{x} - LSL}{S}\right)\right] = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(3\hat{C}_p\hat{C}_a\right) + \frac{1}{2}\Phi\left(3\hat{C}_p(2-\hat{C}_a)\right)\right] \tag{2}$$

where $\hat{C}_p$ is the estimator of C_p and $\hat{C}_a$ stands for a estimator of C_a .By taking the first-order of Taylor expansion, Lee *et al.* (2002) derived a normal approximation to the distribution of $\hat{S}_{pk}$ and can be expressed as :

$$\hat{S}_{pk} \approx S_{pk} + \frac{1}{6\sqrt{n}}\left[\phi(3S_{pk})^{-1}\right]W + O_p(n^{-1}) \tag{3}$$

Where $O_p(n^{-1})$ is the error of expansion having a leading term of order n^{-1} in probability and

$$W = \begin{cases} \sqrt{\frac{n}{2}}\left[\frac{a(S^2-\sigma^2)}{\sigma}\right] - \sqrt{n}\frac{b(\bar{x}-\mu)}{\sigma} & \text{for} \quad \mu < M \\ \sqrt{\frac{n}{2}}\left[\frac{a(S^2-\sigma^2)}{\sigma}\right] + \sqrt{n}\frac{b(\bar{x}-\mu)}{\sigma} & \text{for} \quad \mu > M \end{cases} \tag{4}$$

where *a* and *b* are function of μ and σ (or C_p and C_a) and can be obtained by:

$$\begin{aligned} a &= \frac{1}{\sqrt{2}}\left\{\frac{USL-\mu}{\sigma}\phi\left(\frac{USL-\mu}{\sigma}\right) + \frac{\mu-LSL}{\sigma}\phi\left(\frac{\mu-LSL}{\sigma}\right)\right\} \\ &= \frac{1}{\sqrt{2}}\left\{3C_p\left(2-C_a\right)\phi\left(3C_p\left(2-C_a\right)\right) + 3C_pC_a\phi\left(3C_pC_a\right)\right\} \end{aligned} \tag{5}$$

$$b=\phi\left(\frac{USL-\mu}{\sigma}\right)-\phi\left(\frac{\mu-LSL}{\sigma}\right)=\phi\left\{3C_p\left(2-C_a\right)\right\}-\phi\left(3C_pC_a\right) \tag{6}$$

where ϕ is the probability density function of standard normal distribution. The statistic W follows a normal distribution $N(0, a^2+b^2)$, hence the estimator $\hat{S}_{pk}$ is approximately distributed as:

$$\hat{S}_{pk} \xrightarrow{a} N\left(S_{pk}, \frac{a^2+b^2}{36n\left(\phi(3S_{pk})^2\right)}\right) \tag{7}$$

However, as aforesaid, a manufactured product is not merely inspected by a single quality characteristic, but also being required to be examined with multiple characteristics (e.g. length, width, thickness or weight etc.) for adequate description of product's quality. For the process with multiple quality characteristics, Chen *et al*. (2003) proposed the advanced process yield index S_{pk}^T, which is defined as following:

$$S_{pk}^T=\frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\left[\prod_{i=1}^{v}\left(2\Phi\left(3S_{pki}\right)-1\right)+1\right]\right\} \tag{8}$$

where S_{pkj} denotes the S_{pk} value of the *j*th quality characteristics for *j*=1,2,…,*v*, and *v* is the number of quality characteristics. It is easily to see that S_{pk}^T is a generalization of an individual process yield index S_{pk}. The index S_{pk}^T provides an exact measure of the overall process yield of the inspected item or process itself. Before we go further, we should mention that the quality characteristics are mutually independent and normally distributed. As the convention S_{pk}, S_{pk}^T can also provide a one-to-one relationship between index value and process yield or NCPPM by Eq. (9) as below and we can see some commonly required quality levels in Table 1.

$$\text{Process yield}=\prod_{j=1}^{v}P_j=\prod_{j=1}^{v}\left[2\Phi(3S_{pkj})-1\right]=2\Phi(3S_{pk}^T)-1 \tag{9}$$

Table 1. The corresponding process yield and NCPPM based on given S_{pk}^T value

S_{pk}^T	Process yield	NCPPM
1.00	0.997300204	2699.796
1.10	0.999033152	966.848
1.20	0.999681783	318.217
1.40	0.999973309	26.691
1.60	0.999998413	1.587
1.80	0.999999933	0.067
2.00	0.999999998	0.002

Pearn *et al*. (2006) derived the asymptotic distribution for the natural estimator $\hat{S}_{pk}^T$ and can be defined as:

$$\hat{S}_{pk}^T=\frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\left[\prod_{i=1}^{v}\left(2\Phi\left(3\hat{S}_{pki}\right)-1\right)+1\right]\right\} \tag{10}$$

where $\hat{S}_{pkj}$ is the estimator of S_{pkj} and we need to emphasize that each $\hat{S}_{pkj}$ are all mutually independent. Therefore, the sampling distribution of $\hat{S}_{pk}^T$ can be approximated to an asymptotic normal distribution as:

$$\hat{S}_{pk}^{T} \xrightarrow{a} N\left(S_{pk}^{T}, \frac{1}{36n\phi^{2}(3S_{pk}^{T})} \sum_{j=1}^{v} \left\{ \left(a_j^2 + b_j^2 \left[\frac{\prod_{i=1}^{v} \left(2\Phi\left(3S_{pki}\right)-1\right)^2}{\left(2\Phi\left(3S_{pkj}\right)-1\right)^2} \right] \right) \right\} \right) \tag{11}$$

where a_j and b_j are the function of μ_j and σ_j (or C_{pj} and C_{aj}) of jth quality characteristic which can be shown as Eq.(12) and (13), respectively.

$$\begin{aligned} a_j &= \frac{1}{\sqrt{2}} \left\{ \frac{USL-\mu_j}{\sigma_j} \phi\left(\frac{USL-\mu_j}{\sigma_j} \right) + \frac{\mu_j - LSL}{\sigma_j} \phi\left(\frac{\mu_j - LSL}{\sigma_j} \right) \right\} \\ &= \frac{1}{\sqrt{2}} \left\{ 3C_{pj}\left(2-C_{aj}\right)\phi\left(3C_{pj}\left(2-C_{aj}\right)\right) + 3C_{pj}C_{aj}\phi\left(3C_{pj}C_{aj}\right) \right\} \end{aligned} \tag{12}$$

$$b_j = \phi\left(\frac{USL-\mu_j}{\sigma_j} \right) - \phi\left(\frac{\mu_j - LSL}{\sigma_j} \right) = \phi\left(\frac{d-\left(\mu_j - M\right)}{\sigma_j} \right) - \phi\left(\frac{d+\left(\mu_j - M\right)}{\sigma_j} \right) \tag{13}$$

By the investigation of variance for multiple dimensional processes, Pearn and Cheng (2010) derived a shorter and simpler form of Eq. (11) which can be shown as Eq. (14).

$$\hat{S}_{pk}^{T} \approx N\left(S_{pk}^{T}, \frac{\left(S_{pk}^{T}\right)^2}{2n} \right) \tag{14}$$

3. The proposed variables MQCs sampling plan

Suppose that there are multiple quality characteristics of interest follow a normal distribution and each characteristics have two-sided specification limits (*USL* and *LSL*). The probability of acceptance $P_a(p)$ when the lot fraction of defectives equals to p PPM can be expressed as:

$$P_a(p) = P_a(\hat{S}_{pk}^{T} > c_0 \,|\, \text{submit quality} = p \text{ PPM}) \tag{15}$$

Figure 1 displays the OC curve of the proposed MQCs sampling plan. We can observe that the slope of the OC curve becomes steeper since the number of sample size getting larger. This phenomenon intuitively explains that the larger sample size is the more sample information can be acquired to reflect the actual performance of product or process.

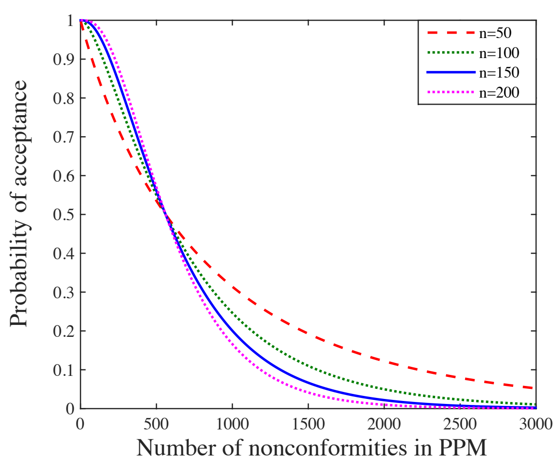

Figure 1. OC curves of the proposed method under $n = 50(50)200$ and $c_0 = 1.15$.

In order to develop a variables MQCs sampling plan for lot sentencing, a well-known method, called "*two-points condition method*" is implemented which requires the OC curve to pass through two designated points, AQL (acceptable quality level) and LQL (limiting quality level). A well-designed sampling plan must provide a guarantee that the probability of acceptance should be at least $1-\alpha$ if the submit quality is at p_{AQL}, on the other hand, in the meantime, the sampling plan must also provide a probability of acceptance no more than β if the quality of submission is at p_{LQL}. That is,

$$P_a(p_{\text{AQL}}) = P_a(\hat{S}_{pk}^T > c_0 \,|\, \text{submit quality} = p_{\text{AQL}}) \geq 1-\alpha \tag{16}$$

$$P_a(p_{\text{LQL}}) = P_a(\hat{S}_{pk}^T > c_0 \,|\, \text{submit quality} = p_{\text{LQL}}) \leq \beta \tag{17}$$

where α represents the producer's risk and β is the consumer's risk. Consequently, the plan parameters *n* and c_0 can be obtained by solving the simultaneous solution of Eq. (16) and Eq. (17).

4. The solved plan parameters and a numerical example

4.1 The solved plan parameters

Table 2 shows the solve plan parameters (n, c_0) based on selected quality requirements and risks. The user can easily determine the required sample size and the corresponding critical value for acceptance. For instance, form the business contract, the quality requirements and tolerable risks are set to $(p_{\text{AQL}} = 100 \text{ PPM}, 1-\alpha = 0.95)$ and $(p_{\text{AQL}} = 1000 \text{ PPM}, \beta\text{=}0.10)$. Then, the needed sample size for inspection and the corresponding critical vale can be found in Table 3 as $(n = 157, c_0 = 1.1763)$. This implies that the lot will be accepted if $\hat{S}_{pk}^T$ value, which calculated by the 157 samples information randomly taken form the entire submission, is larger than 1.1763, otherwise the lot will be rejected.

Table 2. The solved plan parameters based on given quality requirements.

p_{AQL}	p_{AQL}	$(\alpha = 0.01, \beta = 0.05)$		$(\alpha = 0.05, \beta = 0.10)$	
		n	c_0	n	c_0
1	100	158	1.4170	85	1.4245
	200	112	1.3763	60	1.3851
	500	74	1.3177	40	1.3280
100	500	650	1.2132	352	1.2164
	1000	291	1.1717	157	1.1763
500	2000	570	1.0803	308	1.0833
	3000	321	1.0536	173	1.0575
1000	3000	755	1.0311	408	1.0337

4.2 A numerical example

Suppose that the engineering attempts to decide that whether or not the submission should be accepted or rejected by the given quality requirements: $p_{\text{AQL}} = 100 \text{ PPM}$, $p_{\text{LQL}} = 1000 \text{ PPM}$, $\alpha = 0.05$ and $\beta = 0.10$. From Table 3, we know that the required sample size for inspection and the corresponding critical value for acceptance are $(n = 157, c_0 = 1.1763)$. The inspection includes the length (*USL*=23, *LSL*=21), width (*USL*=16, *LSL*=15) and thickness (*USL*=3.5, *LSL*=3.2) of a particular electronic device. Table 4 shows the sample mean, sample standard deviation and the corresponding $\hat{S}_{pk}$ value for each quality characteristic based on the collected data. Then, we can obtain $\hat{S}_{pk}^T = 0.9923$ which is smaller than the critical value for acceptance. Accordingly, the submission is rejected based on the proposed method.

From the above numerical example, we can observe that the overall process yield is 99.7089% which is smaller than the process yields individually calculated of each quality characteristics. Hence, the proposed variables MQCs sampling plan can provide an overall consideration for lot sentencing that the overestimation of overall process yield may be avoided and subsequently make a more accurate and reliable decision.

Table 4. The sample mean, sample standard deviation and the corresponding $\hat{S}_{pk}$ value of each characteristics.

Quality characteristics	Sample mean	Sample standard deviation	$\hat{S}_{pk}$ value	Process yield
Length	$\bar{x}_L = 22.4550$	$S_L = 0.1523$	1.2519	99.9827%
Width	$\bar{x}_W = 15.3325$	$S_W = 0.1183$	1.0089	99.7527%
Thickness	$\bar{x}_T = 3.4711$	$S_T = 0.0372$	1.2151	99.9732%

5. Conclusion

Owing to new technological advances today, the manufactured product involves more than one quality characteristics is quite common. The conventional process capability analysis may be inadequately applied to measure the quality of the items inspected with multiple quality characteristics. The advanced process yield index S_{pk}^T was designed to calculate the overall process yield of products (or the production line) which can avoid the overestimation of true yield. Hence, in this article, a variables MQCs sampling plan is constructed based on S_{pk}^T for the lot consideration which the inspected item is examined with multiple quality characteristics. A numerical example in session four simply shows the suitability of the proposed method when the case is occurred. Further, the framework of the proposed method is expected to extensively apply to different sampling strategies based on the business perspectives which can make a more accurate and reliable on lot determination when the submission is required to be inspected with multiple quality characteristics.

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Biography

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